Analysis on the Muon Track Reconstruction with the PPM-DU

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“Equipped with his five senses, man explores the universe around him and calls the adventure Science.”

Edwin P. Hubble
**Abstract**

KM3NeT is a next-generation neutrino telescope to be constructed in the Mediterranean Sea. Its purpose is to detect high energy neutrinos from astrophysical sources (ARCA) and determine the mass hierarchy of the neutrinos (ORCA). The setup consists of building blocks, each with 115 strings. A single string has with 18 multi-PMT detection units in order to detect the Cherenkov light from passing particles.

A Pre-Production Model of the KM3Net Detection Unit (PPM-DU) has been deployed in May 2014. The PPM-DU offers the opportunity for data analysis, particle tracking and calibration. This thesis presents a tracking algorithm for muons. The algorithm consists primarily of a hit selection based on time causality and an event selection based on the quality criteria. The algorithm also includes multiple techniques to increase the quality of the reconstruction. Among those are the use of the residuals (from the minimization) and weight on the first hit. Although most of the analysis has been performed on the PPM-DU, it can also be used on a full string with 18 detection units.

Using the residuals has revealed that it is not beneficial for the algorithm. However, the addition of the weight on the first hit has improved the fit. Furthermore, the use of the goodness of fit has only selected events with a C.L. of 90%. The results obtained show an event selection of 77% and 70% for the MC-files and data-files respectively, whereas this decreases to 22% for a string with 18 detection units. The resolution for the azimuth angle is around 70°.

The layout of the thesis starts with the introduction (chapter 1) and is followed with a brief motivation of Cosmic Rays and the importance of messenger particles (chapter 2). In chapter 3 the design and detection principle of the neutrino telescopes, Antares and KM3NeT, will be explained. Chapter 4 introduces the algorithm for the track reconstruction. The basics of the algorithm is has already been used in Antares. It is specially modified to for the KM3NeT module. It also includes old and new techniques to improve the quality of the reconstruction. Furthermore, it shows how one of the ambiguities could be resolved; a feature that is only possible due to the unique design of the KM3NeT optical module. The algorithm will be tested on actual data and Monte Carlo. Chapter 5 shows how those Monte Carlo files are generated. Finally, the performance of the algorithm will be discussed in chapter 6.
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1. Introduction

The properties of a neutrino make it an ideal probe to explore the universe. They only interact weakly and thus travel large distances without losing the information of how and where they were produced. This property has consequences for a neutrino detector because it will require a large and dense instrumentation volume. The KM3NeT neutrino telescope uses water (Mediterranean Sea) as an active medium. The detection principle relies on the measurement of Cherenkov photons from charged particles produced in neutrino interactions.

Although construction of the KM3NeT is incomplete, several prototypes have been launched. In May 2014 the pre-production model detection unit (PPM-DU) was deployed. It offers the possibility to perform some preliminary analysis on the muon track reconstruction. This thesis will focus on the analysis of the track reconstruction algorithm and techniques that can improve it. Hit selection, starting values for the minimization, bias on the early hits and the quality of the fit will be evaluated with the use of MC-files. The obtained results will then be compared with the performance on actual data. Finally, the code will be applied on MC-files of an 18DOM string. Its performance will then be compared to those from the PPM-DU.

The layout
2. Research Motivation for Cosmic Rays

In 1912 Victor Hess discovered Cosmic Rays (particles with an astrophysical source) and therefore marked the beginning of astroparticle physics. They are composed mainly of protons and allow us to make observations of the universe up to multiple $10^{20}$eV; far exceeding energies from man-made accelerators [1]. Despite being discovered almost 100 years ago, it remains uncertain where and how cosmic rays are accelerated.

The energy spectrum of the cosmic ray particles (after its propagation) measured by various experiments is shown in Figure 1.

![Figure 1: Energy spectrum of the cosmic ray particles [2]](image_url)

The figure shows that most CR particles are at the lower energy range (1 particle/m$^2$·sec), whereas very few particles are highly energetic (1 particle/km$^2$·century). At high energies the acceleration mechanism might have reached a maximal energy. The slope can be described by a power law following $dN/dE \sim E^y$. Up to $10^{15}$eV, the exponent $y$ is 2.7. Between $10^{15}$eV and $10^{19}$eV (after the knee) the exponent is 3.1. At energies higher than $10^{19}$eV (after ankle) the exponent is slightly lower than 3.1. Particles are believed to be from extragalactic origin because their Larmor radius exceeds that of our galaxy. For a charged particle with $E=10^{20}$eV and an average magnetic field (in our galaxy) of 3µG, the Larmor radius is approximately $r=3.3 \times 10^{20}$eV/(3*10^{-10}T)~100kpc. This is substantially higher than the radius of the galaxy, which is $R_g\sim20$kpc. This radius is calculated with...
the use of Cepheids Variables (stars that change their size and brightness periodically). If one would know what the exact brightness of the star is, then any deviation from it would be caused by a distance change. Mapping all the Cepheids allows the size of the Milky Way to be deduced. [6]

At low energies (~10^9 eV) the particles are most likely accelerated by, for example, stellar winds from the sun. At higher energies (~10^16 eV) cosmic rays are assumed to be produced through supernova remnants (SNR).

![Figure 2: Color Image of Cassiopeia A created with data from the Spitzer Space Telescope (infrared), Hubble Space Telescope (orange) and Chandra Observatory (green and blue) (SNR) [5]](image)

Despite the vast number of CRs in the lower energy range, no conclusions can be drawn as to their exact origin. This is due to the deflection of charged particles in the galactic and intergalactic magnetic fields [1]. Only ultra-high energetic cosmic rays (UHECR) provide this possibility, since they are least affected by the magnetic field. However, the acceleration mechanism/sources of these (UHECR) particles remain a mystery up this day. As a result, it is important to understand the acceleration mechanisms and investigate the use of neutrally charged particles (i.e. photons and neutrinos).
2.1. Gamma Rays

In the early years of astroparticle physics, the sky was investigated at the high energy end of the EM-spectrum. Gamma Rays are not affected by the magnetic field and were used as information carriers. These are produced through the decay of a neutral pion into two photons, synchrotron radiation or inverse Compton Scattering. However, the Earth’s atmosphere absorbs photons with short wavelengths and thus prevents many ground-based experiments. As a result, many experiments are performed on satellites. A successful example would be the Compton Gamma-Ray Observatory (CGRO), which detected 100MeV gamma-rays from Active Galactic Nuclei (AGN) with redshifts of $z = 0.03$ to $z = 2.2$ [7].

Unfortunately, gamma rays cannot travel long distances without scattering or absorption because they interact with the electromagnetic coupling strength [7]. Above 1TeV, the absorption length depends on the optical and IR length, whereas around 100TeV the absorption length depends on the CMBR [7]. This can be seen in Figure 3 (solid line).

![Interaction length of photons (solid) and protons (dashed) as a function of energy](image)

Figure 3: Interaction length of photons (solid) and protons (dashed) as a function of energy [8]
Neutrinos are another candidate as messenger of the universe. They are fundamental particles (produced in every source in the universe), have a neutral charge and interact only weakly (with the W and Z boson). These properties make them an ideal candidate for astrophysics, since they can easily propagate through matter and thus escape high density events. As a result, their properties are (almost) unchanged between creation and detection. Their lack of charge allows them to move in straight lines, pointing back to their origin [7]. Interesting neutrino sources are Supernova remnants (explosion due to core collapse of star), Active Galactic Nuclei (massive black hole with jets at opposite side of the accretion disk) and Gamma-ray bursts (bright flashes of MeV gamma-rays lasting several seconds) [7].

