The Black Hole Binary GRS1915+105 and its Plateau State: is it Canonical?

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Abstract

This is the report for my bachelor project on the black hole binary GRS1915+105. Black hole binaries have been observed from the end of the 1960’s and soon after astronomers found that most of them exhibit similar states, now known as the “canonical” black hole states. Black holes adhering to these states are also referred to as “canonical”. GRS1915+105 is a very peculiar black hole binary, as it presents us with a state not ever seen before in any of the (limited number of) canonical black holes. This state now grows in familiarity as the plateau state. For this project I tried to ascertain how the plateau state relates to the canonical states.

I approached this problem from a multitude of perspectives: through the study of literature and articles, by coding a relatively simple jet model in the C programming language and by using a far more complex code designed by Dr. Sera Markoff in conjunction with the Interactive Spectral Interpretation System (ISIS) to fit real astronomical data from GRS1915+105. This combined X-ray, infrared and radio data was gathered on the 8th of July 1999 by the PCA and HXTE programs on the RXTE satellite, UKIRT and the GBI respectively, when GRS1915+105 is believed to have been in the aforementioned state. The results nominate the canonical Low/Hard state as the most promising candidate for comparison with the plateau state, due to the suspected presence of quasi-steady jets, the relative hardness of the received spectrum and the spectral index obtained from fitting the data.
8 Acknowledgments

A Synchrotron Emulation

B Fitting Results
   B.1 Starting Values for Final Fits
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In contrast to the beliefs of many theoreticians, black holes are not merely static abstract entities only to be described with a mathematical toolbox, but they are real, measurable physical phenomena. Most people have heard of black holes and know that they are objects so dense that not even light can escape from them. This suggests that nothing can be learned from a black hole, since information from cosmological objects travels to us in the form of photons, and photons within the grasp of a black hole could never reach us. While this might be true for information from beyond the event horizon, this certainly does not apply to its surroundings. So we are quite lucky that nature provides us with a means to indirectly study the black hole, to wit, in the form of a binary. The variant found in binary systems is not hidden from sight by the most extreme amounts of gravity known to man. In contrary. The donor star that has fallen prey to the enormous gravity field marks its demise with an extremely energetic and well-measurable death cry, as matter from this companion streams towards the attractor and lights up the binary through the process of accretion.

A particle of mass $m$ in a gravitational field can release a potential energy $\Delta E = \frac{GMm}{R}$ when falling towards an object of mass $M$ and radius $R$. The ratio $M/R$ defines the compactness of an object and determines the amount of energy that can be liberated. A black hole can convert a maximum amount of potential energy equal to 42% of a particle's rest energy (see Table 1.1). Comparing to, for example, fusion of hydrogen to helium, where $\Delta E = 0.007mc^2$, it is clear that for neutron stars or black holes, accretion of matter is a far more efficient process.

Not only the energies involved but also the time scale on which a black hole changes make them very interesting objects to observe. Black hole binaries (BHBs) tend to change state (see Section 1.4.1) on cosmologically negligible intervals of several years to several weeks or...
Introduction

\[
\Delta E = \frac{GMm}{R^2} \quad \text{(in } mc^2\text{)}
\]

<table>
<thead>
<tr>
<th>White Dwarf</th>
<th>Neutron Star</th>
<th>Black Hole</th>
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<tr>
<td>10^4</td>
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<td>10^{-4}</td>
<td>0.15</td>
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Table 1.1: Typical release of energy for different compact objects of 1 M⊙. [3]

...even hours. So it is safe to say that compared to the most astronomical phenomena, the observation of a BHB is an extremely action packed pass-time\(^1\) for an astronomer. However, since Earth’s atmosphere is very effective in absorbing X and \(\gamma\) radiation, equipment had to be brought into orbit first. So while the theoretical foundations describing the formation of a black hole were already laid in 1939 by Oppenheimer & Snyder [5], it was not yet possible to accumulate evidence until techniques for extraterrestrial X-ray observations were developed - in the early sixties. Only in 1972 strong evidence confirming the possible existence of BHBs was first gathered, with optical and X-ray observations of Cygnus X-1. The detection of the periodic modulations in the optical spectrum were needed in conjunction with X-ray data to confirm the fact that the object was indeed a binary, with an (optically) invisible massive object determining the behavior of that binary. At this moment, a total of \(\sim 20\) such systems are known, containing a compact object of more than 3 M⊙. According to current theories, these objects are too massive to be a degenerate star and should thus be black holes. [5] [3]

1.1 Why Black Holes Are Important

Stellar mass black holes are important to astronomy in numerous ways [5]:

- In terms of the amount of energy and physics involved, black holes and BHBs are some off the most extreme objects known to man. Hence they are important tools for testing predictions of fundamental theories on the borderline of their validity; in particular General Relativity.

- On a more practical level, black holes are an endpoint of massive stellar evolution. The collapse of their progenitor stars enriches the universe with heavy elements.

- The mass distribution of even the limited number of currently known BHBs are used to constrain the models of black hole formation and binary evolution.

- BHBs appear to be connected to hypernovae, believed to power gamma-ray bursts.

1.2 Black Hole Binary Accretion

The main method of accretion in a BHB is matter-transfer from the donor star to the compact black hole. Of course accretion from the Inter Stellar Medium (ISM) is also possible, but usually there is not enough of this kind of matter present (anymore) to evoke a significant amount of radiation. Hence I will neglect this type of accretion for this paper.

\(^1\)Probably the only other astrophysical systems comparable in terms of dynamics, are solar flares. They change on time scales of \(\sim 15\) minutes.
When two objects revolve around another, the transferred matter will have a significant amount of angular momentum at its disposition. This angular momentum will prevent the matter from falling directly towards the black hole. Instead, the matter stream will start to revolve around its new attractor in circular orbits, forming a ring. In this ring, dissipative processes, like collisions of gas elements, shocks, viscous dissipation, etc., allow the gas to convert some of its kinetic energy into heat, some of which is radiated. The loss of energy forces the gas into lower circular orbits, spiralling inwards through a series of approximately circular orbits. The slow decrease in distance from the center of mass is only possible by losing angular momentum. Since there are no external torques, angular momentum can only be transferred outwards by internal torques. Because of this transfer, the outer rings gain in angular momentum and spiral outwards, and the inner material is lowered in the black hole’s gravitational well, creating an accretion disk. The accretion disk is instrumental in removing angular momentum from the transferred matter, in the end allowing it to fall into the event horizon [2]. In removing angular momentum from an accretion disk, jets (see section 1.3) are thought to play an important role [3].

Indeed, what we measure on earth from these enormously energetic processes is a flux in a certain part of the electromagnetic spectrum (see section 1.2.2). The luminosity from accretion is determined by the rate at which potential energy is liberated

\[ L_{\text{acc}} = \frac{G M \dot{m}}{R}, \]  

(1.1)

where \( \dot{m} \) is the accretion rate, \( \Delta m/\Delta t \). However, this equation assumes that energy is not only released when matter falls towards the object, but also when it hits the surface. Since a black hole has no surface, matter can disappear before it has radiated all its energy. So for a black hole, a correction is necessary

\[ L_{\text{acc,bh}} = \epsilon L_{\text{acc}} = \epsilon \frac{G M \dot{m}}{R_{\text{bh}}} = 2\eta \frac{G M \dot{m}}{R_{\text{bh}}} = \epsilon \dot{m} c^2, \]  

(1.2)

since a black hole’s radius is defined by its Schwartzschild radius: \( R_{\text{bh}} \equiv 2GM/c^2 \). Here \( \epsilon \) and \( \eta \) are two constants describing the efficiency. For a black hole \( \eta \sim 0.1 \). [3]

1.2.1 Eddington Limit

From equation 1.1 it is clear that the amount of energy released increases with the accretion rate. The Eddington limit sets a maximum for this rate, caused by the radiation pressure of the photons created by the accretion process itself.

When a plasma of ionised hydrogen is accreted, the electrostatic force keeps the plasma neutral by forming electron-proton pairs. Since a proton is \( \sim 1000 \) times more massive than an electron, the pairs behavior due to gravity can be largely accounted for by the proton alone. So neglecting the electron mass and assuming the accretion is spherical, the central force is given by

\[ F_G = \frac{G M m_p}{r^2}. \]  

(1.3)

However, the released photons provide an anti-central force, mostly affecting the electrons. This pressure can be written as a force

\[ F_\gamma = \frac{\sigma_T L}{4\pi r^2 c}. \]  

(1.4)
where \( L \) is a luminosity and \( \sigma_T = 6.7 \times 10^{-25} \text{cm}^{-2} \) is the Thomson cross section; the cross section for photon-electron interactions.

The Eddington limit is defined by equilibrium of \( F_G \) and \( F_\gamma \)

\[
\frac{GMm}{r^2} = \frac{\sigma_T L_{Edd}}{4\pi r^2 c},
\]

yielding for the Eddington luminosity

\[
L_{Edd} = \frac{4\pi GMm_p c}{\sigma_T} \sim 10^{38} \left( \frac{M}{M_\odot} \right) \text{erg/s.}
\]

This result is only a very crude approximation, since it is true only for spherical accretion, while, as explained above, generally the accretion takes place in the form of an accretion disk.

1.2.2 Roche-Lobe Overflow

Matter transfer from a secondary to a primary object can happen in two ways: via accretion of the stellar wind of the companion or through Roche-lobe overflow. The latter allows for the highest accretion rates and was first modelled by Eduoard Roche, in the 19th century.

When a test particle is caught in a gravitational well of a binary system there are three possibilities:

- At a substantial distance from the two components the gravitation felt by the particle will appear to originate from one large mass of mass \( M_1 + M_2 \). The particle will then move in circular orbits around the binary.

- If the particle is closer to one of the objects composing the binary, it will notice little from the other object and move in circular orbits around the first.

- When a particle is at the point in space where the gravity of the two components balances out, the movement of the particle is not pre-determined and can thus be dubbed unstable. This point is known as the inner Lagrange point, usually referred to as “L1”. The distance of this point to each of the objects is determined by the mass ratio \( M_1/M_2 \). Surrounding each component one can imagine an equipotential surface through L1. The volume occupied by this surface is called the objects Roche lobe.

If one of the components in a binary has expanded so much that it has outgrown its Roche-lobe, matter is transferred from one object to the other. This phenomenon is called Roche-lobe overflow. The L1 point then acts as a ”funnel point”, allowing matter to cross from the gravitational pull of the donor to the secondary. The angular momentum of the accreting material will tend to form a differentially rotating disk around the compact object. The material in the accretion disk slowly spirals deeper into the intense gravitational well of the black hole, heating up along the way. To estimate the temperature involved let’s assume

**Figure 1.2:** Artists impression of Roche-lobes, with the Lagrange points.
the energy released through accretion is emitted as black-body radiation. Stefan-Boltzmann’s law then says

\[ T = \left( \frac{L_{\text{acc}}}{4\pi R^2 \sigma} \right)^{1/4}. \]  

(1.7)

Substituting some values expected to correspond to GRS1915+105 (see section 3.1) if it was accreting matter at the Eddington limit (i.e. \( R \sim 40 \) km for a 14 \( M_\odot \) black hole and \( L_{\text{Edd}} \sim 1.8 \times 10^{39} \) [22]) this equation yields a temperature \( T_{\text{Edd}} = 1.48 \times 10^7 \) K. Wien’s displacement law \( (\nu_{\text{max}} = cT/b) \) with \( b = 2.89776853(51) \times 10^{-3} \) mK reveals this temperature corresponds to a black body with a peak frequency of \( \sim 10^{18} \) Hz\( \sim 4 \) keV. So these sources will shine brightly in the X-ray regime. From equation 1.7 it is clear that the temperature only depends weakly on the accretion luminosity, so the source in question doesn’t have to accrete at rates close to the Eddington limit at all, to reach these temperatures. [3] [10]

### 1.3 Bipolar Plasma Outflows: Jets

When Curtis described the optical appearance of M87 in 1918, he wrote about ”a curious straight ray..... apparently connected with the nucleus”. In the 1970’s scientists (e.g. Rees, Scheuer, Blandford) started reflecting on this phenomenon as being connected to a flow of material. The first jet from an X-ray binary was located near SS 433 at the end of that decade. [15] [3]

The most important radiation processes that allow us to observe jets are synchrotron radiation and inverse compton scattering. The former is discussed in section 4.1. The latter was beyond the scope of this project.

Currently a lot of research into jets is being done from the viewpoint that jets emanating from X-ray binaries physically resemble their counterparts found with Active Galactic Nuclei (AGN), or quasars (see figure 1.4). Mirabel and Rodríguez were the first to suggest this analogy. They coined the term micro-quasar, to emphasize the fact that the analogy with quasars is more than just morphological. They expect an underlying unity and scaleability in the physics from stellar mass black holes and the supermassive black holes powering distant quasars. So in spite of the fact that jets found with quasars are multiple (\( \sim 6 \)) magnitudes larger, the physical properties, like e.g. their electron distribution and the way they propagate might be comparable (also see figure 1.4). [13]

Still little is known about how jets are exactly formed. Our best hopes of topping this problem are Magneto-Hydro-Dynamic (MHD) simulations. However, despite the fact that
Figure 1.4: Micro-quasars and quasars powering AGN are thought to have many comparable and hence scalable physical properties. Both phenomena incorporate a spinning black hole, an accretion disk heated by viscous dissipation and collimated jets of relativistic particles. The size of the accretion disk, the jets, and the mass of a microquasar are however only a millionth of that deemed normal for their extragalactic counterpart: $\sim 1000$ km and a few light years and solar masses respectively. Also the characteristic time-scales involved with the flow of matter onto the black hole are proportional to its mass. Microquasars can show variations within minutes, whereas analogous phenomena would persist for thousands of years in a $10^9 M_\odot$ quasar. This allows for the study of events in microquasars that simply have not occurred in quasars, since we started observing them in the late 1950s [14]. In contrast, the temperature of the accretion disk is much lower in quasars; $\sim 1000$ K, while for microquasars $T > 10^6$ K (see section 1.2.2). [13] 

Computers have become exceedingly faster the past decades, the performance is still not sufficient to run ample models for satisfactory analysis of the processes involved. Also, because the chosen initial values determine the outcome, the scientist is expected to get out of a model what is put in, delivering idealized realities from an idealized computer environment.

However, there do appear to be at least three main ingredients for jet formation: [32]

- Some form of collapsing or accreting matter.
- Rotating or spinning plasma.
- In general, the presence of a magnetic field.

Also, in the Kerr metric, the innermost stable circular orbit (ISCO) can lie deep within the
ergosphere, the region where matter is dragged in the spin direction of a black hole. At high accretion speeds this would allow jets to obtain energy from that spin. [3]

1.3.1 Super-Luminal Motion

![Diagram](image)

**Figure 1.5:** When a jet is instigated from a microquasar at distance $d$, light is emitted. After a time $\tau$ the ejecta, expanding with speed $v$, will have crossed a distance $v\tau \cos \theta$ in our direction. Special relativity tells us that the speed of light is limited, so it takes a certain time $t = d/c$ for this light to reach Earth. Here, a radio telescope only directly observes the perpendicular movement $v\tau \sin \theta$. Meanwhile the jet will have expanded over a distance $v\tau = vt$. For light that is emitted from here, it takes less time, $t' = d'/c$ to traverse the remaining distance $d'$ to Earth.