Neutrino interactions are distinctly different depending on the exchanged bosons. They are called Neutral Current (NC) and Charged Current (CC) interactions with their respective bosons Z and W⁺/W⁻, as seen in Error! Reference source not found.. NC interactions result in the production of hadronic showers and an undetected outgoing neutrino. CC interactions occur with an exchange of lepton partners; ie. an antineutrino can be absorbed by a proton (or quark) and create a neutron (or a different flavor quark) and a positron. These differences in final states cause different signals in the detector as discussed in [9].

\[
\begin{align*}
\pi^+/- & \rightarrow \mu^+/- + \nu_\mu/\bar{\nu}_\mu \\
\mu^+/- & \rightarrow e^+/- + \bar{\nu}_e/\nu_e
\end{align*}
\]

(eqns. 1 and 2)

Therefore, for every charged pion production, 3 neutrinos are produced. Neutrinos do not exclusively originate from galactic sources. They can also be produced in the sun or originate from the earth’s atmosphere (atmospheric neutrinos). Charged cosmic rays can interact with the atmosphere, producing a shower of pions, muons and various other particles. These would then decay and produce neutrinos via the channels shown shown in eqn. 1 and eqn. 2.
To calculate the approximate cross section of the neutrinos, it can be assumed that the neutrinos are highly energetic. The involvement of high mass virtual particles ($Z^0$, $W^+$) suppresses an interaction at low energies. Thus the cross sections is given by eqn. 3

$$\sigma_{\text{weak}} \sim 5 \times 10^{-44} \left( \frac{E_{\text{cm}}}{1\text{MeV}} \right)^2 \text{cm}^2$$

(eqn. 3)

For neutrinos above 5MeV the cross section can be further approximated as shown in

$$\sigma_{\nu(e,\mu,\tau)} \sim C_x \times 9.5 \times 10^{-45} \left( \frac{E_{\nu}}{1\text{MeV}} \right) \text{cm}^2$$

(eqn. 4)

Where $C_x$ is a flavor-dependent constant ($C_e=1; C_\mu=C_\tau=1/6.2$) [7]. This scattering cross section is very small. As a result, a large interaction volume and long operation time is necessary to provide a sufficient signal for neutrino detections. The most common technique is based on the facilitation of Cherenkov radiation (see 3.1.2 Cherenkov). Large volume neutrino detectors that use these techniques for the detection of cosmological neutrinos are the Antaeres, IceCube and KM3NeT (currently under development).
Neutrino Telescopes

3. Neutrino Telescopes

Neutrino experiments can be performed via the observation of absorption processes or scattering. Antares, KM3NeT and IceCube are based on the detection of photons via Cherenkov Radiation (see 3.1.2 Cherenkov Radiation). The key feature behind this technique is to know the position and timing of the photons (see chapter 4 for further elaboration). Most neutrino detectors are underground to minimize the background caused by of cosmic rays. Developing a durable apparatus in a completely isolated environment would make it very expensive. To reduce these costs the Earth is simultaneously used as an interaction medium and a filter (for CRs). In addition, the neutrino telescopes need to be situated in a transparent environment (low absorption in the UV-region and optical light spectrum) with a suitable index of refraction otherwise the Cherenkov light cannot propagate to the optical sensors [7]. Antares and KM3NeT use sea water, whereas IceCube uses ice. Their indexes of refraction are 1.39 and 1.31 respectively.

The neutrinos interact with a nucleon (N) via NC or CC and create hadrons (X) and leptons (l).

\[
\nu_l (\bar{\nu}_l) + N \rightarrow \nu_l (\bar{\nu}_l) + X \quad \text{(NC)}
\]

(eqn.5)

\[
\nu_l (\bar{\nu}_l) + N \rightarrow l^- (l^+ + X) \quad \text{(CC)}
\]

(eqn.6)

NC events create a shower that is several hundred meters long (depending on the energy of the hadron). The shower depth (\(\lambda_{max}\)) depends on the center-of-mass energy (E). This can be seen in the following equation

\[
\lambda_{max} = 0.9 + 0.36 \times \ln(E)
\]

(eqn.7)

Eq6 shows that a neutrino could produce an electron, muon or tau. However, electrons interact via electromagnetic interactions. They therefore interact quickly with the surrounding medium and create a shower.

Tau leptons are heavy (1.78GeV) and have a lifetime of 290*10^{-15}s. These particles’ flight paths are too short compared to the hadronic shower in order to identify them with the Cherenkov Radiation technique. Muons are lighter (105.6MeV) and have a longer lifetime (2.2*10^{-6}s) compared to the tau.

The relative angle between the neutrino and muon depends on the energy of the neutrino. For highly energetic neutrinos the muons maintain the same direction as the neutrino due to the large Lorentz boost. Their relative angle can be described with:

\[
\langle \theta_{\nu-\mu} \rangle \leq \frac{1.5}{\sqrt{E_\nu [TeV]}}
\]

(eqn.8)

The direction of a muon also depends on its interaction with matter. They are deflected through multiple scatterings and present a near Gaussian distribution for the deviation from the original muon direction. The scattering effects depend on the energy (E), propagated distance (x) and medium.

\[
\langle \theta_{ms} \rangle = \frac{13.6 MeV}{E_\mu} \frac{1}{\sqrt{x_0}}
\]

(eqn.9)
where $X_0$ is the radiation length of the medium [8]. $\theta_{\mu}$ decreases faster than $\theta_{\nu}$ at higher energies. Due to the energy dependence for very high energies, the angles approach zero. This is advantageous for the reconstruction algorithm, since the direction of a neutrino induced muon simultaneously indicates the direction of the neutrino (provided that its energy is sufficiently high).

3.1. Antares

The Antares Neutrino Detector is located in the Mediterranean Sea at a depth of 2.5km (off the coast of Toulon, France). It uses a network of strings with optical modules holding 10” PMTs attached to them. These PMTs are used for the detection of light through Cherenkov Radiation. The construction of the Antares detector was completed in 2008 (two years after the implementation of the first string).

3.1.1. Design

The final design for the Antares telescope consists of 12 strings with 25 storeys holding 3 optical modules per storey. Each string is 480 m long, anchored to the sea-bottom and pulled up by a buoy on the top. There strings are located 70m apart, whereas the storeys are spaced vertically by 14.5m. Every optical module is a 17” glass sphere with a single 10” PMT. These PMTs are oriented 45 degrees to the vertical axis to improve the detection of upgoing muons (though neutrino interaction) [10]. The time resolution is about ~1 ns.

![Design of a storey used in Antares](image)

Due to the flexibility of the string, a sonar in combination with a compass and tilt-meter are used to determine the exact position and orientation of a module. Each storey also includes four optical beacons (blue LEDs and two green lasers), which serve for time calibration procedures [10]. All the collected data goes to the control module. Finally, the digitized data is sent through an optical cable to shore for further analyses.
3.1.2. Cherenkov Radiation

The creation of Cherenkov light is analogous to the Sonic Boom, where an object travels faster than the speed of sound in a medium. Here a charged particle travels faster than the propagation speed of light in sea-water. The propagation speed of light in any medium can be calculated by dividing c with the index of refraction n.

The charged particle polarizes the material with its electromagnetic field. At low velocities ($\beta < 1/n$) the polarization interferes destructively. As a result, there is no wave front. However, at high velocities ($\beta > 1/n$) the polarization can interfere constructively, since the particle propagation is faster than the reaction time of light in the medium. This phenomenon creates a wave front; as seen in Figure 6 (right) [11].

![Figure 6: Charged particle propagating with $\beta < 1/n$ (left) and $\beta > 1/n$ (right). [11]](image)

The photons are emitted at specific angle to the propagating particle. This is known as the Cherenkov angle. It is described as:

$$\cos(\theta_c) = \frac{1}{\beta n}$$

(eq. 10)

where $\theta_c$ is the Cherenkov angle and $n$ the index of refraction. For a charged relativistic particle ($\beta \approx 1$) in sea-water ($n=1.39$), the Cherenkov angle is $\sim 44^\circ$. Although the Cherenkov cone has a unique geometrical shape, the intensity is low. The number of photons emitted per unit length can be calculated with:

$$\frac{dN}{dx} = \frac{2\pi a^2}{\lambda^2} \left(1 - \frac{1}{n^2\beta^2}\right)$$

(eq. 11)

The PMTs in Antares have a sensitivity in the wavelength of 300nm-600nm [12]. Integrating eqn 11 over the wavelength range leads to eqn. 12.

$$\frac{dN}{dx} = 764 \left(1 - \frac{1}{n^2\beta^2}\right) \text{photons/cm}$$

(eq. 12)
From eqn. 12 it can be concluded that the energy loss due to Cherenkov Radiation is negligible [7]. At energies above 2 TeV the primary energy loss of charged particles in water is caused by Bremsstrahlung. At energies below 2 TeV, energy loss through ionization is dominant.

3.1.3. Muon track reconstruction in Antares

Due to the shape of the Cherenkov cone, the muon track can be reconstructed by using the photon arrival time and position at the storey (further elaboration will follow in 4).

![Figure 7: Impression of the Cherenkov light emitted (blue cone) for a charged particle (e.g. muon) passing through the Antares detector. The yellow strings represent Antares. The red line is the neutrino path before the interaction, whereas the blue line describes the muon path after the interaction. [13]](image)

All hits with a pulse height above the 0.3 p.e. (photo-electrons) are digitized and sent to shore. These hits are called L0 and are subject to multiple trigger algorithms before analysis can be performed. Most L0 are background hits and thus uncorrelated. Among the sources that create these background hits are potassium decay, bioluminescence and the dark current of the PMT.