The apparent velocities of jets on the plane of the sky can appear to supersede the speed of light. When this phenomenon was first discovered in the early 1970’s, it was interpreted as evidence that quasars couldn’t be at cosmological distances from earth. The effect can however easily be explained as a consequence of special relativity, as it is most pronounced in jets propagating in a direction close to our line of sight (see figure 1.6). In our inertial frame of reference, an observer ignoring the movement towards Earth will underestimate the true time interval it took the light to get here as $t' + \tau - t = t' = d'/c = (ct - vt \cos \theta)/c = t(1 - \beta \cos \theta)$ (cf. figure 1.5). Hence the object’s speed is overestimated as

$$V_{app} = \frac{\Delta s}{t'} = \frac{vt \sin \theta}{t(1 - \beta \cos \theta)} = \frac{v \sin \theta}{1 - \beta \cos \theta}. \quad (1.8)$$

This apparent speed can even be many times the speed of light as is clear from the plots of this equation in figure 1.6. GRS1915+105 still owns the super-luminal speed record. [11]

By the same line of reasoning the apparent velocity of a receding “blob” would be $V_{app} = v \sin \theta/(1 + \beta \cos \theta)$. Measuring the proper motions of the approaching and reced-
ing condensations, \( \mu_a \) and \( \mu_r \), in the sky (in \( \text{rad \ s}^{-1} \)) would yield two equations

\[
\begin{align*}
\mu_a &= \frac{v \sin \theta}{(1 - \beta \cos \theta)D} \quad (1.9a) \\
\mu_r &= \frac{v \sin \theta}{(1 + \beta \cos \theta)D}, \quad (1.9b)
\end{align*}
\]

with \( D \) being the distance to the source. These two equations can be rewritten into an equivalent set of equations

\[
\begin{align*}
\beta \cos \theta &= \frac{\mu_a - \mu_r}{\mu_a + \mu_r} \quad (1.10a) \\
D &= \frac{c \tan \theta}{2} \left( \frac{\mu_a - \mu_r}{\mu_a \mu_r} \right). \quad (1.10b)
\end{align*}
\]

Setting \( \beta = 1 \) in (a) gives the maximum angle to the line of sight. The maximum distance is inferred by setting \( \beta = 1 \) in (b) - in convenient units:

\[
D_{\text{max}} \leq 87 \tan \theta_{\text{max}} \left( \frac{\mu_a - \mu_r}{\mu_a \mu_r} \right) \text{kpc.} \quad (1.11)
\]

**Figure 1.6:** The apparent velocity of a jet depends on the angle with the line of sight.

### 1.4 Black Hole Properties

While in general the only parameters defining a black hole are its mass and its spin, black hole binaries can be found in a variety of different states, classified according to the received X-ray spectrum.
Figure 1.7: Diagram illustrating the flow of states for canonical black holes, according to the disc-jet coupling model. The upper panel is a Hardness Intensity Diagram (HID). At the top are the various canonical states. The hardness of the X-rays increases along the horizontal axis, and the intensity along the vertical. Also indicated is the jet line, whose exact locus is still under debate. To the right of this line jets should be able to exist. The bottom panel and the roman numerals give a qualitative impression of the variation of the ejecta’s bulk speed (blue) and the inner disk radius (red) with hardness. [?] GRS1915+105 is thought to only exist in the purple area in the top left corner (from private communication with Dr. M. Klein-Wolt).

1.4.1 The Canonical Black Hole States

The concept of X-ray states was devised in the 1970’s after Cyg-X had showed a global spectral change. Out of the blue, the soft X-ray flux (2-6 keV) decreased by a factor of 4, while the hard flux (10-20 keV) increased by a factor of 2, accompanied by sudden radio emissions. Other sources, like A 0620-00 also showed similar behaviour. Hence a classification into states was devised. The high flux in the soft X-ray is generally observed when a source is very bright, suggesting the name “high/soft state”. The increase in hard flux usually meant a decrease in brightness, and so the name “low/hard state” was adopted. During the late 1980’s and early 1990’s, when X-ray measurements were done with the Large Area Counter (LCA) aboard the Japanese Ginga satellite, a third state was needed to characterize the observations. Because of its relative high luminosity ($\gtrsim 0.1 L_{\text{Edd}}$), it was dubbed the “very high state”. In addition
to the state of low emission, the "quiescent state" a fifth "intermediate state" also exists. This state is mainly used for states that do not fall into any of the above categories. The variability of the X-ray emission has been interpreted as fluctuations in the inner accretion disk radius (cf. figure 1.8). [5] [12]

Each of these “canonical” X-ray states reflects a different accretion system with its own characteristics, to be studied to gain insight into the physics of accretion and BHBs. They are distinguished by: a) The presence or absence of a soft component at energies below 5 keV, generally modelled as a multi-color black body component representing the state of the accretion disk. b) The luminosity c) The spectral slope of the emission at higher frequencies and d) The different shapes and frequencies of Quasi Periodic Oscillations (QPOs) and noise components. [33]

While at first the luminosity was one of the primary criteria defining the state of black hole, [5] are abandoning this concept. Although there are clear correlations between states and luminosity in many sources there are also clear exceptions. It is true that the low/hard commonly occurs at lower luminosity then the high/soft state. Also in some sources the very high state occurs when the luminosity levels are way below that in its high/soft state. Each canonical state has now been observed to span two or more magnitudes in X-ray luminosity.

In [5] four basic properties are used to define the different X-ray states:

- The disk fraction $f$, which is the ratio of disk flux to unabsorbed flux, both without absorption, in the 2-20 keV regime.
- The power law photon index at energies below any break or cutoff.
- The root mean square (rms) power, $r$, in the Power Density Spectrum (PDS) integrated from 0.1-10 Hz, expressed as a fraction of the average source count rate.
- The integrated rms amplitude, $a$, of any QPO detected in the 0.1-30 Hz range.

**Thermal or High/Soft State (HS)**

In this state the flux is dominated by the heat radiation from the inner accretion disk, thought to extend all the way to the last stable orbit [25]. Sources with soft X-ray spectra have a photon index of $\sim 2.5$ and maintain a steep, strong, and unbroken power law component all the way up to $\sim 1$MeV, where the sensitivity of gamma-ray detectors drops off. The integrated power continuum is however faint. Usually there is also a nonthermal component in the spectrum, but it has a low contribution of $\leq 25\%$ to the 2-20keV intensity. In this state, QPOs are absent or very weak. [5]

**Hard or Low/Hard State (LH)**

The state typically has a hard power law component with a low photon index of $1.4 \leq \Gamma \leq 2.1$, that contributes $\geq 80\%$ of the 2-20 keV flux. The power law continuum is bright ($r \geq 0.1$) and QPOs may be present. In this state the accretion disk is usually not observed above 2 keV or appears much cooler and withdrawn from the black hole. Sources in this state appear to suffer an exponential cutoff near 100 keV. In recent years more and more evidence has been gathered with VLBI (Very Long Baseline Interferometer) radio observations that the Low/Hard state is connected to the presence of a compact and quasi-steady radio jet. The
Figure 1.8: Schematic representation of the accretion flow in the canonical black hole states. The accretion rate $\dot{m}$, scaled to the Eddington (mass) limit increases towards the top. The dots represent the Advection Dominated Accretion Flow (ADAF). The different X-ray states are believed to emerge from the variability of the inner disk radius [30]. The "standard" model used in spectral fitting is the Multi-Color Disk (MCD) model that requires a thin accretion disk, represented here by the horizontal bars. Jets are believed to exist in the Low/Hard and the Quiescent state. (From private communication with Dr. S. Markoff)

Evidence for this is threefold: a) During observations of GRS1915+105 and Cyg X-1, spatially resolved jets have been found while a quasi-steady (inverted or flat) radio and hard X-ray spectrum was received. b) Sources that persist in the hard state for more than several weeks are likely to show clear correlations between the radio and X-ray intensities, as well as a flat radio spectrum. c) Usually a quenching of the prolonged radio spectrum is observed as soon as the source retreats into the thermal state. [30] [5]

Steep Power Law (SPL) or Very High State (VHS)

Next to having a relative high luminosity, this state is characterized by the presence of X-ray QPOs in the range 0.1-30 Hz. As with the thermal state, its spectrum consists of both a significant thermal component and a power law component that is much steeper than that of the hard state, $\Gamma \geq 2.4$. Also, in some sources, this power law continues unbroken to energies of $\sim 1$ MeV or higher. However in contrary to the thermal state, this power law is more pronounced and the spectral index isn’t as variable. BHB spectra are generally dominated by the SPL state when they approach the Eddington luminosity, equation 1.6. [5]
**Quiescent State**

Usually an X-ray transient BHB spends most of its time in a quiescent state. This state typically has a very low luminosity of $10^{30.5} \text{ to } 10^{33.5} \text{ erg s}^{-1}$ and a non-thermal, hard spectrum, with a power law component of $1.5 \leq \Gamma \leq 2.1$. This is an important state, because the reduced X-ray luminosity allows for dynamical measurements, since the spectrum of the companion is less masked by copious amounts of X radiation. The inefficient radiation mechanisms observed in this state provide a strong case for the existence of the event horizon. [5]

**Intermediate State**

When the black hole states were first invented, a state classifying the transition between the HS and LS appeared to be needed. This intermediate state occurs at $M$ levels intermediate between the high and the low state, but has the same spectral and timing properties as the VHS, however at a lower count rate. If the high luminosity requirement for the VHS is let go, it can be found at many different flux levels and so the IS and the VHS likely represent the same state. [33]

**1.5 The Problem**

There is a black hole, known as GRS1915+105, which does not seem to adhere to any of the canonical black hole states. All the variations in this BHB appear to move within the VHS [33], when the traditional luminosity based classification is retained. However, observations in radio have revealed a state, known as the plateau state that is believed to harbor jets. Does this state relate to any of the canonical black hole states?
The Project/Project Goal

2.1 Objective

My objectives for this bachelor project are manyfold:

1. Form a basic understanding of the physical processes behind BHBs. What kind of radiation do we pick up from its components, i.e. the accretion disk and possibly jets, and how is this radiation created.

2. Get substantially more experience programming in the C language. Create a code modelling a simple jet, include its optical depth.

3. Fit data gathered from an actual black hole binary using the fortran model agnjet.f, created by Dr. Sera Markoff, and the Interactive Spectroscopy Interpretation System ISIS, http://space.mit.edu/CXC/ISIS/, created at MIT by [35]. Try to get a statistically sound fit, meaning a $\chi^2_{\text{red}} \approx 1$, so statements can be made about the data, backed up by the statistics. From this compare the state of the black hole, at the time of observation, to the canonical black hole states (see section 1.4.1).

4. Try to fit the same black hole data with the program I coded, again using ISIS.

2.2 Approach

In the same order I will state my approach to each of the objectives.

1. Worked through chapters one to seven of [1], skipped chapter five on free-free radiation (Bremstrahlung).

2. From the theoretical foundation laid in [1], created a simple representation of a jet, emitting only synchrotron radiation. Included synchrotron self-absorption to consider the optical depth. Relativistic effects were also planned, but not implemented due to lack of time and gross transgression of time available for project.
3. The selected black hole binary was GRS1915+105. This object was observed on X-ray, infrared and radio bands on the night of July 8th 1999. The X-ray data were taken with PCA and HEXTE (aboard RXTE) as a public Target of Opportunity (ToO) observation, done by the RXTE people, observation ID 40403-01-09-00. The data were processed and reduced by Alberto Castro-Tirado. The $J$, $H$, and $K$ band IR data were taken with the 3.8m United Kingdom Infrared Telescope (UKIRT) on Mauna Kea [29]. The radio data was obtained by Vivek Dhawan, using the Green Bank Interferometer.

To get an indication of its physical properties, I studied research articles about BHB GRS1915+105. From those, I obtained the prevalent values for as many of the parameters used in `agnjet.f` as possible. The `agnjet.f` code was imported into ISIS, using SLIRP [36], the S-Lang [37] Interface Package. After several fitting attempt I acquired one with a satisfactory reduced chi-squared. From this fit I tried to determine how the state GRS1915+105 was in that night\(^1\) (the plateau state, see section 3.2) related to the canonical black hole states.

4. As with `agnjet.f`, my code could be imported into isis using SLANG and SLIRP. However the appropriate language handler appeared not to be available at the time. I decided to forego this interesting but useless pass-time: Since no comptonization was implemented in my code the chance of getting a decent fit in the X-rays was next to nothing.

\(^1\)\(\sim\) 35,000 years ago, depending on the actual distance.
3 Observations

3.1 Black Hole GRS1915+105

Microquasar GRS1915+105 was discovered on the 15th of August 1992, by the Wide Angle Telescope for Cosmic Hard X-rays (WATCH) all-sky monitor, aboard the Russian GRANAT satellite. It is an hard X-ray transient located in the constellation of Aquila, at $l = 45.37^\circ$, $b = -0.22^\circ$. GRS1915+105 is perhaps best known for its remarkable variability in the X-ray band. Observations in the 2-10 keV bands with the All-Sky Monitor (ASM) instrument on board the Rossi X-ray Timing Explorer (RXTE) have shown that the richness in variability distinguishes this source from every other known X-ray source. [7] associated this variability with the dis- and reappearance of the inner region of the accretion disk, caused by the onset of thermal-viscous instabilities. GRS1915+105 is also one of two known binary sources believed to contain a maximally spinning black hole [25]. Black hole spin is said to contribute to the enormous power needed to generate jets.

Due to its residence in the Galactic plane this BHB suffers from a large extinction in the visual band and an optical counterpart has not yet been traced. Since the discovery of (highly variable) counterparts in the infrared and radio [23] the position of GRS1915+105 is accurately known. The distance to GRS1915+105 could first be estimated when the Very Large array (VLA) observed ejections separating from the core between 27 March and 30 April 1994, proper motions of $17.6 \pm 0.4$ and $9.0 \pm 0.1$ mas d$^{-1}$ were revealed, traveling at an angles of $150^\circ$ and $330^\circ$ respectively. From this data, assuming intrinsically symmetric ejection, and using equation 1.11, [18] derived a maximum distance to GRS1915+105 of 13.7 kpc. They then proceeded to get a more accurate distance estimate by determining the kinematic distance$^1$. In order to do this they took 21 cm absorption spectra of atomic hydrogen along the line of sight to GRS1915+105, during a radio outburst. These measurements are found in [19]. To compare, they picked the H II cloud G 45.45+0.06, which is closest on the sky to GRS1915+105 and was determined to have a kinematic distance of $\sim$8.8 kpc by [40] in the 1980’s. The measured H1 column density of $N$(HI)$=1.73 \times 10^{22} T_s/100$ K cm$^{-2}$ was 1.42 times larger than that towards G 45.45+0.06. Assuming a constant column density they were able

$^1$A kinematic distance is determined by taking velocity spectra of a hydrogen cloud, fitting those with gaussians and comparing these to the velocities, expected from the Local Standard of Rest (LSR). [4]
to set the kinematic distance to GRS1915+105, reporting it was as far out as 12.5±1.5 kpc\(^2\). Later measurements of the \(^{12}\text{CO}(J=1-0)\) spectrum, which is linked to the column density of molecular hydrogen, by [41] agree with this value. In this article also an accurate value for the visual extinction, with errorbars, is derived: \(A_v = 26.5 \pm 1.7\) mag. At a distance of 12.5 kpc, the apparent transverse motions of the 1994 outburst, were 1.25\(\pm\)0.15\(c\) and 0.65\(\pm\)0.08\(c\). This made GRS1915+105 the first Galactic source ever exhibiting super-luminal motion\(^3\) (see section 1.3.1). Solving for the proper motions yields a true velocity of 0.92\(c\) \(\pm\)0.08\(c\).

At 12.5 kpc GRS1915+105 should accrete near or even above the Eddington limit \(\dot{M}_{\text{Edd}}\), to account for its luminosity (2.4\times10^{39}\) erg s\(^{-1}\) at peak luminosity - [30]). Since the Eddington luminosity is thought to be an important boundary to accretion processes [3] it may not be surprising that more recent measurements bring the BHB closer and closer, accounting for some of the excess luminosity.