Seawater contains 400ppm of potassium; 0.0117% of it is the radioactive isotope $^{40}$K. This isotope decays into $^{40}$Ca, while emitting an electron (1.3MeV). This electron has sufficient energy to produce L0-hits through Cherenkov radiation at a rate of ~50 kHz for a single PMT [14]. Although sunlight does not arrive at the experimental setup, the water does contain living organisms at a depth of 2.5km. These organisms produce light, bioluminescence, which contributes to the background. Finally, the radioactive decays in the Optical Module-glass (OM-glass) and dark current produce an additional 3-4kHz of background hits [14].

The trigger algorithm only selects hits that are within 20 ns from each other on the same storey, called L1 hits. These L1 hits should also fit the causality criterion. This criterion states that the arrival time between the photons at two different PMTs should be less (or equal) than the propagating time of a hypothetical photon traveling from one PMT to another; i.e.

$$|t_i - t_j| \leq \frac{r_{ij}v_g}{c}$$  \hspace{1cm} (eqn.13)

Here i and j indicate the PMTs, whereas $r_{ij}$ represents the distance (in m) between those PMTs.
3.1.4. Preliminary Results

Although construction of Antares was completed in 2008, 5 strings had already been deployed in 2007. By the end of the same year an additional 5 strings have been anchored [10]. In the Antares data set from February to May 2007 most recorded events are from downward going muons ($90^\circ < \text{zenith} < 180^\circ$) from cosmic ray interactions in the atmosphere above Antares. Muons with a zenith below $90^\circ$ are neutrino-induced but still mostly from atmospheric interactions of cosmic rays at the other side of the earth. This can be seen in Figure 8.

![Zenith distribution of data taken in 2007 (black) compared to the MC of atmospheric muons (red) and neutrino induced muons (blue) [10]](image)

The Monte Carlo (MC) and observed data are in good agreement with each other, suggesting that the reconstruction model has been properly chosen. Figure 9 shows a reconstructed event with a zenith of $52^\circ$ (a neutrino-induce muon) [10]. This event has been observed in December 2007 while only 10 of the 12 strings were deployed.
The angular resolution achieved with the final setup is better than 0.3° for energies above 10TeV. At lower energies this angular resolution deteriorates (see eqn. 8).

3.2. KM3NeT

KM3NeT is a neutrino telescope that is currently under construction in the Mediterranean Sea. It is the successor to the Antares Telescope and will be built in three phases. It consists out of two subprojects, the ARCA and ORCA. The ARCA’s objective is the observation of high-energy neutrino sources, whereas ORCA’s objective is the determination of the neutrino mass hierarchy. Their construction sites are near Toulon (Fr), Sicily (It) and Peloponnese (Gr). The basic principle of KM3NeT is equivalent to Antares, such as the photodetectors are attached to strings and measure the Cherenkov light from a propagating charged particle.

3.2.1. Design

KM3NeT phase-2 is built of 6 so-called building blocks. Each building block consists of 115 strings which are anchored at the seafloor and held upright by three buoys. Similarly to the Antares telescopes, 18 storeys are attached to the string. They each house one digital optical module (DOM), separated by 36 m.

The main difference between the Antares and KM3NeT is the design of the modules holding the PMTs. Antares has a storey with three OMs, each containing the 10” PMT. KM3NeT opted for a module with 31 3” PMTs, each having a 2ns time resolution. The output of those PMTs are amplified
Neutrino Telescopes

and converted into a digital signal. The multi-PMT layout provides for several advantages. First, it yields a larger total photocathode area per OM. Secondly, the small PMTs are insensitive to the magnetic field of the Earth and thus do not require any mu-metal shielding [15]. Another advantage is the redundancy due to the large quantity of PMTs. This means that the loss of a single PMT (10^4 per year) does not affect the performance of an OM significantly. The multi-PMT module is also advantageous for background suppression because the signals from two or more photons can be recognized by counting the hit PMTs. Finally, the PMTs have a gain of 10^6 and a small photocathode area. As a result their anode charge is small [15]. A photograph and technical drawing of the KM3NeT OM can be seen in Figure 10.

Figure 10: Photograph (left) and technical drawing (right) of the KM3NeT OM [16]

A DOM is divided in two hemi-spheres. The lower hemisphere has 19 PMTs, whereas the upper hemisphere has 12. These are held by a 3D-printed support structure. To further increase the photocathode area, a reflective cone surrounding every PMT is added [17]. The curvature of the PMTs creates an empty space between the sphere and the PMT. To prevent light loss through total internal reflection, the empty space is filled with an optical gel.

Besides the PMTs and accompanying electronics, the OM also houses an acoustic sensor and compass. They provide information of the position, rotation and tilt of the OMs on the string (caused by the sea current). Finally, there is a LED beacon which is used for the inter-DOM time calibration (see below).

The acoustic sensor is glued to the lower part of the OM and monitors together with the acoustic emitter in the string base the position of every DOM with a precision of about 10cm. The calculation of the position requires an acoustic transceiver (Long Base-Line) anchored at a well-known position and the receivers at the DOM. The acoustic receivers measure the ATT (Acoustic Transit Time) and calculate their position with respect to the LBL [17]. The compass and tilt-meter are also incorporated in the DOM and provide information of the direction and roll of the storey.

As previously mentioned, every DOM has its own electronics board. Thus every DOM has its own internal clock. However, these clocks have an offset. Knowledge on these offsets is a necessity to make ns precision reconstructions possible (the track reconstruction relies on the concept of an ‘absolute’ time, i.e. all hits share the same clock). Therefore, time calibration becomes a necessity. For this the LED beacon plays an integral part in this process. It is mounted at the top of the DOM and points upward in the vertical direction. The frequency can be varied between 256 Hz and 8 kHz. Flashing the LED illuminate PMTs on its own DOM (DOMN) and PMTs of the DOMs above it (DOMN+1), depending on the chosen light intensity. Due to the known distance between the DOMs
Neutrino Telescopes

(∼36m) and fixed light speed in water, the time difference between the DOMs provides information on any clock offsets. This information is then saved in a file and used during the off-shore data analysis.

3.2.2. Data Acquisition

A single PMT has an average counting rate of about 5kHz. The charge of every hit is digitized and converted into a time over threshold (ToT; 0.3 photo-electrons). In addition to the ToT, the hit-time, tilt, rotation, position and all the other information of an L0-hit are sent to shore. These L0-hits are bundled into a 134 ms ‘time-slice’ and allocated to a processing unit. In other words, a time-slice contains all the information of the neutrino telescope within a 134ms time window [17]. KM3NeT uses the ‘All data to shore’ concept; i.e. there is no offshore trigger. This means that about 80TB of data will be collected every day. Most of these L0 hits are from the same background as Antares (40K and bioluminescence). To reduce the data, trigger and selection algorithms select possible event candidates. All the L0-hits of the candidates will then be saved, including the L0-hits within a +/-2 μs window of the event time, whereas everything else would be discarded.

KM3NeT uses similar triggers as Antares (see 3.1.3 Muon track reconstruction in Antares). L1-hits are created when 2 or more L0-hits are on the same DOM within a 10ns time window. Next, all the L1-hits should meet the requirement of the causality filter (see eqn. 13). Following the conditions, the background hits have been sufficiently reduced for further on-shore analysis.

3.2.3. Pre-Production Model Detection Unit (PPM-DU)

In May 2014 the PPM-DU has been deployed off the coast of Sicily. It is a single string with 3 functional DOMs (spaced by ∼40 m) and serves as the prototype for a KM3NeT string. Goals of the prototype should validate the detection unit structure and deployment procedure, test the software and increase the knowledge of the background (due to bioluminescence). In addition, the various devices required for the time and position calibration will be tested for their functionality.

![Figure 11: Preliminary results of the PPM-DU [22]](image-url)
Neutrino Telescopes

Preliminary results already show that most hits originate from above. This can be seen in Figure 11. It shows the number of hits on every PMT. The blue arrows show the orientation of each PMT. PMTs on the upper hemisphere (blue arrows point upwards) register more hits. This is due to the Cherenkov radiation created by atmospheric muons. The result agrees with the data collected with ANTARES (see Figure 8). The PPM-DU also allows the investigation of coincidence rates on a DOM.