[20] interpreted high resolution radio images of ejecta made by the Multi-Element Radio-Linked Interferometer Network (MERLIN) (see figure 1.3) in October/November 1997. After a plateau state (see section 3.2) of \(\sim\)20 days, four major ejection events were captured, with considerably higher proper motions than those observed with the VLA. Fitting the data (with a \(\chi^2\) \text{red} \leq 1) resulted in proper motions of 23.6\pm0.5 and 10.0\pm0.5 mas d\(^{-1}\), constraining the maximum distance to GRS1915+105 to 11.2\pm0.8 kpc (using equation 1.11). The upper limit of 12.0 kpc is only reached when the jet is traveling at the speed of light, and \(\beta=1\). At this distance \(L_{\text{X-ray}}/L_{\text{Edd}} \sim 1.4\) (with a mass of 10-18 \(M_\odot\), see next section).

A more recent paper, [17], re-evaluates the optical absorption \(A_v\), by more accurately locating the clouds of molecular hydrogen in the direction of GRS1915+105. They took \(^{12}\text{CO}(J=1-0)\) velocity spectra of clouds along the line of sight and also of two HII region close to the object: again G 45.45+0.06 and also G 45.12+0.13 (see figure 3.1) They deduced the total hydrogen column density with the simple relation 

\[N_H = N_{\text{HI}} + 2N_{\text{H}_2},\]

using the total atomic hydrogen column density obtained by [19]. The optical extinction can then be deduced using the empirical relation from [42]

\[A_v = N_H/1.79 \times 10^{21}\] mag.

\(^2\)[18] state this value is consistent with a value for \(A_v\) of \(\sim\)20 mag, referring to a different article by the same chief writer: [23]. However in [23] a value for \(A_v\) of \(\sim\)30 mag. is derived (and in the conclusion even 33 mag is stated!). Also in [18] the reference to [23] is mistyped, dating the article ten years back to 1984. So that’s two typos in one article.

\(^3\)Still only one other Galactic object is known to exert this behavior: GRO J1655-40 [31].

**Figure 3.1:** Map of \(\lambda=21\) cm regions in the direction of GRS1915+105. Two clouds have been renamed after the creation of this map, due to more accurate localization: G45.45+0.06 is now G45.45+106 and G45.13+0.14 is now G45.12+0.13. The figure shows the contours of the -4, 4, 6 ,8, 10, 20, 40, 80, 160, and 320 times 3.0 mJy beam\(^{-1}\), and the half power contour of the beam in the top left corner. The three HII regions shown here are within 30' from GRS1915+105. Sources away from phase center may appear weaker then they are, since the image is not corrected for primary beam response.
With a total column density of molecular hydrogen of \( N_{H_2} = 8.8 \pm 1.4 \times 10^{21} \text{ cm}^{-2} \), they calculated the total column density of hydrogen to be \( N_H = 3.5 \pm 0.3 \times 10^{22} \text{ cm}^{-2} \), yielding an optical extinction of \( A_v = 19.6 \pm 1.7 \) mag. A lower optical extinction can put GRS1915+105 closer, since there are less molecular clouds between us then thought before.

The distance calculated by [18] is based on the fact that they assume it to be behind the molecular hydrogen region G 45.45+0.06. While in 1994 this cloud was still believed to be at \( \sim 8.8 \) kpc, more recent findings support a distance of 6.6 kpc [21], placing GRS1915+105 at a distance of 7-12 kpc. Because of the high H1 column density to the source, according to [18] and [19], there would still be one more molecular cloud between G 45.45+0.06 and GRS1915+105, so [20] decide to adopt the conservative distance of 11kpc. [17] suggest that all the clouds (G 45.12+0.13, G 45.45+0.06 and the cloud behind that one), all belong to the same complex, which they locate at the tangent point of LSR velocity \( 41 \pm 6 \) km s\(^{-1}\), or equivalently, at a distance of 6 kpc. They state GRS1915+105 is behind this complex, that has to contribute to the HI extinction because of its large contribution of 6 mag., with the tangent point as the closest lower limit. Then using the upper limit of [20] as their upper limit they put GRS1915+105 at a distance of 9.0\( \pm 3.0 \) kpc (so this 3 kpc interval is not the one sigma confidence level). Together with the lower \( A_v \) this supports the conclusion that GRS1915+105 is indeed less luminous then expected, in the optical and infrared regimes. Considering all the assumptions made by [17] and the huge errorbars, for this project I will stick with the more conservative value of 11 kpc distance to GRS1915+105, also adopted by [20].

The Mass

To set a lower limit to the mass of the compact object in a binary, the parameters in the mass function

\[
f(M) = \frac{P_{\text{orb}} K_d^2}{2 \pi G} = \frac{M_{BH}^3 \sin^3 i}{(M_{BH} + M_{\text{donor}})^2} = \frac{M_{BH} \sin^3 i}{(1 + q)^2},
\]

with \( q \equiv M_d/M_{BH} \) the ratio of the masses of the components, \( K_d \) the velocity amplitude and \( P_{\text{orb}} \) the orbital period, need to be determined.

The orbital period \( P_{\text{orb}} \) of the binary system is deduced from the periodogram, figure 3.2, top panel, which clearly shows a peak at 33.5 days. [6] constructed this periodogram by cross-correlating the \(^{12}\)CO and \(^{13}\)CO band heads of 16 different spectra taken between April and August 2000, indicating the radial velocities involved in GRS1915+105. The mean of these \( K \) band observations is shown in figure 3.3, bottom curve. From the lower panel figure 3.2, the velocity amplitude \( K_d \) can be set at 140\( \pm 15 \) km s\(^{-1}\). However the flux in the infrared is dominated by emissions from the accretion flow or jet, in stead of the secondary star, so this value might introduce a systematic error. Above values give a mass function \( f(M) = 9.5 \pm 3.0 \) M\(_{\odot} \), leaving only the inclination \( i \) and the mass of the donor star (see section 3.1) in order to derive GRS1915+105’s mass.

Assuming the binary plane is the same that of the accretion disk, the orbital inclination of GRS1915+105 can be deduced from the orientation of the jets. Ever since its discovery this inclination has been a constant, indicating there is no measurable precession, so the jets can be assumed to be perpendicular to the accretion disk and orbital plane. The exact inclination is however still open to debate, since it is determined from the brightness and velocities of
Figure 3.2: Velocity amplitude (bottom panel) obtained by cross-correlation of the CO band heads - e.g. figure 3.3, top curve - with a template spectrum of the K2III star HD202135. The curve plotted in the bottom panel denotes the best fit period of $P_{\text{orb}} = 35.5$ days, with a semi-amplitude $K_d = 140 \pm 15$ km s$^{-1}$. The systemic velocity is $-3 \pm 10$ km s$^{-1}$, implying a kinematic distance to GRS1915+105 of $12.1 \pm 0.8$ kpc. The value of "stat" in the periodogram (top panel) is a measure of the significance of the obtained period. [6]

both the approaching and receding blobs. Using

$$\theta = \tan^{-1} \left[ 1.16 \times 10^{-2} \left( \frac{\mu_a \mu_r}{\mu_a - \mu_r} \right) D \right]$$

(3.3)

[18] calculated an inclination of $70^\circ \pm 2^\circ$, using a distance of 12.5 kpc. At a distance of 11 kpc however, this reduces to $66^\circ \pm 2^\circ$ [20]. From this the mass of GRS1915+105 is estimated to be at least $14 \pm 4 M_\odot$, making it the most massive Galactic low-mass X-ray binary known to date. [6]

Properties of the donor star

A first rough identification of the companion of GRS1915+105 was done by [24]. In $K$ band observations (e.g. figure 3.3) they identified $^{12}\text{CO}$ absorption band heads characteristic of a low ($<7000$ K) temperature star. They also found weak $^{12}\text{CO}$ (2,0) and $^{13}\text{CO}$ (3,1) transitions, associated with luminosity class III or brighter. In addition they identified the Na doublet and possibly the Ca triplet, Al I, and the Mg doublet. The CN doublet was not found, which excludes supergiants, since in those stars this line is usually more prominent then Al/Mg. Furthermore they analyzed the $H$ band they identified Mg I, $^{12}\text{CO}$ (6,3) and $^{12}\text{CO}$ (8,5) in a ratio which is consistent with M-K standards. From this analysis they conclude the companion as a class III K-M giant. From a method proposed by Meyer et al. (1998), considering the veiling-independent temperature/luminosity discriminant, they estimate the
Figure 3.3: [22] analyzed the same VLT spectra of GRS1915+105 as [6], but used a different KIII template star, HD 138185, multiplied by a veiling factor of 0.14 (bottom curve). Above this curve is a Doppler corrected spectrum that is the average of 16 individual spectra, followed by the difference between these first two spectra, representing the accretion disk spectrum. The top curve is the difference between the bottom template and the Doppler corrected spectrum of GRS1915+105 at binary phase $\sim 0.75$. The presence of the $^{13}$CO isotope and the equivalent width ratio of $^{12}$CO to $^{13}$CO suggests a late type giant companion, that, judging from the faintness and the small width of the CO band heads, only contributes a few percent to the K band brightness.

temperature to be $\sim 4800^{+200}_{-500}$K, suggesting a late G or K type star. However, because the CO bands in their spectra are weak, they are not confident in this result. As a representative of the aforementioned classes, they proceed to compare the star to a K2 III star. Scaled to a suitable brightness, this would imply a donor magnitude of 14.5-15.0, uncorrected for extinction. At a distance of $\sim 11$ kpc and an extinction correction of $A_K = 3$ mag, they find an absolute magnitude of $M_K = -2$ to $-3$, agreeing with the giant classification. If the donor star is indeed a K2 III star, its mass would be $1.2±0.2$ M$_\odot$ and its temperature would be $4455±190$ K [26]. As there is still a lack of a more accurate classification of GRS1915+105’s companion I will employ this temperature for the project.

[22] further analyzed the same spectra from the VLT as [6] (figure 3.3), deducing the mass ratio $q$ of GRS1915+105 and its companion. This yielded the mass of the companion and a more refined result for the black hole mass. From the width of the photospheric $^{12}$CO and $^{13}$CO absorption lines, the rotational broadening, $v\sin i$, was derived. Assuming the donor is tidally locked to the attractor and completely fills its roche-lobe surface (section 1.2.2) lead to a $q$ of $0.058±0.033$. Using the equation

$$\frac{v\sin i}{K_d} = 0.46 \left[ (1 + q)^2 q \right]^{1/3}$$

(3.4)

together with equation 3.2 a donor mass of $0.81±0.53$ M$_\odot$ and a black hole mass of $14±4.4$
M⊙ were obtained. The large uncertainty in black hole mass came from in the semi-amplitude radial velocity of 15 km s⁻¹.

From the preceding papers it is clear that the companion has a mass of ~ 1 M⊙. For stars of this mass range the mass loss rate ṁ by stellar winds cannot account for the the high accretion luminosity found in GRS1915+105. So accretion should occur via roche-lobe overflow [24]. [22] concur to this fact. Using Eggleton’s approximation from [27]

\[ r_L = \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1 + q^{1/3})} \]  

they find a Roche lobe / donor size of 19 R⊙, and using his approximation for the density

\[
P(\rho)^{1/2} = 0.1375 \left(\frac{q}{1+q}\right)^{1/2} r_L^{-3/2} \]

they get a density \( \rho = 2 \times 10^{-4} \text{ g cm}^{-3} \) (using \( P=33.5 \text{ days for the orbital period} \)). As the radii and density for classes KIII 1-5 should lie within the regimes 11-28 R⊙ and \( \rho = 12 - 4 \times 10^{-4} \text{ g cm}^{-3} \) these values support the prevalent donor classification, as well as the claim that in GRS1915+105 accretion happens through Roche lobe overflow. Since the star’s envelope is contained within its Roche lobe, evolutionary expansion of the donor can sustain the mass transfer.

The reduced mass of 0.81 M⊙ could imply that GRS1915+105 has proceeded quite far in its evolution. More then a solar mass may already have been transferred to the black hole. In time the orbital period will increase to ~ 45 days. The companion will then reach a mass equal to that of its helium core. At this stage mass transfer is said to cease. A helium core mass of 0.27 M⊙ has a size of ~18.6 R⊙, so compared to the 19 R⊙ radius mentioned before, this might not be far into the future. [22]

### 3.2 GRS1915+105 and the Canonical States

As noted in section 3.1 GRS1915+105 has a remarkable X-ray variability. There has been much discussion about wether GRS1915+105 actually fits any of the canonical black hole states. The condition in which GRS1915+105 exhibits steady radio and X-ray emission has an average luminosity of \( L_X \approx 10^{38} \text{ erg s}^{-1} \) - a factor 100 higher then for most black holes in the hard state. Also the photon index in the X-ray is steeper then usual, \( \Gamma \sim 2.2 \), meaning a particle distribution index of \( p = 2\Gamma - 1 = 3.4 \) [30]. So at first sight the states found in GRS1915+105 do not match the canonical states. Therefore [8] chose to commonly (and unimaginatively) refer to the states found in this object as state A, B and C. State A and B display a soft spectrum, but state B has a slightly harder spectrum and lower flux. The flux in state C is even lower and and this condition instigates a hard spectrum. [8] analyzed 349 observation intervals and encountered observations more then a year apart almost indistinguishable from another. So despite GRS1915+105’s extreme variability they were able to classify the data into only twelve classes, based on their color-color diagram (see figure 3.4) and light curves. Ten of the twelve classes can be understood as the interplay of two or three of the basic states. Repeating patterns of are found and transitions between individual states can occur within seconds. State B never lasts for more then a few hundred seconds, while states A and C may appear for short intervals (¡ 100 s) or can persist for days.
Figure 3.4: Color-Color diagram indicating the locus of the three basic states A, B and C and their transitions. All the different state can change into one another, but state C will never change into state B. The two X-ray colors are defined as \( HR_1 = B/A \) and \( HR_2 = C/A \), with A:2-5 keV, B:5-13 keV and C:13-60 keV.

Two classes, however, do not show state transitions. Class \( \phi \) and \( \chi \) are reserved for state A and C respectively.

[33] looked at observations from RXTE’s PCA that represent the three basic states. They found that the canonical HS is never seen in GRS1915+105 since the power-law tail is always present. Normally its absence characterizes the HS in canonical black holes, where a change in the innermost radius of the accretion disk determines the transitions between those states. They conclude that although GRS1915+105 does reveal canonical hallmarks, the source is somewhat of an odd-ball, spending most of its time in a state that resembles the VHS and includes the display of QPOs\(^4\). Sometimes and for extended episodes of tens of days it can go into state C, that resembles a hard state, without the complex periodicity, but this state can certainly not be called low\(^5\). This state is the same as variability class \( \chi \) mentioned before, or more specifically, the \( \chi_1 \) and \( \chi_3 \) states in [8]. It is growing in familiarity as the plateau state, from its appearance in radio (see figure 3.5). The plateau state was first described by [38] and this description was more refined later by [20]. The plateau state is characterized by a flat topped light curve in radio, with a rapid (as short as a day) onset and decrease in flux density, always coinciding with hard X-ray outbursts. At peak level the flux density increases to \( \sim 100 \) mJy. The radio emission is generally optically thick, but sometimes major radio flares are superimposed, adding optically thin radiation. According to [20] a the plateau state is accompanied by quasi-stable X-ray emission and a significant hardening of the spectrum. The plateau state has also been called the radio loud, radio-plateau low/hard and type II hard steady, see [39] and references therein.

The optically thick radio emission originates from compact jets [34]. This can be understood from the fact that a flat or inverted radio spectrum is likely to arise from absorbed emission, as can be seen in section 4.2, where the absorption arises from synchrotron self-

\(^4\)Quasi periodic oscillations have been observed in GRS1915+105 ranging from 0.001 to 67 Hz.

\(^5\)This is one of many reasons for McClintock and Remillard to abandon the luminosity for classification of X-ray states.
Figure 3.5: Radio and X-ray flux measurements done by MERLIN from September 9th to December 8th 1997. The top panel shows flux densities from the Ryle Telescope (RT) and Green Bank Interferometer (GBI). As is clear from this panel, the plateau state is characterized by a flat topped light curve in radio. The middle panel represents the radio spectral index (at 13.3 and 3.6 cm), which is $\sim 0$ during this state pointing at optically thick emission. The dashed line in this panel denotes a spectral index of $\alpha \sim -0.8$, indicating that during the flaring states the emission is optically thin. The bottom panel shows the RXTE ASM counts in the 2-12 keV bands.

absorption. Of course the traditional classification of states is not done from radio observations, but in other BHBs jets are only found during the traditional LS. So although for many reasons state C cannot be called a LS, it does bear characteristics of this condition.

The plateau state appears to precede another radio state: the flaring state. This state is characterized by a short rise time of less than a day and an optically thin exponential decay of radio flux. During this decay the hard X-ray emission decreases [38]. The outbursts of superluminal ejections from [18] and [20], from which they deduced the distance to GRS1915+105, appear to have followed extended plateau states. The plateau state is thus of great interest for understanding these plasma outflows [39]. It is still unclear if the radio flare preceding the plateau state in figure 3.5 also corresponded to a plasma ejection [20].
4

Model/Model Grid

In this section the different models and the fitting process are described, starting with a discussion of synchrotron radiation and synchrotron self-absorption. Both my code and dr. Markoff’s model include this form of radiation, that allows us to observe black hole jets, primarily in radio.

4.1 Synchrotron Radiation [1]

When a magnetic field accelerates a particle, that particle will radiate. The non-relativistic version of this process, cyclotron radiation, was first observed in particle accelerators in the 1930’s. [28]

An electron in a magnetic field twists around the field lines due to the Lorentz force $\mathbf{v} \times \mathbf{B}$. At non-relativistic speeds the frequency of emission is just this frequency of gyration (or gyro-frequency) in the magnetic field

$$\omega_B = \frac{qB}{\gamma mc},$$

where $q$ and $m$ are the charge and mass of the particle, respectively, and $\gamma$ is the Lorentz factor $\gamma = \sqrt{1 - \beta^2}$ with $\beta = v/c$.

For relativistic particles the frequency spectrum is much more complex. Here the acceleration causes radiation to be emitted at harmonics of the gyro-frequency (also see figures in section 4.1.1). The gyro-frequency is linearly proportional to the critical frequency

$$\nu_c = \frac{3}{4\pi} \gamma^3 \omega_B \sin \alpha = \frac{3qB\gamma^2 \sin \alpha}{4\pi mc} = 4.21 B \gamma^2 \text{ MHz.}$$

$\alpha$ is the so-called pitch-angle; the angle between the magnetic field and the velocity. Assuming $\alpha = \pi/2$ and using $E = \gamma mc^2$ yields

$$\nu_c = 16.08 \times 10^6 BE^2 \text{ MHz,}$$

where $E$ is the electron energy in GeV en $B$ is the magnetic field strength in Gauss. This result can be used to determine the characteristic energies of the electrons responsible for the
radiation, once the magnetic field can be measured or estimated. [3] [1]

To deduce the total power emitted by an electron by synchrotron radiation I’ll start of with the tensor equation of motion for a charged particle:

\[
a^\mu = \frac{e}{mc} F^\mu_\nu U^\nu. \tag{4.3}
\]

Here \(m\) is the particle mass, \(F^\mu_\nu\) is the Lorentz four-force and \(U^\nu\) is the particle four-velocity. This equation gives two equation for a charged particle in a magnetic field:

\[
\frac{d}{dt}(\gamma m v) = \frac{q}{c} v \times B \tag{4.4}
\]

\[
\frac{d}{dt}(\gamma mc^2) = q v \cdot E = 0 \tag{4.5}
\]

Because the last equation implies that either \(\gamma\) or \(|v|\) is constant, it follows that

\[
m \gamma \frac{dv}{dt} = \frac{q}{c} v \times B. \tag{4.6}
\]

Or, separating into components along and normal to the field

\[
\frac{dv_\parallel}{dt} = 0, \quad \frac{dv_\perp}{dt} = \frac{q}{\gamma mc} v_\perp \times B \tag{4.7}
\]

So \(v_\parallel\) is constant and, because \(|v|\) is constant, so is \(v_\perp\). These equations describe the helical motion of the particle (see figure 4.1), with the frequency of gyration given by equation 4.1.

Now, to determine the radiation emitted by a relativistic particle, move into an instantaneous rest frame \(K'\), where the particle has zero velocity at a certain time. Since the particle will move non-relativistically for adjacent moments on the timeline, the emitted radiation can be calculated from the dipole formula, also known as Larmor’s formula. If a total amount of energy \(W\) is emitted in \(K'\), in a time \(dt'\), the momentum of this radiation will be zero, because the emission is symmetrical with respect to any direction and its opposite direction. So the energy in frame \(K\) is

\[
dW = \gamma dW'. \tag{4.8}
\]

The time intervals between the different frames are related by

\[
dt = \gamma dt' \tag{4.9}
\]

with \(dt'\) the proper time of the particle. For the total power we can know that

\[
P = \frac{dW}{dt}, \quad P' = \frac{dW'}{dt'}, \tag{4.10}
\]

proving that the total emitted power is Lorentz invariant for any emitter with aforementioned symmetry: \(P = P'\). Larmor’s formula in covariant form is

\[
P = \frac{2q^2}{3c^2} \vec{a} \cdot \vec{a}, \tag{4.11}
\]
From this power can be evaluated in any frame by computing $\vec{a}$ in that frame and squaring it. Using the transformation rules for acceleration

$$a_x = \frac{a'_x}{\gamma^3\sigma^2},$$  
$$a_y = \frac{a'_y}{\gamma^3\sigma^2} - \frac{u_y v}{c^2} \frac{a'_x}{\gamma^3\sigma^2},$$  
$$a_z = \frac{a'_z}{\gamma^3\sigma^2} - \frac{u_z v}{c^2} \frac{a'_x}{\gamma^3\sigma^2},$$

(4.12)

where

$$\sigma = 1 + \frac{vu'x}{c^2}.$$  

If $K'$ is the instantaneous rest frame of the particle, $u'x = u'y = u'z = 0$ and $\sigma = 1$, yielding the the acceleration components parallel and perpendicular to $v$

$$a'_\parallel = \gamma^3 a_\parallel$$  
$$a'_\perp = \gamma^2 a_\perp$$

(4.13)

Equation 4.11 can be rewritten in terms of these components:

$$P = \frac{2q^2}{3c^2} a' \cdot a' = \frac{2q^2}{3c^2} (a'^2_\parallel + a'^2_\perp) = \frac{2q^2}{3c^2} \gamma^4 (\gamma^2 a'^2_\parallel + a'^2_\perp)$$

(4.14)

From equations 4.1 and 4.7 it is clear that the acceleration normal to the velocity has magnitude $a_\perp = \omega_B v_\perp = \omega_B v \sin \alpha$ and $\beta = v/c$ the total emitted power can be written as

$$P = \frac{2q^4 \gamma^2 B^2 \beta^2 \sin^2 \alpha}{3m^2 c^3}$$

(4.15)

The power emitted at a certain frequency can also be written as a constant of proportionality times a function of $\omega$. This function can be made dimensionless using $\omega_c$ for the scaling.

$$P(\omega) = C F \left( \frac{\omega}{\omega_c} \right)$$

(4.16)

The trick is now to determine $C$. This can be done by comparing the integral over $\omega$ to equation 4.15.

$$P = \int_0^\infty P(\omega) d\omega = C \int_0^\infty F \left( \frac{\omega}{\omega_c} \right) = \omega_c C \int_0^\infty F(x) dx,$$

(4.17)

using $x \equiv \omega/\omega_c$. Of course $\int_0^\infty F(x) dx$ is unknown until $F(x)$ is specified. But since $F(x)$ is dimensionless and its value is arbitrary only a convention for the normalization of $F(x)$ is needed. The dependence of $C$ on all physical parameters can be found from equations 4.2, 4.15 and 4.17. Obeying the conventional choice for the normalization of $F(x)$ to $\sqrt{3}$ finally yields for the power emitted by one electron, in terms of frequency $\nu$

$$P(\nu) = \frac{\sqrt{3} q^3 B \sin \alpha}{mc^2} F(x).$$

(4.18)
The power as a function of angular frequency relates to the power as a function of frequency as \( P(\nu) = 2\pi P(\omega) \). For small and large values of \( x \), \( F(x) \) can be approximated by asymptotic forms

\[
F(x) \sim \frac{4\pi}{\sqrt{3(t_\delta^2)}} \left( \frac{x}{1} \right)^{1/3}, \quad x \ll 1, \quad (4.19a)
\]
\[
F(x) \sim \left( \frac{\pi}{2} \right)^{1/2} e^{-x}, \quad x \gg 1. \quad (4.19b)
\]

![Figure 4.2: Synchrotron function F(x), computed with the Gnu Scientific Library function gsl_sf_synchrotron_1 [16]. The peak is at x = 0.29.](image)

Eventually we’ll want to know the total power emitted by a source, so first we need to get from equation 4.18 to the total emitted power per unit volume per unit frequency. For this we will need to know the electron distribution.

**Power Law**

From equation 4.18 it is clear that the power from one electron only depends on \( \gamma \) through \( \omega_c \). This leads to an important conclusion about synchrotron radiation. Over certain ranges of energy, the spectra can often be approximated by a power law. It is therefore convenient to define the spectral index, \( \alpha \), which gives the slope on a log \( P(\nu) \) - log \( \nu \) plot

\[
P(\nu) \propto \nu^{-\alpha} \quad (4.20)
\]

Also particle distributions of relativistic electrons can sometimes be described by a power law. The number density of particles with energies between \( E \) and \( E + dE \), or \( \gamma \) and \( \gamma + d\gamma \) can than be expressed as

\[
N(E) = CE^{-p}dE, \quad E_1 < E < E_2 \quad (4.21a)
\]

or

\[
N(\gamma)d\gamma = C\gamma^{-p}d\gamma, \quad \gamma_1 < \gamma < \gamma_2. \quad (4.21b)
\]
Now to get to the total radiated power per unit volume per unit frequency for such a distribution, we only need to multiply equation 4.18 by one of these formulas and integrate over all $\gamma$’s or energies.

$$P_{\text{tot}}(\nu) = C \int_{\gamma_1}^{\gamma_2} P(\nu) \gamma^{-p} d\gamma \propto \int_{\gamma_1}^{\gamma_2} F\left(\frac{\omega}{\omega_c}\right) \gamma^{-p} d\gamma$$  \hspace{1cm} (4.22)

Changing variables to $x \equiv \omega/\omega_c$ and noting that $\omega_c \propto \gamma^2$ yields

$$P_{\text{tot}}(\nu) \propto \nu^{-(p-1)/2} \int_{x_1}^{x_2} F(x)x^{(p-3)/2} dx.$$  \hspace{1cm} (4.23)

If the energy limits are wide enough, the boundaries can be approximated as $x_1 \approx 0$ and $x_2 \approx \infty$, producing an almost constant integral. Then the end result is

$$P_{\text{tot}}(\nu) \propto \nu^{-(p-1)/2},$$  \hspace{1cm} (4.24)

relating the spectral index to the particle distribution index as

$$\alpha = \frac{p - 1}{2}.$$  \hspace{1cm} (4.25)

Furthermore, the spectral index is related to the photon index $\Gamma$ as $\Gamma = \alpha + 1$ I will now quote the exact result for the total volume power per unit volume per unit frequency, for a power-law distribution of electrons, equation (6.36) in [1].

$$P_{\text{tot}}(\nu) = \frac{\sqrt{3}q^3 CB \sin \alpha}{mc^2(p+1)} \Gamma\left(\frac{p}{4} + \frac{19}{12}\right) \Gamma\left(\frac{p}{4} - \frac{1}{12}\right) \left(\frac{2\pi mc\nu}{3q B \sin \alpha}\right)^{-(p-1)/2}$$  \hspace{1cm} (4.26)

However, as equation 4.24 this result is derived with energy (or $\gamma$) limits from 0 to $\infty$, while in reality $\gamma$ cannot be smaller then one or equivalently, $E$ cannot be smaller then $mc^2$. In section 4.2 I will demonstrate that under these conditions this approximation breaks down.

### 4.1.1 Cyclotron and Synchrotron Radiation Compared

As mentioned before, synchrotron radiation is the relativistic counterpart of cyclotron radiation. The frequency of cyclotron radiation only depends on the gyro-frequency, at which also the electric and magnetic components vary. Thus the fourier transform of this sinusoidal time dependent signal reveals the spectrum as a single spike at $\omega_B$, equation 4.1 (cf. figure 4.3a). As $\beta = v/c$ increases, only harmonics of this fundamental frequency will start to contribute, because the basic periodicity remains $T = 2\pi/\omega_B$. The spectrum will expand with spike at these higher frequencies, contributing at a strength proportional to increasing powers of $v/c$ for $v/c \ll 1$ (cf. figure 7b).

As $\beta$ increases to highly relativistic speeds, the sinusoidal signal changes to a series of sharp pulses, still repeated at the basic $\omega_B$ frequency (cf. figure 4.3d). The envelope of the spikes in the resulting spectrum resembles the shape of the function $F(x)$. The gaps between individual spikes can be filled by several physical broadening mechanisms, e.g.:

- Since the gyro-frequency is proportional to $1/\gamma$, for a (relativistic) distribution of particle energies the spectra of individual particles do not fall on the same lines.
- Different regions in physical phenomena emitting synchrotron radiation, for example jets, also have different magnetic field strength and direction. This causes the harmonics to fall on different places in a spectrum.

An observer will see the superposition of all these different pulses: a continuous spectrum.
Figure 4.3: (a) At sub-relativistic speeds particles only radiate at the gyro-frequency $\omega_B$. (b) Increasing the particle speed higher harmonics of the gyro-frequency will start to contribute (denoted here as spike 2). (c) While the time dependency of the electric field is still shaped like a sinoid for a low speed particle, at high speeds the field is only observed in sharp pulses, separated by interval $2\pi/\omega_B$. The spectrum for these pulses will look like (d) At highly relativistic speeds the envelope of the individual spikes, representing different harmonics of the gyro-frequency, will take the shape of the synchrotron function $F(x)$, cf. figure 4.2.

4.1.2 Synchrotron Self-Absorption

Synchrotron emission is creation of photons from accelerated charges in a magnetic field. Photons also interact with charges through the electromagnetic interaction. Thus in a medium that creates synchrotron radiation a variety of absorption processes occur, and because it is the medium itself that creates and absorbs the radiation this is called self-absorption. Another possibility is negative absorption or stimulated emission. When this happens a particle will radiate more strongly in a direction and at a frequency where photons are already present.