![Figure 12: Coincidence hit rate of a KM3NeT DOM measured in a 134ms time window [18]](image)

The coincidences in Figure 12 are defined as 2 (blue) or 3 (black) L0-hits within a 20ns window. There are about 150kHz of single hits (31 * 5kHz), 2kHz of twofold and ~500Hz of threefold coincidences. The single hits and twofold coincidences show multiple increased rates (spikes). This is due to the enhanced random coincidences [18]. However, the effect reduces significantly with the threefold coincidences. Other results have also shown that most L1-hits occur between two neighboring PMTs (small angular separation), whereas the amount of L1-hits decreases with larger angular separation [18].
4. Reconstruction

The information of the timing and position of the hits on the PMTs of the PPM-DU allows us to reconstruct the track of the muons. Note the reconstruction algorithm is only valid for a single string and does not include the rotation and tilt of the strings, i.e. it assumes that the string is rigid. In the following paragraphs the formula is derived that translates the particle’s parameters (eqn. 14) into time and position of the hits on the DOMs (eqn. 23).

The purpose of the muon reconstruction is to find the best possible fit with a good resolution. Therefore, many techniques and constraints are introduced to improve the fit resolution.

The position of any particle at any given time can be described with

\[ \vec{p}(t) = \vec{q} + v(t - t_0) \times \vec{u} \]  

where \( p(t) \) denotes the particle’s position (in Euclidean coordinates) at a specific time \( t \), \( q \) denotes the particle’s position at \( t_0 \) (i.e. \( p(t=t_0) = q \)), whereas \( u \) is the orientation and \( v \) the velocity of the particle. The orientation depends on the zenith (\( \theta_{\text{zenith}} \)) and the azimuth angle (\( \phi_{\text{azimuth}} \)).

\[ \vec{u} = \{ \sin(\theta) \times \cos(\phi), \sin(\theta) \times \sin(\phi), \cos(\theta) \} \]  

This means that a full description of any particle requires at least 5 parameters (2 angles and 3 for \( q \)) [19]. These parameters can be reparametrized into more practical parameters.

The particle track and string can be simplified as two lines in 3D space. Those lines (vectors) will have a unique point where they are closest two each other. This position can be calculated by minimizing the distance between the vector of the particle track and the vector of the string. That specific time will be denoted as \( t_c \). Furthermore, the closest horizontal distance will be described as \( d_c \), whereas the height will be known as \( z_c \). In conclusion, the reconstruction will look for the optimal values for \( t_c, d_c, z_c, \theta_{\text{zenith}} \) and \( \phi_{\text{azimuth}} \).

The single string offers two methods two for a track reconstruction. However, neither of them can provide the values for all 5 parameters. The most common reconstruction method is the ‘time-based’ track reconstruction (TB), which uses the height (\( z \)) as input parameter. The second method is the ‘angle-based’ track reconstruction (AB), which also uses the height as input parameter. However, both methods (TB and AB) can only reconstruct 4 and 3 parameters respectively [19].

4.1. Time-based track reconstruction

Minimizing the distance between the string (approximated by a vertical line at position \( L_x \) and \( L_y \)) and particle track (eqn. 14) reveals that the critical time is defined as:

\[ \frac{d}{dt}(d) = \frac{d}{dt}|\vec{p} - \vec{s}_{\text{string}}| = 0 \]  

\[ 0 = \dot{\vec{q}} \times \vec{u} + v \times \vec{t}_c - \dot{\vec{s}}_{\text{string}} \times \vec{u} \]  

\[ \vec{t}_c = t_0 - (\dot{\vec{q}} \times \vec{u} - \ddot{\vec{s}} \times \vec{u})/v \]
Reconstruction

Note that eqn. 18 still describes the velocity of a particle as \( v \). The velocity cannot have an arbitrary value. The Cherenkov radiation sets the minimum velocity at 0.73c. Anything slower would not create Cherenkov Radiation. Furthermore, muons can only travel multiple kilometers through atmosphere and water if they have a very high energy. In other words most muons travel with \( v=c \). Therefore, the MC and data are reconstructed with the assumption that all particles travel at a speed \( c \).

With eqn. 18 the critical height \( z_c \) can be calculated:

\[
\begin{align*}
p_x(t_c) &= z_c = q_x + c \cdot (t_c - t_0) \cdot u_x \\
z_c &= q_z - (\vec{q} \cdot \vec{u}) \cdot u_z + (L_x \cdot u_x + L_y \cdot u_y) \cdot u_z + z_c \cdot u_z^2
\end{align*}
\]

(eqns.19, 20)

For simplicity \( L_x \) and \( L_y \) are set to zero, i.e. the origin of the coordinate system is at the bottom of the string. This reduces eqn. 20 to:

\[
z_c = \frac{q_z - (\vec{q} \cdot \vec{u}) \cdot u_z}{1 - u_z^2}
\]

(eqnn.21)

Finally, the critical distance can be described as:

\[
d_c = \sqrt{p_x(t_c)^2 + p_y(t_c)^2}
\]

(eqnn.22)

Figure 13: Geometric drawing of a single string (black), a propagating particle (red) and Cherenkov cone (purple) with the reconstruction parameters [19]
Reconstruction

Figure 13 shows that the arrival time of a photon does not depend on the azimuth angle, i.e. it is invariant around the z-axis [19]. This drawback is the reason why a time-based reconstruction can only reconstruct 4 parameters. With those 4 parameters the arrival time of the photon is described by:

\[ t_\gamma(z) = (t_c - t_0) + \frac{1}{c} ((z - z_c) * u_z + \frac{n^2 - 1}{n} * d_\gamma(z)) \]  

(eq.23)

where

\[ d_\gamma(z) = \frac{n}{\sqrt{n^2 - 1}} * \sqrt{d_c^2 + (z - z_c)^2 (1 - u_z^2)} \]  

(eq.24)

In eqn. 23 and 24 \( n \) and \( u_z \) denote the index of refraction (1.38 for sea-water) and zenith angle (see eqn. 15) respectively [19], whereas \( t_\gamma \) and \( z \) are the time and position of the hit on the PMT. Eqn. 23 has already been developed for Antares because the basic experimental setup (strings with optical modules) is equivalent to KM3NeT. However, the optical modules of the KM3NeT each have 31 PMTs at different positions. In this thesis all hits of an event are treated independently; i.e. a hit on top of an optical module is at a higher position compared to a hit on the bottom of an optical module. This choice provides more detailed information for eqn. 23 and thus more accurate information for the reconstruction.

4.2 Angle-based track reconstruction

The angle (relative to the string, see figure 14) at which the photon arrives is calculated with

\[ \cos(\theta_{photon}) = (1 - u_z^2) * \frac{z - z_c}{d_\gamma(z)} + \frac{u_z}{n} \]  

(eqn.25)

[19]. This formula had also been developed for Antares. However, the optical modules of Antares cannot exploit it as well as those from the PPM-DU. The PPM-DU has a DOM with 31 PMTs pointing in different directions. This means that every hit on the DOM has (besides hit-time) a distinct zenith and azimuth angle (due to the orientation of the small PMTs). This provides the opportunity to reconstruct the muon track up to 3 parameters. However, the field of view of every PMT is 86°, i.e. the angular resolution of a single PMT is about 43°. The low angular resolution may degrade the resolution of the angle-based reconstruction. Nevertheless, this feature has not been tested on the optical modules of the PPM-DU and will be invested in this thesis. Furthermore, the angle-based reconstruction resolution will be investigated for its compatibility to the time-based reconstruction method.

4.3 \( \chi^2 \) Minimization

To find the best fit, the least squared error had been used. Furthermore, the MINUIT package in ROOT has been used to increase the computation speed of the reconstruction. MINUIT calculates \( \chi^2 \) with the formula

\[ \chi^2 = \sum \frac{(t_i^{th} - t_i^{exp})^2}{\sigma_i^2} \]  

(eqn.26)

and tries to find the minimal value for it. Here \( t_i^{th} \) is the theoretical arrival time for the photon, whereas \( t_i^{exp} \) is the timestamp of the hits (from data or MC). \( \sigma_i \) denotes the 2ns time resolution of the measurement. However, this Minimization procedure has several drawbacks. It is sensitive to the starting values for the parameters and for large residuals. A single data point may have a large
residual, whereas all other data points do not. Since $\chi^2$ is the sum of all residuals, it is important to develop a boundary that would remove those anomalies.

For this procedure the time-slices (see 3.2.2), which contain the hits of potential events are used. In addition, all L0 hits +/-2μs to the event time are included. Again, the L1 trigger and causality filter are applied on the time-slice. Furthermore, only three-DOM coincidences are accepted. Hits that merely occur on two DOMs (or less) are discarded. This creates multiple possibilities for an event. For example, if DOM1 has two L1, DOM2 three L1 and DOM3 two L1, then there are maximal 12 events (provided that causality is not violated). A loop is taken over all these 12 events and selects the most probable event (the procedure for the selection will be further elaborated in 4.7Goodness of Fit).