The individual processes are interrelated by Einstein coefficients. From radiative transfer theory the absorption coefficient $\alpha_\nu$ can be deduced. However, due to time-limitations for this project I will account for self-radiation in a jet, using a simpler model. I will thus forego the complete derivation of the absorption coefficient which is discussed at thoroughly in [1] pages 186-190 and just start by quoting their general result for the absorption coefficient, eq
\[ \alpha_{\nu} = -\frac{c^2}{8\pi\nu^2} \int dE P(E, \nu) E^2 \frac{\partial}{\partial E} \left[ \frac{N(E)}{E^2} \right]. \]  
(4.27)

For a power law distribution of particles we then have
\[ -E^2 \frac{d}{dE} \left[ \frac{N(E)}{E^2} \right] = (p + 2)CE^{-(p+1)} = \frac{(p + 2)N(E)}{E}. \]  
(4.28)

allowing equation 4.27 to be rewritten as
\[ \alpha_{\nu} = \frac{(p + 2)c^2}{8\pi\nu^2} \int dE P(\nu, E) \frac{N(E)}{E}. \]  
(4.29)

This result can be verified with the following, eq (6.53) from [1]
\[ \alpha_{\nu} = \sqrt{\frac{3q^3}{8\pi\hbar}} \left( \frac{3q}{2\pi\hbar^3c^3} \right)^{p/2} C(B\sin\alpha)^{1/2} \Gamma \left( \frac{3p + 2}{12} \right) \Gamma \left( \frac{3p + 22}{12} \right) \nu^{-(p+4)/2}. \]  
(4.30)

### 4.1.3 Skin Depth

The absorption coefficient describing the self-radiation allows us to calculate the jet flux we should receive on Earth. Actually this intensity follows from radiative transfer theory, but since this is far beyond the scope of this project, we approximated it using the following equation:
\[ F_\oplus = \pi rh(1 - e^{-\tau}) \frac{P_{\nu}}{\alpha_{\nu}} \times \frac{1}{4\pi D^2}, \]  
(4.31)

where \( r \) is the radius of the jet cone, \( h \) is the height of the cone component, \( P_{\nu} \) is the power per volume per Hz and \( D \) is the distance. \( \tau \) can be estimated from dimensional arguments. The absorption coefficient has units cm\(^{-1}\), so this can be regarded as a skin depth, giving a measure of how far into the jet we can see. Since \( \tau \) is unitless and the larger the radius of the jet is, the bigger the chance that a photon gets absorbed again, it can be expressed as
\[ \tau \sim r\alpha_{\nu}. \]  
(4.32)

Due to the absorption, the jet is *optically thick* and we only see radiation coming from the outer 'layer', or its 'skin'. This is seen from the two limiting cases \( \tau \to 0 \) and \( \tau \to \infty \):

- For \( \tau \to 0 \) the medium can be called *optically thin*, since for small \( \tau \), \( e^{-\tau} \approx 1 + \tau \), so \( (1 - e^{-\tau}) \to \tau \). The received flux will thus be
  \[ F_{\oplus, \tau \to 0} = \pi rh(r\alpha_{\nu}) \frac{P_{\nu}}{\alpha_{\nu}} \frac{1}{4\pi D^2} = \pi r^2 hP_{\nu} \frac{1}{4\pi D^2}. \]

  There is no absorption and all the emitted flux will be received.

- For \( \tau \to \infty \) the medium is called *optically thick*, since for \( \tau \to \infty \), \( e^{-\tau} \to 0 \), therefore \( (1 - e^{-\tau}) \to 1 \). The received flux on Earth will change to
  \[ F_{\oplus, \tau \to \infty} = \pi rh \frac{P_{\nu}}{\alpha_{\nu}} \frac{1}{4\pi D^2} \]

  Since \( \alpha_{\nu} \propto \tau \), \( F_{\oplus, \tau \to \infty} \to 0 \): the emitted flux will be absorbed within the effective volume \( \frac{\pi rh}{\alpha_{\nu}} \tau \).
4.2 My Code

This chapter comprises the results of my simple conical self-absorbed jet model. The detailed code can be found in appendix A. The code starts by computing the synchrotron radiation emitted by one electron (figure 4.4). From this the synchrotron radiation emitted per unit volume by a power law distribution of electrons is calculated (figure 4.5). Finally the optical depth of the jet is considered by means of the synchrotron self-absorption effect. The end result (figure 4.6) is a flat radio spectrum, which starts to look like what we pick up from BHBs when they have formed jets. In reality the radio spectrum is inverted ($\alpha > 0$) but for this, relativistic effects need to be taken into account, as well as a weak acceleration component.

Figure 4.4: The 10-log of the spectrum emitted by one electron under synchrotron with a magnetic field of $10^5$ Gauss, at three values of gamma: 10, 100 and 1000. Clearly the energy of the radiation increases with gamma.

The code assumes equipartition of the power $Q_j$ going into the jet: it is divided equally between the magnetic energy density ($B^2/8\pi$) and the total particle energy density $\int CE^{-p}EdE$. Dimensional analysis suggests that the power (erg s$^{-1}$) can be converted into an energy density (erg cm$^{-3}$) using

$$\langle U \rangle = \frac{Q_j}{\pi r_0^2 v},$$

where $r_0$ is the local cone radius, $Q_j$ is set at a value of $10^{-3}\dot{m}c^2$ erg s$^{-1}$, with $\dot{m} \sim 10^{18}$ gm s$^{-1}$ and the particle speed is set at $v = 0.3c$. Furthermore the jet cone starts $5 R_g = 5GM/c^2$ from the black hole and has the same base radius. The cone has an opening angle of $10^\circ$ and a total length of $10^{14}$ cm. For the particle distribution index $p$ a value of 2.2 is used.

The jet is divided in fifty steps. Under the assumption of equipartition, the constant $C$ in the power law electron density $N(E) = CE^{-p}$, see equations 4.21, is normalized. Using this constant the code calculates the self-absorption coefficient of every jet component and
then its flux received on Earth. Adding the fifty components resulted in the flat spectrum of figure 4.6. A discussion of why this spectrum is flat and all the components contribute the same flux can be found in section 5.1.

4.3 Agnjet.f

As stated in section 1.3 a lot of research into jets is currently done from the belief that jets found in BHBs resembles the jets found in AGN and the underlying physics are scaleable. For my research I too used a code, *agnjet.f*, that was also first designed for the analysis of AGN jets and later modified for BHB jets. Completely describing this code isn’t part of the project’s objective but code I will state a few include effects: Special relativity including effects such as beaming - making the jet coming towards us brighter than the one moving away. Inverse Compton - upscattering synchrotron pre-shock photons to the hard X-ray regime. It also includes an accretion disk based on the multi-color disk model. This is the ”standard“ model representing a thin accretion disk where each disk segment contributes blackbody radiation of a temperature $T \propto r^{-3/4}$. However this only a weak element in the spectrum.

When Agnjet is ”SLIRP“-ed into ISIS, it constitutes a 39 dimensional parameter space. Although most of these parameters are frozen, around 15 remain variable for the fitting. All the parameters are found in appendix B.2, where I included the final parameter file, *start.par* that delivered the best fit. The details on all the individual parameters are found in the appendix of [32]. Here I will only quote the five basic assumptions on which the code is built, section 2.1 from the same paper.

**Figure 4.5:** The 10-log of the spectrum emitted by a power-law electron density, integrated from $\gamma = 10^{-15}$ to $\gamma = 10^{300}$, and $\gamma = 1$ to $\gamma = 10^{300}$. While the graph integrated from $\gamma \approx 0$ agrees completely with equation 4.26, clearly this approximation breaks down in real life where $\gamma \geq 1$. 
Figure 4.6: Total synchrotron flux received on Earth. The diagram clearly shows optically thick radio emission, amounting in a flat spectrum ($\alpha \sim 0$). Also shown are several components representing the flux coming from a certain volume $\sim \Delta z \pi r_0^2$ along the length $z$ of the jet. The flux coming from the base of the jet is found on the right, and from the end is most left.

1. The total power in the jet scales with the accretion power at the inner edge of the disk, $\dot{M}c^2$

2. The expanding freely and is only weakly accelerated via the resulting pressure gradient

3. The jet contains cold protons carry most of the kinetic energy while leptons do most of radiating

4. Particles are eventually accelerated into a powerlaw distribution

5. The power law is maintained along length of the jet thereafter. In this section we only summarize the key parameters and assumptions of the jet model.

In table 4.1 are the name of the variables in agnjet.f that were not frozen, including a short description. These parameters could be adjusted in order to get a better fit, first by hand using a wrapper for the code, then later with ISIS.

4.4 Fitting GRS1915+105

4.4.1 Fitting outside ISIS

Before I could proceed to fit the GRS1915+105 data with the agnjet.f model, I needed to gather values for some of the parameters used in this code. Since letting ISIS fit the data with all the agnjet parameters unfrozen would yield a multitude of possible solutions.
Table 4.1: Free parameters in agnjet.f and short description. Also their equivalent symbol in [32] is denoted, for comparison with table 5.1.

All these outcomes would be valid in the 39 dimensional parameter space comprised by the ISIS/agnjet combination, however of no scientific use. So to limit the possible outcomes, I used the following values (from section 3.1).

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>error (±)</th>
<th>units</th>
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</tr>
</thead>
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<td>4</td>
<td>M⊙</td>
<td>mbh</td>
</tr>
<tr>
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<td>2</td>
<td>◦</td>
<td>incl</td>
</tr>
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<td>distance</td>
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<td>kpc</td>
<td>dkpc</td>
</tr>
<tr>
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<td>4455</td>
<td></td>
<td>K</td>
<td>tbb2</td>
</tr>
</tbody>
</table>

Table 4.2: Frozen values used for fitting, obtained from literature. Errors for distance and donor temperature are uncertain and, since they are frozen anyway, not specified.

The values of table 4.2 were entered into a file (grs1915_in.dat), that was read by jetwrap.f - a wrapper that entered these values into agnjet.f and then ran the model. This resulted in ASCII data files of all the different components, which I plotted using gnuplot (see fig 4.7). The real data was also added, allowing me to eyeball an according approximate curve. To make this a little easier I decided not to use all the data points in the infrared. There were ∼ 850 data points in this regime and gnuplot could not plot all these points in a way that the general shape could easily be made out. So I needed to reduce the number of points. At first re-binning was considered, however with mostly errors of less than 1%, the quality of the data was too good - re-binning would result in such tiny error bars that it would be impossible for ISIS to get a decent fit. So for the initial fitting I picked seven points spread throughout the infrared, with the smallest error bars. Once I was satisfied with the results I copied the values from grs1915_in.dat (manually) into an ISIS parameter file (e.g. start.par, see appendix B.2), which could be loaded by ISIS. ISIS then supplied me with an estimate of goodness of the fit, quantified with the reduced chi squared (χ₂red).
method. The objective was to try and get a decent $\chi^2_{\text{red}}$ value of $\lesssim 10$, before letting ISIS fit the available data. The value needed to be of this order of magnitude as to avoid false local minima that fitting with such a complicated model sometimes produces [32].

![Figure 4.7: Example of an outcome combining jetwrap.f with agnjet.f and gnuplot. The separate components plotted are the synchrotron radiation before and after the jet shockwave and the comptonization, i.e. the result of the Compton up-scattering of the pre-shock synchrotron. Also the totals of the separate and combined components are shown. This plot represents the starting values used for the final three fits.](image)

4.4.2 Fitting with ISIS, the Interactive Spectral Interpretation System.

The main difference between fitting with the wrapper and ISIS, was the fact that ISIS unfolds the model through the detector response matrices, compensating for the fact that actual detectors would measure data slightly different, due to physical limitations of the equipment. ISIS also incorporated absorption effects, which were most important at lower frequencies, as can be seen from e.g. figure 4.10, cf. the caption. ISIS only used the PCA data in the 3-22 keV range. For higher energy X-rays - between 20-200 keV - HEXTE data was used.

The first run I manually got the $\chi^2_{\text{red}}$ as low as 30 and decided to let it run in ISIS, starting with a tolerance level of five. When ISIS couldn’t find a better fit anymore I reduced the tolerance level to three and then to one. After 10 days and with a tolerance of .5 the $\chi^2_{\text{red}}$ reached its lowest value of $\sim 3.9$:

Parameters[Variable] = 39[15]
- Data bins = 123
- Chi-square = 425.5925
- Reduced chi-square = 3.940671

When I ran the acquired values in the wrapper again to see the separate components, a discrepancy surfaced between the curve drawn by ISIS and the wrapper/gnuplot (cf. plots in
Figure 4.8: The normalization from the wrapper deviated by a factor of two from that coming out of ISIS. The top panel and the bottom panel are screenshots of gnuplot and ISIS respectively (note the different units on the horizontal axes). From the bottom panel it is also clear that the residuals in the infrared were largest.
Figure 4.9: Estimating the donor temperature from the approximate slope of a blackbody. In the Rayleigh-Jeans regime, a black body has a slope of $\sim 2$ in a frequency-intensity (in mJy) plot. Plotting this slope through the infrared data (the purple line parallel to the black line of slope 2) and estimating the peak (red line), yields an according frequency of $\sim 10^4.5$. Using Wien’s displacement law (in terms of frequency: $v_{\text{max}} = 2.82 kT/h$), gives a temperature $T$ of $\sim 5.4 \times 10^3$ K. This is an increase of $\sim 20\%$ over $4455$ K. In this plot the total and synchrotron dots are not really relevant, but give an indication of their general shapes.

factor two difference had only been present when slurping agnjet.f into more recent versions of ISIS and the bug had yet to be located and removed.

After this first run the residuals in the infrared showed that this region would be hardest to fit. Usually the donor star adds a black body curve to the intensity in the infrared forcing the curve to slope upwards. However - estimating from the slope, see figure 4.9 - the datapoints indicated that to agree with this shape, the donor star would require a surface temperature of $\sim 5.4 \times 10^3$ K. This value exceeds that expected from a class K-M III giant, as proposed by [24], by $\sim 20\%$. We decided to run three more fits:

1. A fit using only two of the lowest points in the infrared, thereby neglecting the total shape/slope here, but including infrared from the jets or possibly the accretion disk. I will refer to this run as fit_two_low. The results of this run are shown in figure 4.10.

2. A fit with the temperature and normalisation of the donor temperature as free parameters. An extra data point in the upper end of infrared was included to see what value ISIS required to fit the slope. I will refer to this fit as fit_free_t. The results of this run are shown in figure 4.11.

3. A fit excluding all the infrared data. This component could always be added later. Since the X-ray and radio data are the most important regimes when examining the black
hole states, fitting these regions was instrumental in getting a global impression. I will refer to this fit as fit_no_ir. The results of this run are shown in figure 4.12.

Before these runs were started I went back to fitting with the wrapper and gnuplot. The results of this effort are shown in figure 4.7. I included the thereby obtained starting values in appendix B.1 since they eventually determine what local minimum we end up in. They gave the following $\chi^2_{\text{red}}$ when entered into ISIS:

\[
\begin{align*}
\text{Parameters}[\text{Variable}] &= 39 \{14\} \\
\text{Data bins} &= 120 \\
\text{Chi-square} &= 2358.986 \\
\text{Reduced chi-square} &= 22.25458
\end{align*}
\]

After many days of calculations, again reducing the tolerances from 40 to a minimum of 0.5, the errorbars ISIS had found at 90% confidence level would be in a .save file. The three different runs gave the following values for $\chi^2_{\text{red}}$:

\[
\begin{align*}
\text{fit_two_low:} \\
\text{Parameters}[\text{Variable}] &= 39 \{13\} \\
\text{Data bins} &= 118 \\
\text{Chi-square} &= 261.3726 \\
\text{Reduced chi-square} &= 2.489263
\end{align*}
\]

\[
\begin{align*}
\text{fit_free_t:} \\
\text{Parameters}[\text{Variable}] &= 39 \{15\} \\
\text{Data bins} &= 119 \\
\text{Chi-square} &= 428.7876 \\
\text{Reduced chi-square} &= 4.122958
\end{align*}
\]

\[
\begin{align*}
\text{fit_no_ir:} \\
\text{Parameters}[\text{Variable}] &= 39 \{13\} \\
\text{Data bins} &= 116 \\
\text{Chi-square} &= 256.863 \\
\text{Reduced chi-square} &= 2.493816
\end{align*}
\]

The according obtained parameter files with values and errorbars are included in section B.2.
Figure 4.10: Results of fit_two_low. The entire spectrum (top) and only the X-ray (bottom). The separate components are data files created by the wrapper, and included for clarity. Notice that in certain regions the total intensity, as calculated by the wrapper, overshoots the ISIS value. This indicates the difference between values acquired by using only the model, and those by forward-folding the model through the detector response matrices. The 'wiggles' in the bottom residuals, between 5 and 20 keV, might be helped by using models that include more GR effects.
Figure 4.11: Results of fit_free_t. The entire spectrum (top) and only the X-ray (bottom). Also see the caption of figure 4.10.
Figure 4.12: Results of fit_no_ir. The entire spectrum (top) and only the X-ray (bottom). Also see the caption of figure 4.10.
Results and Discussion

Although the values of the $\chi^2_{red}$ from the fits in the previous chapter are still quite high ($\gtrsim 2.5$), we have to bear in mind that the data we are fitting here is fairly complex. To improve upon these fits more time is needed: first, to try different starting points, since for the moment we seem to be stuck in a (false) local minimum, and second, to try different models which include more GR effects. The lack of certain GR effects in the used models can cause the ‘wiggles’ as those seen in the residuals of the bottom panel of figure 4.10.

To verify if the fits have resulted in realistic values, the distance between locus of the pre-shock synchrotron and (up-scattered) compton peak can be compared. According to [45], eq (2.9), the energy of a scattered photon can only be greater by a maximum factor of $\sim 4\gamma^2$. $\gamma$ can be deduced from the obtained electron temperature $T_e \sim 3 \times 10^{10}$ K (cf. table 5.1). Most electrons in a Maxwell-Boltzmann distribution are at a temperature of $3kT \sim 3 \times 1.38 \times 10^{-16} \times 3 \times 10^{10}$ K $\sim 7.8 \times 10^3$ keV. As the electrons have a rest mass of 511 keV, $\gamma \sim 15$ and $\gamma^2 \sim 225$. Looking at the plots, the synchrotron and compton peaks are at $\sim 10^{-2}$ and $\sim 2$ keV respectively, giving a separation of $\sim 200$, so no problems here.

As indicated in the previous section, the temperature needed to fit the slope portrayed by the infrared data, exceeds that of a main sequence class K-M III giant. This no reason to doubt the classification done by [24]. When ISIS was fitting fit_free_t for comparison, another value for the donor temperature was obtained - valid for the starting values in B.1. This value was $\sim 4.9 \times 10^4$ K and is an order of magnitude larger than the manual estimate. However, the quality of this fit was very low, with a $\chi^2_{red} \sim 4$. Also ISIS was not able to reduce the error bars any further then the range $4.8 \times 10^3 - 10^5$ K. So, although this range does exclude the main sequence value and includes the estimate value, it can hardly be regarded as evidence.

While the temperature of 4455 K is the predominant value for a class 2 star of this type - on the main sequence - GRS1915+105’s companion is in a state of Roche lobe overflow. Although research into stars in this state and the effects on their properties is still in its infancy, related evidence has been found. [44] also found discrepancies in the effective temperature of the donor star in high mass X-ray binaries. Here the deviation could be resolved by increasing the effective temperature with 10-25%, coinciding with the aberration of the estimate.
### Results and Discussion

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<td>4.4 - 9.1</td>
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<td>1740$^{+1}_{-1}$</td>
<td>1776$^{+10}_{-10}$</td>
<td>1707$^{+10}_{-10}$</td>
</tr>
<tr>
<td>$L_{\text{disk}}$</td>
<td>$10^{-3} L_{\text{Edd}}$</td>
<td>0.1, 0.33, 99</td>
<td>2.4, 0.8</td>
<td>49.5$^{+0.7}_{-0.6}$</td>
<td>50.3$^{+0.7}_{-0.6}$</td>
<td>48.6$^{+0.7}_{-0.7}$</td>
</tr>
<tr>
<td>$T_{\text{disk}}$</td>
<td>keV</td>
<td>0.06, 0.36, 1.53</td>
<td>0.71 - 0.98</td>
<td>1.289$^{+0.011}_{-0.010}$</td>
<td>1.299$^{+0.011}_{-0.010}$</td>
<td>1.290$^{+0.010}_{-0.010}$</td>
</tr>
<tr>
<td>Goodness of fit</td>
<td>$\chi^2_{\text{red}}$</td>
<td>unknown</td>
<td>unknown</td>
<td>2.489263</td>
<td>4.122958</td>
<td>2.493816</td>
</tr>
</tbody>
</table>

**Table 5.1:** Values found for variables in the agnjet.f code using ISIS. The specified ranges for GX339-4 and Cyg X-1 were found in [32]. For both BHBs, the values represent three different observation dates (Observation ID 20181-01-02, 40108-02-01 and 40108-02-03 for GX 339-4, and 40099-01-19, 60090-01-26 and 60090-01-41 for Cyg X-1), during which they are believed to have been in the hard state. The variables denoted with a $\dagger$ are the mean of these observations, since these ranges were very narrow. The error bars have been resolved at 90% confidence level.
Before comparing the results to previous results obtained with the same code, it also should be noted that, for final refinements, the length of the jets, e.g. $\sim 10^{14}$ cm, should also be considered and another special relativistic effect comes into play. This could be compensated for by using jet data that was taken $10^{14}/c \approx 3$ hours later than that coming from the accretion disk.

Comparing the values in table 5.1 for GRS1915+105 to Cyg X-1 and GX 339-4 it immediately becomes clear that regarding this threesome, GRS1915+105 is the most extreme:

- The luminosity of GRS1915+105 is strikingly larger than that observed from the other two black holes. The value for $N_j$ is two to three magnitudes larger and also $L_{\text{disk}}$ greatly exceeds that found in its relatives, save during one observation of GX 339-4. Canonical black holes displaying these luminosity levels would long have shut down their jets, yet somehow GRS1915+105 seems to be able to sustain them.

- GRS1915+105 is much larger then the other two BHBs: This is obvious from the radius of the base of the jet $r_0$ and the location of the jet shockwave $z_{\text{acc}}$. While the former is two times larger in GX 339-4 then for Cyg X-1, GRS1915+105’s jet base is even six times larger then that of GX 339-4. Looking at the location of the shockwave the differences are even more outspoken: revealing a ratio of 1:10:150 in the same order. GRS1915+105 is undeniably a huge structure and also the biggest black hole in a binary known to date (cf. figure 5.1). This size is probably directly linked to its luminosity.

- Another remarkable result is the high value of $k$, indicative of extreme magnetic energy domination in the jets of GRS1915+105. For the other holes, this constant remains around two or lower, which agrees far better with the assumption of equipartition between magnetic and kinetic energy. I propose that this magnetic domination is required to sustain the jets at high luminosity.

- Also $f_{\text{sc}}$ is quite high - yet still feasible - moving the synchrotron cut off to the left in the spectrum. The comptonization can dominate the harder x-rays allowing a fit for the extremely steep x-ray data in the plateau state.

- Lastly, the accretion disk temperature seems to be a bit higher then for the other two black holes (accept on one occasion for GX 339-4). This corroborates the higher luminosity found in GRS1915+105

![Figure 5.1: Schematic figure of size and distance to the donor star of the black holes compared in this section, according to [5] (created by J. Oros). The tilt of the accretion disks and the donor color represent the estimated inclination and surface temperature, respectively.](image-url)
Results and Discussion

On the other hand, not all values describing GRS1915+105’s properties surpass those of the other two black holes.

- Concerning the temperature of the electrons going into the jet, GRS1915+105 seems to have a much lower value. This can be understood considering its size. The electrons have much more time to cool before they get pumped into the jet.

- Also the particle distribution index \( p \) is a little lower. Converted to the photon index, \( p \sim 2 \) gives a value of \( \Gamma \sim 1.5 \), within the values that [5] state for the Hard State, and just within those for the Quiescent State. However the flux level implied by the extremely high \( N_j \) contradicts the latter state. \( p \) is an important parameter and it value remained more or less the same throughout the fitting process. The final value did not diverge much from the value it had from the beginning, when \( N_j \) was first adjusted to get a high enough normalization. If \( p \) was increased to \( \sim 2.4 \), while simultaneously boosting \( N_j \) to retain get a high enough normalization, another decent (maybe even better) fit might be found. Exploring more parameter space like this was neglected due to the limited time available for the project.

Also some values seem to coincide with earlier results. \( pl_f \) was frozen for these runs but the value used was 0.79 (see section B.2). Although this value was obtained by eyeballing the data, it seems to be consistent with with the values in [32], supporting their decision to freeze this value. The value is indicative of efficient processes, as a high fraction of the thermal distribution is put into the power law.

5.1 Why the Flat Spectrum is Flat

All the peaks from all the components throughout the jet calculated in my code, save the one from the base of the jet, result in an equal level of flux, yielding a perfectly flat spectrum. This fact is supported by a dimensional analysis that I will submit in this section.

Since the magnetic \( B^2/8\pi \) and particle energy density \( \int C E^{-p} EdE \) are conserved, they decrease as \( 1/r^2 \). This gives for the magnetic field a dependency of \( 1/r \). According to equation 4.30

\[
\alpha_\nu \propto CB^{(p+2)/2} \nu^{-(p+4)/2}.
\]

Setting a value for \( p = 2 \), the particle density integral yields a constant, indicating \( C \) is proportional to \( B^2 \). Putting all this into the previous equation yields

\[
\alpha_\nu \propto B^4 \nu^{-3} \propto r^{-4} \nu^{-3}.
\]

From equation 4.32, \( \tau \sim r \alpha_\nu \), so that at the photosphere, where \( \tau = 1 \) - per definition - \( 1 \propto r r^{-4} \nu^{-3} = r^{-3} \nu^{-3} \) and thus \( r \propto \nu^{-1} \). From equation 4.26, the emitted power per volume per Hz is

\[
P_{tot}(\nu) \propto CB \left( \frac{\nu}{B} \right)^{(p-1)/2},
\]

meaning that the flux at the photosphere is (multiplying by the emitting volume)

\[
F_\nu \propto B^3 \left( \frac{\nu}{B} \right)^{-1/2} r^3 \propto (r \nu)^{-0.5}
\]
and therefore, employing $r \propto \nu^{-1}$, a constant and independent of frequency. This shows that the flux received on earth from each component should be the same, as this only adds a factor of $1/(4\pi D^2)$ to the photosphere flux. As the magnetic and particle energy density decrease along the jet, the emitting volumes increase in size, just enough to keep the flux coming from one component steady.
Conclusions

The most remarkable results from the fits are the exorbitant values for the luminosity and the partition of jet-energy. As jets in the canonical black holes would not be able to persist at these luminosity levels, the implication is that the magnetic energy dominance has something to do with this. Although far more research needs to be done before any strong conclusions can be made, this is an interesting result to say the least.

Clearly we are still far removed from completely understanding GRS1915+105. Indeed, from the steepness of the x-ray data and the presence of optically thick radio emissions, GRS1915+105 was in the plateau state on the night of July 8th 1999. Despite the persistence of the power law tail in the plateau state, this state bears many characteristics of the hard state in the canonical black holes. This can mainly be inferred from the fact that radio emissions imply the presence of quasi-steady jets, and from the obtained flat value for the spectral index.

These observations support the recent trend of abandoning the classification of black hole states considering their luminosity. This previous, largely phenomenological approach to the description of X-ray states does not provide a satisfactory classification of the states displayed by GRS1915+105. Because of its high luminosity all the states are lifted into the Very High State. However some leeway is gained, when releasing the classic method, allowing the plateau state to be interpreted as GRS1915+105’s version of the Hard State.
Future Work

As the results in this paper show that a lot more work needs to be done before the combined data of July 8th 1999 can be understood. Before we come close to this goal at we need to delve deeper into the following points, as for this project there wasn’t any more time available.

- The parameter space needs to be further explored: Some values available in `agnjet.f` were frozen from the start, and set according to previous experiences. GRS1915+105 is in many ways a peculiar BHB and it is therefore not far fetched to state that this black hole might require entirely different values to get the model and the data to agree.

- More fitting needs to be done using the parameters that were left unfrozen, e.g. a higher value for $p$ could be explored, adjusting $N_j$ to suit the normalization needs of the data.

- What is the temperature of the donor star? What results do we get with the estimated temperature of $5.4 \times 10^3$ K?

The results of all these future tasks will amount in an article - probably in the form of a letter - written by Sera Markoff, me, Castro-Tirado, Harlaftis and Dhawan.
I would like to sincerely thank the following people:

Sera Markoff, for showing me that science and astronomy can indeed be combined with rock & roll and thereby inspiring me to go get my masters degree.

Dipankar Maitra for the plotting script to plot the components in gnuplot, and for his extensive help. Even during the hectic days before his wedding Dipankar made himself available to me. Again, congratulations, and may you have a happy life and grow old together.

Arjan Verhoeff for the discussion about rebinning, although this did not appear to be required in the end.

Martin Heemskerk and Lianne Muijres, for letting me run ISIS on their computer, in spite of the fact that they were using it at the time.
Appendix A
Synchrotron Emulation

Different subroutines were placed in different files, and included from main.h:

Main include file main.h

```c
#include <stdio.h>
#include <math.h>
#include "params.h"
#include "constants.h"
#include "splineunitoffset.h"
#include "splint.h"
#include "plot.h"
#include "writedata.h"
#include "getdata.h"
#include "qromb.h"

//numerical recipes//
#include "nr.h"
#include "nrutil.h"
```

Parameters params.h

```c
#define ALEN 280
#define BINS 100
```
**Constants constants.h**

#define PI 3.141592653589793
#define ME 9.108E-28 //grams
#define QE 4.803E-10 //statCoulomb
#define CSPEED 2.998E10 //cm per s
#define KBOLTZ 1.38E-16 //erg per K
#define GCONST 6.67259E-8 //dyne-cm^2 gm^-2
#define MSOLAR 1.99E33 //gm
#define PARSEC 3.086E18 //cm
The main code main.c

#include "main.h" //every c file has its own h file and these are all included

#define pspec 2.2
#define alfa PI/2.
#define B 1E5 //Gauss
#define gammaonethird 2.678938534707747633655692940974677644128689377957301100
#define mJy 1E-26
#define JETSTEP 50
#define EPSILON 1E-3
#define PRECISION -35

// the main program

int main() {
  int i,j;
  double numin=1.E7,numax=1.E20,nu[BINS]; //frequency range
  double Pconst=sqrt(3.)*QE*QE*QE*B*sin(alfa)
       /(ME*CSPEED*CSPEED); //the constant in P(nu)
  double empty[BINS]; //empty array for plotting

  //filling nu array evenly over frequency range
  nu[0]=log10(numin);
  double stepsize=log10(numax/numin)/(BINS-1);
  for(i=0;i<BINS;i++)
  {
    nu[i+1]=nu[i]+stepsize;
    //printf("i=%d,nu=%f,xnu=%e\n",i,nu[i]);
  }

  //getting data from pre-stored table and splining
  //to get smooth synchrotron F(X), for x~1
  float xget[ALEN],fxget[ALEN]; //array of second derivatives
  float *xp,*fp; //changing null-based to unit-based arrays
  getdata(xget,fxget); //read data from table "fxtable.dat"
  xp=xget-1;
  fp=fxget-1;
  for(i=1;i<ALEN;i++) //for testing values picked up by getdata
  {
    // printf("xp[%d]=%g,fp[%d]=%g\n",i,xp[i],i,fp[i]);
  }
  spline(xp, fp, ALEN, 0, 0, y2);
  init_splint_pieter(xp,fp,y2, ALEN);

  double power[BINS];
}
double nu_c;
float x; //LAAT DEZE OP FLOAT STAAN
double gam;
double logpow_tot[BINS];

//function describing electron power per particle as a function of wavelength nu
//for every nu, x is calculated for all values of gamma (through nu_c/integration)

float FofX(float gam)
{
    nu_c=3*QE*B*sin(alfa)*pow(gam,2)/(2*PI*2*ME*CSPEED);
    x=pow(10,nu[j])/nu_c;
    //printf("nu_c=%e x=%e\n",nu_c,x);
    if(x>0)
    {
        //printf("test x x=%e\n",x);
        if(x<=1E-2)
        {
            //printf("using x<1 x=%e\n",x);
            return pow(x/2.,1./3.) * (4*PI/(sqrt(3.)*gammaonethird));
        }
        else if(x>=1E2)
        {
            //printf("using x>1 x=%e\n",x);
            return sqrt(PI/2.)*exp(-x)*sqrt(x);
        }
        else
        {
            //printf("using spline x=%e\n",x);
            return splint_pieter(x);
        }
    }
    else{
        return 0.0;
    }
}