The L1-hits can be further refined through the addition of an L2-trigger. The basic principle behind this trigger is that photons from the same wave front can only illuminate one side of the DOM (as illustrated in the figure below). This trigger looks into the L1-hits and confirms if they are within 180° of each other. It loops over every L1-hit and calculates the direction of the affected PMT with the azimuth and the zenith. This direction can be described with a vector ($\mathbf{u}_1$). This will be also done with the other L1-hit ($\mathbf{u}_2$). Finally, the angle between the vectors can be calculated with

$$\cos(\beta) = \frac{\mathbf{u}_1 \cdot \mathbf{u}_2}{|\mathbf{u}_1| \cdot |\mathbf{u}_2|}$$

(eqn. 27)

, where $\beta$ denotes the angle between the vectors (i.e. L1-hits). If the other hit is within +/-90° range, then it is added as part of the L2. Note that with a +/-90° range an effective region of 180° around the L1-hit is investigated. This process is repeated for every L1-hit. This means that there could be multiple L2 for a single L1; similar to having multiple L1 for the L0.

The drawback of the method above is that L1 with 2 or 3 hits are always within 180° of each other but do not necessarily pass the boundary set. For example, an L1 could have two opposite hits (180°). This can be calculated with eqn. 27. However, it would not pass through the boundary condition and thus will be rejected. This is not intended, therefore the all L1 with 2 or 3 hits will be considered as L2-hits.

Only after 4 or more hits in the L1 does this trigger become effective. It is therefore useful for events with many hits, because the influence of a background should be reduced and thus improve the track reconstruction. This assumption (improving track reconstruction with L2) will be investigated and compared with the performance of the reconstruction with L1-hits.
4.4. Starting values

As previously mentioned, the starting values are crucial for the fit because a wrong starting point could cause MINUIT to find a local minimum. Most hits occur at the point of closest approach [19] because the optical path is shortest. Thus decreasing any influences on the photons (scattering, absorption, etc…). It has therefore been chosen to set the starting values for $t_c$ and $z_c$ at the mean of the hit-times and height positions. With this concept, the range has been set at $t_{\text{mean}} \pm 1000\text{ns}$ and from -300 to 300m for the height. The starting value of the critical distance is not significant to the fit [19]. It is therefore set at 10m with a range of 0m to 100m.

The final variable, the zenith angle, is fixed during the Minimization process. Therefore, a loop is created with 180 iterations (one for every degree). MINUIT will calculate the best fit for every loop and select the most probable one. It will then compare the probabilities of every possible event in a summary-slice and choose the event with the highest probability.

However, prior to the calculation of the probability, the fit will be evaluated based on the time residuals.
4.5. Residuals

The sum of the time residuals is always 0 (i.e. Minuit found a minimum) and are calculated with

\[ t_{\text{res}} = t_i^{th} - t_i^{exp} \]

(eqnn.28)

In an ideal situation, a perfect fit would have no time residuals, i.e. \( t_{\text{res}} = 0 \text{ns} \), resulting in \( \chi^2 = 0 \) and create a delta-peak. However, with many data points, the distribution should have a Gaussian shape with a certain width \( \sigma_{\text{gauss}} \). Due to the sensitivity of \( \chi^2 \) for large time residuals, a good fit may be unnecessarily discarded (\( \chi^2 \) has a large value). The time residuals would reveal this by presenting a hit far from its peak. Consider the hypothetical situation in Table 1:

A possible event has x-hits with the following preliminary residuals after the first iteration

<table>
<thead>
<tr>
<th>Time Residual [ns]</th>
<th>-1</th>
<th>12.7</th>
<th>-0.3</th>
<th>-0.5</th>
<th>+0.7</th>
<th>+1.1</th>
<th>+0.2</th>
<th>-0.4</th>
<th>-11.7</th>
<th>-0.8</th>
</tr>
</thead>
</table>

Table 1: Residuals of the first iteration of a hypothetical fit

Note that the sum of all the residuals is 0. Minuit considers this a good fit because it has found a minimum. According to eqnn. 26 the \( \chi^2 \) is 300.1. Initially, it may be concluded that \( \chi^2 \) is too large and should be discarded. However, a closer look reveals that the second and third residual are 12.7ns and -11.7ns respectively. Compared to the other residuals, these are much higher and thus affect the quantitative interpretation of fit (i.e. Is \( \chi^2 \) too large or too low?). Therefore, the residuals are evaluated with a two-step process. Step one; a histogram (with a 1ns bin size) containing all the residuals is created. Next, a Gaussian fit is performed and reveals what the width \( \sigma_{\text{gauss}} \) is of the distribution (in Table 1 \( \sigma_{\text{gauss}} = 0.42 \)). The second step uses the width \( \sigma_{\text{gauss}} \) and checks if there are any residuals that exceed the \( 3\sigma_{\text{gauss}} \) boundary condition. Every residual (and corresponding hit) that exceed this boundary will be discarded. Finally, Minuit will recalculate the fit without the hits that have been discarded. Note that these hits are not erased.

The second iteration could reveal the

<table>
<thead>
<tr>
<th>Time Residual [ns]</th>
<th>0.2</th>
<th>15.7</th>
<th>-0.3</th>
<th>-0.5</th>
<th>+0.5</th>
<th>+0.7</th>
<th>+0.3</th>
<th>-0.4</th>
<th>-4.7</th>
<th>-0.3</th>
</tr>
</thead>
</table>

Table 2: Residuals of the second iteration of the hypothetical fit. The residuals marked in red were discarded in the previous iteration, whereas the residuals marked in black are used for the fit in the second iteration

Table 2 indicates that Minuit has found another fit because the residuals have changed. Now, the \( \chi^2 \) is 1.46 with a smaller width \( \sigma_{\text{gauss}} = 1.24 \). Note that the residuals of the two previously discarded hits have also changed. They were not included into the second iteration but with the parameters of the new fit their residuals could be calculated. This step should not be ignored because it reveals if one of the discarded hits could be potentially used for a possible third iteration.

At the end of the second iteration, it can be concluded that no additional hits need to be discarded. This means that that a third iteration is not possible because nothing will change between the second and third iteration. However, there are two problems that could occur. The primary problem would be that too many hits may be discarded. This effectively reduces the quality of the fit because most
information of the event is not taken into account. The second problem would be that it could change an L1-hit (or L2-hit) into an L0-hit, which is not intended. To prevent these problems, two constraints are applied before discarding a hit. For the former problem (too many hits discarded), the constraint states that only 25% of all the available hits may be discarded. The previous example discards 20% of its hits and thus meets the requirement. For the latter problem (changing an L1-hit into an L0-hit), the constraint states that the L1 (or L2) should always have 2 or more hits. If the second residual in Table 1 were part of an L1 with only 2 hits, then discarding it would not meet the criteria. The program would revert back to its first iteration, not discard anything and consider this fit as the final/best fit (for that specific event).

4.6. Weight of first hit

In addition to the hit rejection, a weight is applied on the hit that arrives at the DOM first. The motivation for this weight is that the first photon is most likely from direct Cherenkov light, whereas the latter ones are probably delayed due to scattering or background. In this thesis, it was chosen to describe the weighted function as an exponential function and modifies the \( \chi^2 \) –formula (eqn. 26) into

\[
\chi^2 = \frac{\sum(t_i^{th} - t_i^{exp})^2}{\sigma_i^2} \cdot \exp\left(\frac{t_{first} - t_i}{\sigma_i}\right) \tag{eqn. 29}
\]

where \( t_{first} \) denotes the arrival time of the first photon. The intention of eqn. 29 is to improve the fit. Therefore its performance will be compared to the fits performed with eqn. 26.

The resulting \( \chi^2 \), fit results \((t_0, d_0, z_0 \text{ and } \cos 0)\) and time residuals are then stored in vectors to analyze their goodness of fit.

4.7. Goodness of Fit

A good fit would have small time residuals and thus a small \( \chi^2 \). Therefore, one would simply select fits with small \( \chi^2 \) values. However, there is a flaw with this type of selection. Events that contain many hits could create large \( \chi^2 \) values, whereas other events with few hits would have low \( \chi^2 \) values. Therefore, good events may be rejected (type 2 error) or bad events may be accepted (type 1 error) respectively.

The goodness-of-fit test uses the \( \chi^2 \) and the \( ndf \) (defined as number of hit subtracted by the free parameters; \( ndf = N_{hits} - 4 \)) to quantify whether a fit is good or not. The basic principle of this states that if \( \chi^2 \) is much larger than \( ndf \), that it can be concluded that the fit is bad (assuming that the model and \( \sigma \) are correct).