/* some testplots for the power emitted by one electron
   at certain value of gamma and B=10^5 gauss */
double gamma,gamE;
double logpow_one_gam10[BINS],logpow_one_gam100[BINS],logpow_one_gam1000[BINS];
j=0;
//gamma=10;
logpow_one_gam10[0]=log10(Pconst*FofX(10)/mJy);
for(j=0;j<BINS;j++)
{ 
    if(logpow_one_gam10[j]>PRECISION) 
    { 
        logpow_one_gam10[j]=log10(Pconst*FofX(10)/mJy); 
    } 
    else 
    { 
        logpow_one_gam10[j]=0.0; 
    } 
}

//gamma=100; 
logpow_one_gam100[0]=log10(Pconst*FofX(100)/mJy); 
for(j=0;j<BINS;j++) 
{ 
    if(logpow_one_gam100[j]>PRECISION) 
    { 
        logpow_one_gam100[j]=log10(Pconst*FofX(100)/mJy); 
    } 
    else 
    { 
        logpow_one_gam100[j]=0.0; 
    } 
}

//gamma=1000; 
logpow_one_gam1000[0]=log10(Pconst*FofX(1000)/mJy); 
for(j=0;j<BINS;j++) 
{ 
    if(logpow_one_gam1000[j]>PRECISION) 
    { 
        logpow_one_gam1000[j]=log10(Pconst*FofX(1000)/mJy); 
    } 
    else 
    { 
        logpow_one_gam1000[j]=0.0; 
    } 
}

/*power of one electron, testplot*/ 
plot1name(BINS,nu,logpow_one_gam10,"log power one electron"); 
plot3name(BINS,nu,logpow_one_gam10,"log power one electron at gamma=10" 
 ,logpow_one_gam100,"log power one electron at gamma=100" 
 ,logpow_one_gam1000,"log power one electron at gamma=1000");
float powerintFofX(float gammac)
{
    gam=exp(gammac);
    return gam*FofX(gam)*pow(gam,-pspec);
}

//Here the total power from a power law distribution, at a given frequency 
// is calculated
double gamma_1=1,gamma_2=1.E300; //removed gamma_1=1E-15=>gamma=0
for(j=0;j<BINS;j=j+1)
{
    logpow_tot[j]=log10(qromb(powerintFofX,log(gamma_1),log(gamma_2))*Pconst/mJy);
    printf("nu[%d]=%e logpow_tot=%e\n",j,nu[j],logpow_tot[j]);
}

/*/total power testplot*/
plot1name(BINS,nu,logpow_tot,"total power emitted");


double Bfield[JETSTEP];

float FofXE(float E)
{
    //Epass=exp(E); //already expanded E in functions calling this function
    gamE=E/(ME*CSPEED*CSPEED);
    nu_c=3*QE*Bfield[i]*sin(alfa)*pow(gamE,2)/(2*PI*2*ME*CSPEED);
    x=pow(10,nu[j])/nu_c;
    //printf("nu_c=%e x=%e\n",nu_c,x);
    if(x>0)
    {
        //printf("test x x=%e\n",x);
        if(x<=1E-2)
        {
            //printf("using x<1 x=%e ",x);
            return pow(x/2.,1./3.) * (4*PI/(sqrt(3.)*gammaonethird));
        }
        else if(x>=1E2)
        {
            //printf("using x>>1 x=%e ",x);
            return sqrt(x*PI/2.)*exp(-x);
        }
        else
        {
            //printf("using 1<<1 x=%e ",x);
            return pow(x/2.,1./3.) * (4*PI/(sqrt(3.)*gammaonethird));
        }
    }
}

//printf("using spline  x=%e ",x);
    return splint_pieter(x);
}
}
else{
    return 0.0;
}
}

double Ebot=ME*CSPEED*CSPEED; //before Etop=1.30;
double Etop=1E20*ME*CSPEED*CSPEED; //zero~1E-20
double Ecalc;

float absorpint(float E)
{
    Ecalc=exp(E);
    gamE=Ecalc/(ME*CSPEED*CSPEED);
    return pow(Ecalc,-pspec)*gamE/Ecalc*FofXE(Ecalc); //extra factor E for log
    //integral cancels out
    //1/E under N(E) in (6.52)
}

//calculating absorption at a given frequency
// printf("calc absorp\n");
double absorp[BINS],logabsorp[BINS];
for(j=0;j<BINS;j++)
{
    absorp[j]=qromb(absorpint,log(Ebot),log(Etop))*(pspec+2)
        *(sqrt(3.)*QE*QE*QE*B*sin(alfa))*(CSPEED*CSPEED/(8*PI*pow(10,2*nu[j])));
    logabsorp[j]=log10(absorp[j]);
    printf("nu[%d]=%e absorp_nu=%e logabsorp=%e
",j,nu[j],absorp[j],logabsorp[j]);
}

/*/plotting absorption coefficient*/
plot1name(BINS,nu,logabsorp,"absorption coefficient");

///////////
/*jet model*/
///////////

double M=14*MSOLAR;
double rg=GCONST*M/(CSPEED*CSPEED);
double r[JETSTEP],z[JETSTEP];
double magnergdens[JETSTEP];
double U[JETSTEP]; //total particle erg dens
double C[JETSTEP];
double vspeed=.3*CSPEED;
double zmax=1E14;
double r0=5*rg,z0=5*rg;  //cm
double mdot=1E18;
double opangle=10*PI/180;

double Qjet=EPSILON*mdot*CSPEED*CSPEED;  //power going into jets:
double powperjet=Qjet/2.;  //there are two jets
double powforpart=powperjet/2.;  //half jetpower-> particles
double powformagn=powperjet/2.;  //half jetpower-> magnetic

/*integral over all electron energies*/
double Emin=ME*CSPEED*CSPEED;  //before Emax=1E300;
double Emax=1E20*ME*CSPEED*CSPEED;  //before zero~1E-25
double Epass;
float allelerg(float E)
{
    Epass=exp(E);
    return Epass*pow(Epass,-pspec)*Epass;  // # of electrons in powerlaw,
                                        //times their energy
    }  // + extra factor E for logint

r[0]=r0;
z[0]=log(z0);
printf("z[0]=%g\n",z[0]);
magnergdens[0]=powformagn/(PI*r[0]*r[0]*vspeed);
Bfield[0]=sqrt(magnergdens[0]*8*PI);
U[0]=powforpart/(PI*r[0]*r[0]*vspeed);
C[0]=U[0]/qromb(allelerg,log(Emin),log(Emax));

double zstep=log(zmax/z0)/(JETSTEP-1);

printf("M=%.3e gr rg=%.3e cm r0=%.3e r[0]=%.3e cm\n",M,rg,r0,r[0],zmax);
printf("z[0]=%.3e cm zstep=%.3e Qjet=%.3e erg/s \n",exp(z[0]),zstep,Qjet);

for(i=1;i<JETSTEP;i++)
{
    z[i]=z[i-1]+zstep;
    r[i]=r[i-1]+(exp(z[i-1]+zstep)-exp(z[i-1]))*tan(opangle);
magnergdens[i]=magnergdens[i-1]*r[i-1]*r[i-1]/(r[i]*r[i]);
    Bfield[i]=sqrt(magnergdens[i]*8*PI);
    U[i]=U[i-1]*r[i-1]*r[i-1]/(r[i]*r[i]);
    C[i]=U[i]/qromb(allelerg,log(Emin),log(Emax));
}

/*/to check the numberdensity*/
double N[JETSTEP];

float numdens(float E)
{
    Epass=exp(E);
    return Epass*pow(Epass,-pspec); // # of electrons in powerlaw, times
} // their energy + xtra factor Epass
    // for logscale => N(E)=C[i]*E^-pspec

for(i=0;i<JETSTEP;i++)
{
    N[i]=C[i]*qromb(numdens,log(ME*CSPEED*CSPEED),log(Emax));
}

float powerintFofXE(float E)
{
    Epass=exp(E);
    // gamE=Epass/(ME*CSPEED*CSPEED); // do not need gamma yet. Do this in FofXE.
    return pow(Epass,1-pspec)*FofXE(Epass); // put the extra factor E for log integral
} // in the powerlaw

double opdepth,h;
double synchro[BINS],logsynchro[BINS];
double flux[BINS];
double D=11.E3*PARSEC;
double synchroatz[JETSTEP][BINS],fluxatearth[JETSTEP][BINS],absorpC[JETSTEP][BINS];

/* check the above arrays*/
for(i=0;i<JETSTEP;i++)
{
    h=(exp(z[i]+zstep)-exp(z[i]));
    // printf("dz=%e
",h);
    printf("dz=%e z[%d]=%.3e cm r=%.3e cm Bfield=%.3e G U=%.3e C=%.3e N=%.3e\n"
        ,h,i,exp(z[i]),r[i],Bfield[i],U[i],C[i],N[i]);
}

/////////////////////////////////////////////////////////////////////////////////
// setting optical/skin depth
/////////////////////////////////////////////////////////////////////////////////

double fluxatE(double r,double abs,double z,double Pnu)
{
    h=(exp(z+zstep)-exp(z));
    opdepth=r*abs;
    return PI*r*h*(1-exp(-opdepth))*Pnu/abs*(1/(4*PI*D*D*mJy)); // exp long double!!
}
/* filling absorption, synchrotron and flux arrays with zeroes */ 
for(i=0;i<JETSTEP;i++)
{
  for(j=0;j<BINS;j++)
  {
    absorpC[i][j]=0.;
    synchroatz[i][j]=0.;
    fluxatearth[i][j]=0.;
  }
}

printf("Calculating synchrotron radiation/evolution throughout jet\n");

/* calculating absorption throughout jet*/
for(i=0;i<JETSTEP;i++)
{
  for(j=0;j<BINS;j++)
  {
    absorpC[i][j]=C[i]*((pspec+2)*CSPEED*CSPEED/(8*PI*pow(10,2*nu[j]))
      *qromb(absorpint,log(Ebot),log(Etop))
      *(sqrt(3.)*QE*QE*Bfield[i]*sin(alfa)));
    //printf("i=%d j=%d abs=%e\n",i,j,absorpC[i][j]);
  }
}

/* calculating synchrotron throughout jet*/
for(i=0;i<JETSTEP;i++)
{
  for(j=0;j<BINS;j++)
  {
    synchroatz[i][j]=sqrt(3.)*QE*QE*Bfield[i]*sin(alfa)*C[i]
      *qromb(powerintFofXE,log(ME*CSPEED*CSPEED),log(1E30))
      /(ME*CSPEED*CSPEED);
    //printf("i=%d j=%d abs=%e synchro=%e\n",i,j,absorpC[i][j],synchroatz[i][j]);
  }
}

/* calculating flux arriving at earth from different places in jet*/
for(i=0;i<JETSTEP;i++)
{
  for(j=0;j<BINS;j++)
  {

{  
  fluxatearth[i][j]=fluxate(r[i],absorpC[i][j],z[i],synchroatz[i][j]);  
}

;/* checking absorption, synchrotron and earth flux*/
double logfluxbase[BINS],logflux1[BINS],logflux2[BINS],logflux3[BINS],logfluxend[BINS];
double logsynchrobase[BINS],logsynchro1[BINS],logsynchro2[BINS],logsynchro3[BINS],logsynchroend[BINS];
double logabsbase[BINS],logabs1[BINS],logabs2[BINS],logabs3[BINS],logabsend[BINS];
double logabsorptimesr[BINS];
double exbase[BINS],ex1[BINS],ex2[BINS],ex3[BINS],exend[BINS];
double exbaseb[BINS],ex1b[BINS],ex2b[BINS],ex3b[BINS],exendb[BINS];

for(j=0;j<BINS;j++)
{
  i=0;
  logfluxbase[j]=log10(fluxatearth[i][j]);
  i=1.*(JETSTEP-1)/6.;
  printf("%d\n",i);
  logflux1[j]=log10(fluxatearth[i][j]);
  i=3.*(JETSTEP-1)/6.;
  printf("%d\n",i);
  logflux2[j]=log10(fluxatearth[i][j]);
  i=5.*(JETSTEP-1)/6.;
  printf("%d\n",i);
  logfluxend[j]=log10(fluxatearth[i][j]);

  logsynchrobase[j]=log10(synchroatz[0][j]*(1/(4*PI*D*D*mJy)));
  logsynchro1[j]=log10(synchroatz[1][j]*(1/(4*PI*D*D*mJy)));
  logsynchro2[j]=log10(synchroatz[2][j]*(1/(4*PI*D*D*mJy)));
  logsynchro3[j]=log10(synchroatz[3][j]*(1/(4*PI*D*D*mJy)));
  logsynchroend[j]=log10(synchroatz[JETSTEP-1][j]*(1/(4*PI*D*D*mJy)));

  logabsbase[j]=log10(absorpC[0][j]);
  logabs1[j]=log10(absorpC[1][j]);
  logabs2[j]=log10(absorpC[2][j]);
  logabs3[j]=log10(absorpC[3][j]);
  logabsend[j]=log10(absorpC[JETSTEP-1][j]);
  //logabsorptimesr[j]=log10(absorpC[0][j]*r[i]);
}

;/* adding it all up to get total synchrotron received at Earth*/
double totsynchro[BINS],logtotsynchro[BINS];
for(j=0;j<BINS;j++)
{
  


totsynchro[j]=0;
for(i=0;i<JETSTEP;i++)
{
    totsynchro[j]=totsynchro[j]+fluxatearth[i][j];
    //printf(" i=%d totsyn[%d]=%e\n",i,j,totsynchro[j]);
}
logtotsynchro[j]=log10(totsynchro[j]);
//printf("j=%d logtotsynchro=%e\n",j,logtotsynchro[j]);

/*various plots / data table writes*/
plot5name(BINS,nu
    ,logtotsynchro,"total synchrotron on earth from one jet, divided in 50 steps"
    ,logfluxbase,"synchrotron power recieved on earth from base of jet"
    ,logflux1,"synchrotron power recieved on earth from 1/3 along length of jet"
    ,logflux2,"synchrotron power recieved on earth from 2/3 along length of jet"
    ,logfluxend,"synchrotron power recieved on earth from end of jet");

plot1name(BINS,nu
    ,logtotsynchro,"log total synchrotron on earth from jet, divided in JETSTEP steps");

plot5name(BINS,nu
    ,logfluxbase,"z=0",logflux1,"z=2",logflux2,"z=4",logflux3,"z=6",logfluxend,"z=8");

plot5name(BINS,nu
    ,logabsbase,"z=0",logabs1,"z=2",logabs2,"z=4",logabs3,"z=6",logabsend,"z=8");

writedata("testfile.dat",BINS,nu,logsynchrobase,logsynchro2,logsynchroend);

return 0.0;
The spline routines \texttt{splineunitoffset.c} and \texttt{splint.c}

Copyright Numerical Recipes [43].