\( \chi^2 \) has a probability distribution with \( v \) degrees of freedom \((ndf)\)

\[
f(\chi^2) = \frac{1}{2^{v/2} \Gamma(v/2)} \cdot e^{-\frac{\chi^2}{2}} \cdot (\chi^2)^{v/2 - 1} \tag{eqn. 30}
\]

where \( v \) is always a positive integer [20].
Reconstruction

Figure 15 shows three examples of the $\chi^2$ distribution (left) and the normalized $\chi^2$ (right).

Figure 15: Left: $\chi^2$ distribution for events with $\nu = 2$, 4 and 10. Right: $\alpha$ vs $\chi^2/\nu$. $\alpha$ denotes the probability that a sample will be larger than $\chi^2_\nu$. Each curve is denoted with its corresponding number of degrees of freedom [20].

Note that the $\chi^2$ distribution looks similarly to a Poisson distribution for small $\nu$, whereas it resembles more a Gaussian at high $\nu$, which is in accordance with the Central Limit Theorem [20].

A $\chi^2$ distribution implies that repeating an event multiple times (with the same $\nu$) would provide the distribution as seen above. Furthermore, if the model for the track reconstruction were chosen correctly, then about half of the calculated $\chi^2$ should be before and after the median (i.e. the $\chi^2$ is clustered around the median) [20]. Therefore, a good fit has a $\chi^2$ that is close to the median; i.e. a probability $\alpha$ of about 50% to have a higher/lower $\chi^2$ if the event were repeated. This implies that the reduced $\chi^2/\nu$ should be 1 (as can be seen in Figure 15). This is often the general rule of thumb. However, it is inaccurate for low number of degrees of freedom because there the median differs from the mean.

The events occurring at the PPM-DU usually have about 8-10 degrees of freedom. Figure 15 shows that the distribution is skewed. Therefore, the calculation of the probability should be opted instead of the reduced $\chi^2/\nu$. Nevertheless, both are analyzed and compared on their performance during the fit selection/rejection. The calculation of the reduced $\chi^2/\nu$ is simple. However, the calculation of the probability $\alpha$ uses the incomplete gamma function. This can be found in the ROOT library and is called with TMath::Prob($\chi^2$,ndf). Note the probability calculated with the gamma function is not the $\chi^2$-probability.

In both cases fits within a specified boundary are accepted. For the reduced $\chi^2/\nu$, fits with a value around ~1 are accepted. Similarly, boundaries are also set on the probability. The exact values for the boundaries will be discussed in 4.
4.8. Azimuth angle reconstruction

As previously mentioned, the time-based and angle-based track reconstructions are independent of the azimuth angle. However, the KM3NeT DOM design provides the opportunity to determine the azimuth angle. All 31 PMTs point in a different direction and thus observe different regions around the string. This means that it should be possible to differentiate the azimuth angle of the muon track (with respect to the string line). This feature has been developed as part of the thesis and is illustrated for the PPM-DU in Figure 16.

![Figure 16: Example of a down-going muon illuminating 3 DOMs via Cherenkov radiation. The example shows that the PMTs in the upper-left corner are most likely to have a hit, whereas there should not be any hits in the lower-right corner.](image)

If a photon arrives at the PMT, then the direction of PMT (described by its azimuth and zenith) should be pointing to the muon. Mathematically, the direction of the PMT is exactly opposite to the direction of the photon (photons comes from the track, whereas the PMT looks toward the track).

The relation between the photon and muon is described with

\[
\cos(\theta_c) = \frac{\mathbf{u}_{\text{muon}}^\ast \mathbf{u}_{\text{photon}}}{|\mathbf{u}_{\text{muon}}||\mathbf{u}_{\text{photon}}|} \quad \text{(eqn. 31)}
\]

Due to the opposite direction of the photon and PMT, eqn.31 can be written as

\[
\cos(\theta_c) = -\frac{\mathbf{u}_{\text{muon}}^\ast \mathbf{u}_{\text{PMT}}}{|\mathbf{u}_{\text{muon}}||\mathbf{u}_{\text{PMT}}|} \quad \text{(eqn. 32)}
\]

, where \(\theta_c\) is the critical angle at ~43°. Note that \(\mathbf{u}_{\text{muon}}\) is the same vector specified in eqn. 15. Also, \(\mathbf{u}_{\text{PMT}}\) has use the same format as \(\mathbf{u}_{\text{muon}}\) (i.e. vector structures are the same but the zenith and azimuth
are different). The time-based (or angle-based) reconstruction provides the zenith of the muon track, whereas the PMT provides the zenith and azimuth for $\mathbf{u}_{PMT}$. With all the known variables, eqn.32 reduces into an equation with only one unknown variable; the azimuth of the muon.

Despite this simple calculation for the azimuth, it is currently not very accurate. As previously stated, the PMT has a field of view of about 86°. Photons arriving with an angle +/- 43° around the axis of the PMT will be denoted as $-\mathbf{u}_{PMT}$ (i.e. photons within this range will receive the same opposite orientation of the PMT). To compensate for this effect, the zenith and azimuth of all the hits on a DOM are averaged. The mean zenith and azimuth is then implemented into $\mathbf{u}_{PMT}$ eqn. 32. Following this step, the azimuth of the muon track is calculated according to the DOM. This process is repeated for all the other DOMs; each DOM calculating the azimuth. Finally, they are averaged and considered as definitive azimuth of the muon track (with respect to the string).

The only drawback of this technique is that it heavily relies on an accurate zenith value from the track reconstruction. Therefore, the azimuth reconstruction will be evaluated with a zenith value from the Monte Carlo and from the track reconstruction. The outcome should provide insight in the potential modifications that could be applied to eqn. 23 and eqn. 25.

4.9. Ambiguities

The bottleneck of the reconstruction process lies in the many ambiguities. One of them is the invariance of the azimuth. Fortunately, the problem should be resolved with the steps explained in the previous 4.8 Azimuth angle reconstruction. However, it cannot differentiate between events within a certain phase-space.

Consider a down-going muon, which passes along the string (shown in Figure 17). The Cherenkov radiation from that specific muon would irradiate the upper DOM, followed by the DOMs below it. However, as Figure 17 points out, another muon track can irradiate the DOMs at the exact same time and location. In the example shown in figure 17, the zenith angle between both tracks differs by 20°.

Figure 17: Example of two muon tracks (red) with photons (blue and green) that arrive at the DOMS at the same time, position and angle.
Reconstruction

Having only two possible outcomes for a certain event would not be very problematic because a MC study would reveal two distinct peaks for their zenith, separated by $2\theta_c$. However, due to the conical shape of the Cherenkov radiation, $2\theta_c$ is actually the upper limit between the zenith angle of both tracks. This means that the zenith resolution should present a smear between $0^\circ$ and $2\theta_c$.

It is important to note that this phenomenon only occurs if the distance of closest approach ($d_0$) is above or below the highest or lowest DOM respectively. If $d_0$ were between one of those DOMs, then the fit is not ambiguous anymore. Due to the small setup of the PPM-DU, the ambiguities are more frequent compared to a full string with 18 DOMs.

It is possible to suppress the ambiguities by changing the calculation of $\chi^2$ into

$$\chi^2 = \sum \frac{(t_i^{th} - t_i^{exp})^2}{\sigma_i^2} \ast (1/\cos(\theta_{zenith-fit}))$$  \hspace{1cm} (eqn.33)

or

$$\chi^2 = \sum \frac{(t_i^{th} - t_i^{exp})^2}{\sigma_i^2} \ast \exp(t_{first} - t_i^{exp}) \ast (1/\cos(\theta_{zenith-fit}))$$  \hspace{1cm} (eqn.34)

depending on whether the weight lies on the first hit (eqn. 34) or not (eqn. 33). $\theta_{zenith-fit}$ is the same zenith value used for the calculation of $t_i^{th}$, i.e. it is one of the free parameters, which Minuit tries to calculate. This bias can successfully suppress ‘fake’ muons propagating in the horizontal direction. However, it also does the same thing for ‘real’ muons propagating in the horizontal direction. Those muons will then be incorrectly fitted as up-/down-going muons. This should not happen very often because most muons are atmospheric muons. However, the use of this suppression should be used with caution. A flow chart of the reconstruction procedure can be seen in Figure 18.
Figure 18: Flow chart of the Muon track reconstruction
Monte Carlo Production

5. Monte Carlo Production

The performance studies of the track reconstruction with simulations from MUPAGE. It can generate multiple atmospheric muons for underwater/ice neutrino telescope with little computation time. The kinematics of the events are user dependent and are defined at the top of a virtual volume for the proposed detector [21]. Note that it is only useful for detectors at a depth between 1500m and 5000m. Furthermore, it can only generate events with a zenith angle of 180° up to 95°.

MUPAGE can be used for the PPM-DU because the setup is located at a depth of 3500m. With parametric formulas the muon flux, angular distribution and energy spectrum are generated [21]. The kinematics then become the input for PPM-DU MC simulation files, which include the Cherenkov radiation and the arrival times of the photon at the PMTs [21].

To produce the MC simulation files for the correct detector, the user has to provide a detector file; i.e. a file with the geometric features of the DOMs. For PPM-DU the detector file should include three DOMs, with their respective height and PMT orientation.

MUPAGE does not create a simulation of a near infinite volume. It creates two virtual cylindrical volumes surrounding the string. The z-axis of the PPM-DU (or an 18 DOM string) would simultaneously be the z-axis of the virtual cylinders. Only events that hit the top of the outer cylinder are accepted, whereas Cherenkov radiation is only included if they pass through the inner cylinder. A schematic structure of this can be seen in the following figure.

![Schematic of the virtual cylinders used for the MUPAGE](image)

Figure 19: Schematic of the virtual cylinders used for the MUPAGE [21]

The dimensions can be defined by the user. However, their default values are 512.12m for $H_{\text{can}}$, -278.15m for the position of the lower disk (Z$_{\text{min}}$), 313.97m for the upper disk (Z$_{\text{max}}$), 238.61m for $R_{\text{can}}$ and 338.61m for $R_{\text{ext}}$ [21]. Note that the positions of those disks are with respect to the seafloor. Furthermore, those values were used as the boundary values for MINUIT.

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6. Results of MC/Data; Comparison

In this chapter the results of all the filters/constraints will be shown and discussed. The analysis below will be performed with the same MC-files to maintain consistency. In the beginning, MC-files are used without $^{40}$K background. This gives a better view of the effect of the filter/constraints on muon events. In the end, a comparison will be shown between reconstructed MC-events (including $^{40}$K) with and without all the filters/constraints. Initially, all filters/constraints will be disabled to explicitly show the differences achieved with the filters/constraints. After each paragraph a filter/constraint will be enabled and included in the following paragraph.

6.1. Coincidence and Causality Cuts

The MC-files used (PPMDU_mod_sb_mu10G[1...5].km3_v5r1.JTET1T4pmteff.root) contain 11040 events (without the addition of $^{40}$K). The L0-hits are bundled into L1 (hits within 10 ns of each other) and later into L2 (hits within 180° of each other). The criteria set before any reconstruction are that the hits must be causally correlated and that there should be a coincidence between 3 or more DOMs. For the PPM-DU this translates into: all DOMs must have causally correlated L1s.

Figure 20 shows the distribution of L1 hits on the DOMs.

![Figure 20: L1 coincidences on 1 DOM, 2 DOMs and 3 DOMs (labeled with their respective numbers).](image)

Figure 20 shows that most L1 coincidences occur on a single DOM, whereas the frequency decreases with increasing number of DOMs. This is in agreement with earlier studies, as can be seen in Figure 12. Out of 11040 events, only 499 are three-DOM coincidences (~4.5%). Although, the frequency of three-DOM coincidences is low the quality of the fit will improve due to the supply of hits from more...
DOMs (i.e. a fit with 5 DOMs is much more accurate compared to a fit with 3DOMs). This is the reason why the cut has been set at coincidences between 3 or more DOMs.

Almost all L1 combinations (~95%) satisfy the causality criteria. Those will then proceed to MINUIT. Note that only the best L1 combination for an event will be selected (as has been explained in the previous chapters).

### 6.2. Preliminary Results of Angle-Based and Time-Based Reconstruction

The initial result of the reconstruction have shown the following zenith resolution for the angle-based and time-based track reconstruction respectively. The MC creates events which have a zenith angle between 120° and 180°. The starting values and range of the fit are important. The height \( z_0 \) and time \( t_0 \) use their respective mean value of the hits. In addition, the range is set at 0 to 100m, -300m to 300m, 0° to 180° and mean_time +/-2μs for the \( d_0, z_0, \) zenith and \( t_0 \) respectively (the starting values are the same for time based and angle-based minimization).

![MC zenith distribution](image1.png) ![zenith distribution after reconstruction](image2.png)

**Figure 21:** MC zenith distribution (left) and the zenith distribution after the reconstruction with the angle-based technique
However, none of the reconstructed events are close to the MC distribution. This can be attributed to the few free parameters (it only requires 3 free parameters). Furthermore, the angle-based minimization is done between the theoretical and measured arrival angle of the photons. The problem with the latter value is the large field of view (86°) of the PMT. In other words, the input values of the photon angle have a large variance.

Due to the few free parameters, the angle-based reconstruction was intended to compliment the time-based track reconstruction. In the most opportunistic case, the angle-based and time-based reconstruction could be combined to find the best fit. Unfortunately, this does not seem to be the case due to the already different shape. It was therefore chosen to only proceed with the time-based reconstruction.

The time-based reconstruction has the additional time parameter $t_0$. It uses the arrival times of the photons for the minimization, which have a time resolution of 2ns for each PMT. The initial result of the time-based reconstruction has two peaks. This can be seen in Figure 22.

![Figure 22: Zenith residuals between the reconstructed and MC values for the time-based reconstruction. Two peaks can be recognized. They reveal a zenith resolution of 20.4° (low zenith residual) and 34.3° (high zenith residual)](image)

The first peak in Figure 22 is where the zenith residual is minimal (0°). There the zenith resolution is 20.4°. The second peak is lower than primary peak and lies at a zenith residual of 40°. The zenith resolution of the second peak is 34.3°. The second peak and large zenith residual (-50° up to 90°) are caused by the degeneracy, which allows for a reconstruction with a zenith difference up to ~90°. Note that Figure 22 agrees with this boundary because there are almost no reconstructed events which differ more than 90° from the MC event.
Results of MC/Data; Comparison

It is not possible to evade the degeneracy with statistics or anything similar. However, it is possible to narrow the range of the zenith. The MC events are between 120° and 180°; a 60° range. Therefore, reconstruction will be done between 0°-60°, 60°-120° and 120°-180°. All other ranges remain the same. This gives a better indication of the zenith resolution.

For the time-based reconstruction with a zenith range between 120°-180° the distribution can be seen in Figure 23.

![Figure 23: Zenith residuals between the reconstructed and MC values. The zenith parameter is constrained between 120° and 180°. The Gaussian fit reveals that μ=1.71° and σ=25.1°](image)

For the zenith range between 60°-120° the zenith distribution can be seen in Figure 24.
Results of MC/Data ; Comparison

Finally, the distribution of the zenith residual between 0°-60° can be seen in Figure 25.

Figure 24: Zenith residuals between the reconstructed and MC values. The zenith parameter is constrained between 60° and 120°. The Gaussian fit reveals that \( \mu=47.2° \) and \( \sigma=23.5° \)

Figure 25: Zenith residuals between the reconstructed and MC values. The zenith parameter is constrained between 0° and 60°. The Gaussian fit reveals that \( \mu=91.8° \) and \( \sigma=15.1° \)
Dividing the zenith range has shown that the zenith residual is best between 120° and 180°. Although not presented in a figure, the normalized $\chi^2$ (commonly used to indicate the quality of a fit) are higher for the latter two regions. Note that Figure 23 shows an asymmetrical residual distribution. This is most likely also caused by the degeneracy. The mean zenith value for the MC is about ~150°. This means that a degenerate event can only occur between 150° and 180° or between 60° and 150°. The latter zenith region is larger. Therefore it is more likely to fit a degenerate event, as opposed to the former zenith region. If the mean zenith value for the MC would be around 90°, then the zenith resolution would have a more symmetrical shape.

Figure 8 showed that the most events (more than 99.99%) are generated from atmospheric muons above the setup. As a result, we know that almost all events on the PPM-DU are generated from the muons with a zenith loser to 180°. It has therefore been chosen to fit all events with the zenith being constrained between 120° and 180°. This is beneficial for the MC studies but disadvantageous for reconstructions performed on the data. However, most muons are predominantly from the atmosphere. Therefore, the selected choice should not compromise the track reconstruction on the PPM-DU data (or data from a string with 18 DOMs).

6.3. Performance of hit selection (L1 & L2)

Until now the reconstruction has been performed with L1s. However, it is possible to refine them into L2 (hits that are also within 180° of each other). Analogous to L1 with L0, there could be multiple L2 possibilities for any given L1 (although that is most likely if L1 contains many hits). Including this condition prior to the reconstruction reveals a different zenith distribution.