#include \texttt{"splineunitoffset.h"}

void spline(float x[], float y[], int n, float yp1, float ypn, float y2[])
{
    int i,k;
    float p,qn,sig,un,*u;
    //printf("n=%d,x[1]=%g\n",n,x[2]);
    u=vector(1,n-1);
    if (yp1 > 0.99e30)
    {
        y2[1]=u[1]=0.0;
    } else
    {
        y2[1] = -0.5;
        u[1]=(3.0/(x[2]-x[1]))*((y[2]-y[1])/(x[2]-x[1])-yp1);
    }
    for (i=2;i<=n-1;i++)
    {
        sig=(x[i]-x[i-1])/(x[i+1]-x[i-1]);
        p=sig*y2[i-1]+2.0;
        y2[i]=(sig-1.0)/p;
        u[i]=(y[i+1]-y[i])/(x[i+1]-x[i]) - (y[i]-y[i-1])/(x[i]-x[i-1]);
        u[i]=(6.0*u[i]/(x[i+1]-x[i-1])-sig*u[i-1])/p;
    }
    if (ypn > 0.99e30)
    {
        qn=un=0.0;
    } else
    {
        qn=0.5;
        un=(3.0/(x[n]-x[n-1]))*(ypn-(y[n]-y[n-1])/(x[n]-x[n-1]));
    }
    y2[n]=(un-qn*u[n-1])/(qn*y2[n-1]+1.0);
    for (k=(n-1);k>=1;k--)
    {
        y2[k]=y2[k]*y2[k+1]+u[k];
    }
    free_vector(u,1,n-1);
}
#include "splint.h"

void init_splint_pieter(float* xa, float* ya, float* y2a, int n) {
    splint_state.xa = xa;
    splint_state.ya = ya;
    splint_state.y2a = y2a;
    splint_state.n = n;
}

float splint_pieter(float x) {
    float* xa;
    xa = splint_state.xa;
    float* ya;
    ya = splint_state.ya;
    float* y2a;
    y2a = splint_state.y2a;
    int n = splint_state.n;

    void nrerror(char error_text[]);
    int klo,khi,k;
    float h,b,a;
    klo=1;
    khi=n;

    while (khi-klo > 1)
    {
        k=(khi+klo) >> 1;
        if (xa[k] > x) khi=k;
        else
            klo=k;
    }

    h=xa[khi]-xa[klo];
    if (h == 0.0) nrerror("Bad xa input to routine splint");
    a=(xa[khi]-x)/h;
    b=(x-xa[klo])/h;

    return a*ya[klo]+b*ya[khi]+((a*a*a-a)*y2a[klo]+(b*b*b-b)*y2a[khi])*(h*h)/6.0;
}

/*
void splint(float xa[], float ya[], float y2a[], int n, float x, float *y) {
    void nrerror(char error_text[]);
    int klo,khi,k;
    float h,b,a;
    klo=1;
    khi=n;
    while (khi-klo > 1)
*/
\{
    k=(khi+klo) >> 1;
    if (xa[k] > x) khi=k;
    else
        klo=k;
    }
    h=xa[khi]-xa[klo];
    if (h == 0.0) nrerror("Bad xa input to routine splint");
    a=(xa[khi]-x)/h;
    b=(x-xa[klo])/h;
    *y=a*ya[klo]+b*ya[khi]+((a*a*a-a)*y2a[klo]+(b*b*b-b)*y2a[khi])*(h*h)/6.0;
}\
The plotting routine plot.c

#include "plot.h"

ẩm://double arrays were not accepted by gplot routine/
iedades/and are therefore first converted to float  ///

/*plotting one function with name*/
void plot1name(int ALEN, double x[], double y[], char *string)
{
    int i;
    float xplot[ALEN], yplot[ALEN], y2plot[ALEN], y3plot[ALEN];
    for (i = 0; i < ALEN; i++){
        xplot[i] = (float) x[i];
    }
    for (i = 0; i < ALEN; i++){
        yplot[i] = (float) y[i];
    }
    gpl_style(LINES);
    gpl_data(ALEN, xplot, yplot, string);
    gpl_show();
}

/*plotting 3 function*/
void plot3(int ALEN, double x[], double y[], double y2[], double y3[])
{
    int i;
    float yplot[ALEN], xplot[ALEN], y2plot[ALEN], y3plot[ALEN];
    for (i = 0; i < ALEN; i++){
        xplot[i] = (float) x[i];
    }
    for (i = 0; i < ALEN; i++){
        yplot[i] = (float) y[i];
    }
    for (i = 0; i < ALEN; i++){
        y2plot[i] = (float) y2[i];
    }
    for (i = 0; i < ALEN; i++){
        y3plot[i] = (float) y3[i];
    }
    gpl_style(LINES);
    gpl_data(ALEN, xplot, yplot, "");
    gpl_data(ALEN, xplot, y2plot, "");
    gpl_data(ALEN, xplot, y3plot, "");
    gpl_show();
}

/*plotting 3 functions with name*/
void plot3name(int ALEN, double x[], double y[],
              char *string1, double y2[], char *string2
              , double y3[], char *string3)
{
    int i;
    float yplot[ALEN], xplot[ALEN], y2plot[ALEN], y3plot[ALEN];
    for(i=0; i<ALEN; i++){
        xplot[i] = (float) x[i];
    }
    for(i=0; i<ALEN; i++){
        yplot[i] = (float) y[i];
    }
    for(i=0; i<ALEN; i++){
        y2plot[i] = (float) y2[i];
    }
    for(i=0; i<ALEN; i++){
        y3plot[i] = (float) y3[i];
    }
    gpl_style(LINES);
    gpl_data(ALEN, xplot, yplot, string1);
    gpl_data(ALEN, xplot, y2plot, string2);
    gpl_data(ALEN, xplot, y3plot, string3);
    gpl_show();
}

/*plotting five functions with name*/
void plot5name(int ALEN, double x[], double y[], char *string1
              , double y2[], char *string2, double y3[], char *string3
              , double y4[], char *string4, double y5[], char *string5)
{
    int i;
    float yplot[ALEN], xplot[ALEN], y2plot[ALEN], y3plot[ALEN], y4plot[ALEN], y5plot[ALEN];
    for(i=0; i<ALEN; i++){
        xplot[i] = (float) x[i];
    }
    for(i=0; i<ALEN; i++){
        yplot[i] = (float) y[i];
    }
    for(i=0; i<ALEN; i++){
        y2plot[i] = (float) y2[i];
    }
    for(i=0; i<ALEN; i++){
        y3plot[i] = (float) y3[i];
    }
    for(i=0; i<ALEN; i++){
        y4plot[i] = (float) y4[i];
    }
    for(i=0; i<ALEN; i++){
        y5plot[i] = (float) y5[i];
    }
}
y5plot[i] = (float) y5[i];
}
gpl_style(LINES);
gpl_data(ALEN, xplot, yplot, string1);
gpl_data(ALEN, xplot, y2plot, string2);
gpl_data(ALEN, xplot, y3plot, string3);
gpl_data(ALEN, xplot, y4plot, string4);
gpl_data(ALEN, xplot, y5plot, string5);
gpl_show();
}
File writing routine for making tables writedata.c

#include "writedata.h"

/////////////////////////////////////////////////////////////////////////
/*routine to write max five columns of data or less if not all supplied*/
/////////////////////////////////////////////////////////////////////////

void writedata(char *writefile, float *rows, double col1[], double col2[], double col3[],
               double col4[], double col5[], double col6[])
{
    int i;
    FILE *wfijl = fopen(writefile, "w");

    for (i=0; i<rows; i++)
    {
        if(col6[i]==0)
        {
            if(col5[i]==0)
            {
                if(col4[i]==0)
                {
                    if(col3[i]==0)
                    {
                        if(col2[i]==0)
                        {
                            fprintf(wfijl, "%.5f\n", col1[i]);
                        }
                        else
                        {
                            fprintf(wfijl, "%.5f %.5f\n", col1[i], col2[i]);
                        }
                    }
                    else
                    {
                        fprintf(wfijl, "%.5f %.5f %.5f\n", col1[i], col2[i], col3[i]);
                    }
                }
                else
                {
                    fprintf(wfijl, "%.5f %.5f %.5f %.5f\n", col1[i], col2[i], col3[i], col4[i]);
                }
            }
            else
            {
                fprintf(wfijl, "%.5f %.5f %.5f %.5f %.5f\n", col1[i], col2[i], col3[i], col4[i], col5[i]);
            }
        }
        else
        {
            fprintf(wfijl, "%.5f %.5f %.5f %.5f %.5f %.5f\n", col1[i], col2[i], col3[i], col4[i], col5[i], col6[i]);
        }
    }
}
Routine to get data from synchrotron table getdata.c

#include "getdata.h"

///////////////////////////////////////////
/*getting synchrotron function table data*/
///////////////////////////////////////////

void getdata(float x[], float fx[])
{  
    int i;
    FILE* fijl = fopen("fxtable.dat", "r");
    for (i=0; i<ALEN && !feof(fijl); i++)
    {  
        fscanf(fijl, "%f %f", &x[i], &fx[i]);
        //printf("i=%d, x=%f, fx=%f\n", i, x[i], fx[i]);
    }
    return;
}
Romberg integration routine qromb.c

Copyright Numerical Recipes [43].

#include "qromb.h"

#include "qromb.h"

//Romberg integration

#define EPS 1.0e-5
#define JMAX 20
#define JMAXP (JMAX+1)
#define K 5

/*polint*/

void polint(float xa[], float ya[], int n, float x, float *y, float *dy)
{
    int i,m,ns=1;
    float den,dif,dift,ho,hp,w;
    float *c,*d;
    dif=fabs(x-xa[1]);
    c=vector(1,n);
    d=vector(1,n);
    for (i=1;i<=n;i++) {
        if ( (dift=fabs(x-xa[i])) < dif) {
            ns=i;
            dif=dift;
        }
        c[i]=ya[i];
        d[i]=ya[i];
    }
    *y=ya[ns--];
    for (m=1;m<n;m++) {
        for (i=1;i<=n-m;i++) {
            ho=xa[i]-x;
            hp=xa[i+m]-x;
            w=c[i+1]-d[i];
            if ( (den=ho-hp) == 0.0) nrerror("Error in routine polint");
            den=w/den;
            d[i]=hp*den;
            c[i]=ho*den;
        }
        *y += (*dy=(2*ns < (n-m) ? c[ns+1] : d[ns--]));
    }
    free_vector(d,1,n);
free_vector(c,1,n);
}

/*trapzd*/

#define FUNC(x) (*((func)(x))
float trapzd(float (*func)(float), float a, float b, int n)
{
  float x,tnm,sum,del;
  static float s;
  int it,j;
  if (n == 1) {
    return (s=0.5*(b-a)*(FUNC(a)+FUNC(b)));
  } else {
    for (it=1,j=1;j<n-1;j++) it <<= 1;
    tnm=it;
    del=(b-a)/tnm;
    x=a+0.5*del;
    for (sum=0.0,j=1;j<=it;j++,x+=del) sum += FUNC(x);
    s=0.5*(s+(b-a)*sum/tnm);
    return s;
  }
}

/*qromb*/

float qromb(float (*func)(float), float a, float b)
{
  void polint(float xa[], float ya[], int n, float x, float *y, float *dy);
  float trapzd(float (*func)(float), float a, float b, int n);
  void nrerror(char error_text[]);
  float ss,dss;
  float s[JMAXP],h[JMAXP+1];
  int j;
  h[1]=1.0;
  for (j=1;j<=JMAX;j++) {
    s[j]=trapzd(func,a,b,j);
    if (j >= K) {
      polint(&h[j-K],&s[j-K],K,0.0,&ss,&dss);
      if (fabs(dss) <= EPS*fabs(ss))
        return ss;
    }
    h[j+1]=0.25*h[j];
  }
  nrerror("Too many steps in routine qromb");
  return 0.0;
}
### Appendix B

**Fitting Results**

#### B.1 Starting Values for Final Fits

<table>
<thead>
<tr>
<th>param</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>mbh</td>
<td>14</td>
</tr>
<tr>
<td>eddrat</td>
<td>0.065</td>
</tr>
<tr>
<td>jetrat</td>
<td>0.65</td>
</tr>
<tr>
<td>pspec</td>
<td>2.07</td>
</tr>
<tr>
<td>zsh</td>
<td>2000</td>
</tr>
<tr>
<td>r0</td>
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</tr>
<tr>
<td>outfac</td>
<td>500</td>
</tr>
<tr>
<td>hratio</td>
<td>1.5</td>
</tr>
<tr>
<td>visco</td>
<td>0</td>
</tr>
<tr>
<td>incl</td>
<td>66</td>
</tr>
<tr>
<td>ush</td>
<td>0.6</td>
</tr>
<tr>
<td>tin</td>
<td>2e+07</td>
</tr>
<tr>
<td>eltemp</td>
<td>3.11e+10</td>
</tr>
<tr>
<td>plfrac</td>
<td>0.79</td>
</tr>
<tr>
<td>bbsw</td>
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</tr>
<tr>
<td>mxsw</td>
<td>1</td>
</tr>
<tr>
<td>dkpc</td>
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</tr>
<tr>
<td>dopsw</td>
<td>1</td>
</tr>
<tr>
<td>comsw</td>
<td>1</td>
</tr>
<tr>
<td>fsc</td>
<td>1875</td>
</tr>
<tr>
<td>zfrac</td>
<td>150</td>
</tr>
<tr>
<td>zmax</td>
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</tr>
<tr>
<td>equip</td>
<td>9</td>
</tr>
<tr>
<td>tbb2</td>
<td>4455</td>
</tr>
<tr>
<td>bbf2</td>
<td>0.01</td>
</tr>
<tr>
<td>gamfac</td>
<td>37</td>
</tr>
</tbody>
</table>
### Fitting Results

#### B.2 Final Versions of start.par

In the following three parameter files the value for \texttt{jetrat} has to be multiplied by two.

**Two Low Infrared Points: fit_two_low**

```plaintext
constant(Isis_Active_Dataset)*(phabs(1)*reflect(1,(agnjet(1)+gaussian(1))))
```

<table>
<thead>
<tr>
<th>idx</th>
<th>param</th>
<th>tie-to</th>
<th>freeze</th>
<th>value</th>
<th>min</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>constant(1).factor</td>
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<td>0</td>
<td>1</td>
<td>0.9</td>
<td>1.2</td>
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<td>2</td>
<td>phabs(1).nH</td>
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<td>1</td>
<td>3.5</td>
<td>0.5</td>
<td>5</td>
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<td>3</td>
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<td>1</td>
<td>0</td>
<td>1e+10</td>
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<td>agnjet(1).mbh</td>
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<td>1</td>
<td>14</td>
<td>10</td>
<td>18</td>
</tr>
<tr>
<td>5</td>
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<td>0.04923862</td>
<td>0.0486059</td>
<td>0.04989326</td>
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<tr>
<td>6</td>
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<td>0.2698582</td>
<td>0.2696105</td>
<td>0.269952</td>
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<td>agnjet(1).pspec</td>
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<td>2</td>
<td>2.00041</td>
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<td>1928.115</td>
<td>2427.346</td>
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<td>agnjet(1).r0</td>
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<td>0</td>
<td>128.6017</td>
<td>128.5569</td>
<td>128.7819</td>
</tr>
<tr>
<td>10</td>
<td>agnjet(1).outfac</td>
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<td>1</td>
<td>500</td>
<td>1</td>
<td>10000</td>
</tr>
<tr>
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<td>agnjet(1).hratio</td>
<td>0</td>
<td>1</td>
<td>1.5</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>12</td>
<td>agnjet(1).visco</td>
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<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
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<tr>
<td>13</td>
<td>agnjet(1).incl</td>
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<td>1</td>
<td>66</td>
<td>0</td>
<td>180 deg</td>
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<tr>
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<td>agnjet(1).ush</td>
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<td>1</td>
<td>0.6</td>
<td>0.5</td>
<td>0.9 c</td>
</tr>
<tr>
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<td>agnjet(1).tin</td>
<td>0</td>
<td>0</td>
<td>1.495678e+07</td>
<td>1.483834e+07</td>
<td>1.508597e+07</td>
</tr>
<tr>
<td>16</td>
<td>agnjet(1).eltemp</td>
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<td>0</td>
<td>2.853852e+10</td>
<td>2.852017e+10</td>
<td>2.854553e+10</td>
</tr>
<tr>
<td>17</td>
<td>agnjet(1).plfrac</td>
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<td>0.79</td>
<td>0.0001</td>
<td>1</td>
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<td>agnjet(1).bbsw</td>
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<td>1</td>
<td>0</td>
<td>1</td>
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<td>agnjet(1).mxsw</td>
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<td>1</td>
<td>0</td>
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