Figure 26: Residuals of all parameters (zenith, t0, z0 and d0) between the reconstructed and MC values. The reconstruction uses L1 (red) and L2 (blue) hits and is fixed within a zenith of 120° and 180°.
Of the 499 events, only 484 events have L1 that pass the condition imposed by the L2 (~97%). Figure 26 shows that the residuals between events reconstructed with L1 or L2 are similar. It can therefore be interpreted that most hits of the L1 occur close to each other. However, although not clearly visible in Figure 26, the reconstructions using the L2 have a better fit. This can be seen in Figure 27.

![Figure 27: Normalized $\chi^2$ distribution. The reconstruction uses L1 (red) and L2 (blue) hits and is fixed within a zenith of $120^\circ$ and $180^\circ$]

The figure shows the normalized $\chi^2$ for all the reconstructed events. The rule of thumb is that a fit with a normalized $\chi^2$ of about ~1 is a good fit. Fits performed with the L2 show a significant better result. This improvement is most likely caused by the improvement of z0. A first look at Figure 26 shows that the z0 distribution has not changed significantly. However, there is a distinct peak at a z0 residual of 0 m. This means that more events that with the L2 there more fits with a near perfect reconstruction of z0. With the L2 about ~40% of the fits are considered good and only about ~20% for fits performed with the L1. Therefore, it has been chosen to proceed with L2s for the track reconstructions.

### 6.4. Performance of fit with the emphasis on first hit

Theoretically, the first arriving photon should be from direct Cherenkov radiation (as is travels the shortest path). Scattering effect, showers or background could be sources of later photons. Selecting the first fit should then represent the arrival times of photons. This concept might work with MC events, where background ($^{40}$K) could be removed. However, the background could create a photon prior to the arrival of a Cherenkov photon and accidentally decrease the quality of the fit. There is a possibility to counter this drawback. This would be to emphasize the earlier hits (through the addition of a weight factor), compared to the latter hits (as shown in eqn. 29).
Comparing both techniques reveals

![Graphs showing improvements in parameters](image)

**Figure 28**: Residuals of all parameters (zenith, t0, z0 and d0) between the reconstructed and MC values. The distribution of the weighted-method (blue) is compared to the no-weight-method (red).

Figure 28 shows improvements in two parameters: zenith and z0. The zenith distribution has become narrower, whereas the peak (vaguely visible in Figure 26) has increased for the z0 distribution. This means that the quality of the fit should have improved. A brief look into the quality of the fit confirms this.

![Graph showing normalized χ2 distribution](image)

**Figure 29**: Normalized $\chi^2$ distribution for reconstructions using all hits equally (red) and using the weighted function (blue)

indicates that almost ~40% of the fits are considered good if all hits have an equal contribution in the fit. It also shows that this has increased to 73% and 75% for the first-hit reconstruction and weighted-$\chi^2$ reconstruction respectively. Applying the weighted function into the calculation improves the quality of the fit. Therefore, it is beneficial for the track reconstruction.
6.5. Residuals

The residuals were used to eliminate possible outliers in the reconstructions. This is done by discarding all hits that are more than $3\sigma$ apart from the reconstructed value. It is a valuable tool to remove background hits. However, it could possibly remove hits that are part of a muon event. The implications of these boundaries are shown in Error! Reference source not found. and Error! Reference source not found..

![Residuals of all parameters (zenith, t0, z0 and d0) between the reconstructed and MC values.](image)

**Figure 30**: Residuals of all parameters (zenith, t0, z0 and d0) between the reconstructed and MC values. The MC reconstruction, which uses the $3\sigma$ cut (blue), is compared to the reconstruction without the $3\sigma$ cut (red).
Results of MC/Data ; Comparison

Figure 31: Normalizes $\chi^2$ distribution for the reconstruction, which does (not) use the 3*σ cut; blue (red)

Error! Reference source not found. shows a small change in the residuals of the zenith and $z_0$. The 3*σ-cut has increased the peak of the zenith distribution, whereas the peak for the $z_0$ distribution has decreased. This means that some parameters have improved, whereas others did not. The 3*σ-cut affected the $z_0$ parameter most. As a result, it is expected that the quality has decreased. This is also reflected in Error! Reference source not found.; less events have a normalized $\chi^2$ of ~1. It can therefore be concluded that applying this filter compromises the quality of the track reconstruction. Adding $^{40}$K to the MC reveals similar results. This raises the question of the contribution of this technique to the quality of the reconstruction. There are two interpretations to this. The first (trivial) interpretation would be that there are not sufficient hits. This has already been accounted for (varying bin-sizes to fit a Gaussian). Nevertheless, the fit strongly depends on the distribution of the residuals. If the distribution does not look Gaussian, the fit fails to perform properly. The only solution would be to have more hits.

The second interpretation would be that the 3*σ method overcompensates. This can be recognized by the fits with low normalized $\chi^2$ (below 1). The L2 (and L1) has a time window of 10ns. The probability of measuring a background hit in the time window is about 0.47% (=31*3*10ns/5000Hz). It may be unnecessary to include this for the PPM-DU.

In conclusion, it is currently not recommended to use the 3*σ boundary on the PPM-DU. However, it can be used for a track reconstruction on the full string (where you have more hits). The probability of a $^{40}$K increases to 2.79%. Therefore, the PPMDU track reconstruction will proceed without using the cuts derived from the residuals. It will be later included for the 18DOM string.

6.6. Goodness of Fit

The previous results used the normalized $\chi^2$ as a rule of thumb to evaluate the quality of the fit. The goodness of fit is the actual origin of this rule and it is used to select/reject fits based on their quality. The quality is derived from the probability of having a larger $\chi^2$ in another measurement (see chapter 4.7 for further elaboration). If the probability is close to 50%, then it is considered a good hit (because it is close to the median), whereas a probability $\alpha$ of 0% or 100% indicates a bad fit.

The probability distribution of all 484 events can be seen in Figure 32.
Figure 32: The figure displays the result from the inverse-gamma function for a given $\chi^2$ and ndf. The inverse-gamma function calculates what the probability is for a larger $\chi^2$ if the event with the same ndf would occur.

Figure 32 shows a peak at 50%, i.e. most fits have a good quality. The distribution is concentrated between 48% and 52%. These values can be used to apply a cut. Choosing a range from 47.5% to 52.5% would then imply that the fits have a confidence level of 95%. With a range of 45% to 55% the confidence level would decrease to 90%. This is advantageous for the zenith resolution because fits with a lower quality will be rejected. Some of those 'bad' fits can be seen on the far left of Figure 32. Their probability is 0% and thus implicate that their $\chi^2$ is too large. The figure below shows two reconstructed events.

Figure 33: Left: A successfully reconstructed event. Right: An unsuccessfully reconstructed event. Each plot shows the arrival time (x-axis) vs the vertical height (y-axis) of each string.

The event on the right has a $\chi^2$ of 3237.59 (whereas most well reconstructed events have $\chi^2$ of about ~10-20). As a consequence the fitted parameters (including the zenith) become unreliable. This decreases the resolution and should therefore be rejected.
Applying the goodness of fit changes (with a confidence level of 90% and 95%) changes the zenith resolution.

![Graph showing zenith resolution with and without goodness-of-fit selection](image)

**Figure 34**: Zenith resolution without the goodness-of-fit selection (blue), with a confidence level of 90% (red) and 95% (green)

The goodness-of-fit selection has decreased the number of reconstructed events from 484 to 374 and 312 for a confidence level of 90% and 95% respectively. This equates to a fit selection of 77% and 65% respectively.

The zenith resolution does not indicate that the resolution has improved. The first impression shows that some 'good' events are rejected (due to the lower peak at 0°). This is actually not the case because it is possible that the zenith can be estimated correctly, whereas other parameters are completely wrong (i.e. MINUIT has found a local minimum). As a result, the peak of the zenith can decrease slightly.

In conclusion, the goodness-of-fit selects the good fits successfully. However, it does not improve the zenith resolution for the PPM-DU. This is probably due to the degeneracy, which the goodness-of-fit cannot correct for (and will never be able). Nevertheless, the fit selection procedure will be included because all events will be within a confidence level of 90% or 95% (depending on the choice). Here a confidence level of 90% has been chosen. The zenith resolution achieved at this point is about 17°.

### 6.7. Azimuth reconstruction

In 4.8 a possibility was shown to reconstruct the azimuth with the orientation of the 31PMTs on the DOM. The main problems were the wide field of view of the PMTs and their relative angle to each other (the closest PMTs are 60° apart). Furthermore, it strongly depends on a well reconstructed zenith angle. In addition the zenith and azimuth of the arriving photons are used.

There have been two possibilities to select the zenith (azimuth) of the photon. Either the average photon zenith (azimuth) will be calculated for each DOM or the zenith (azimuth) of the first arriving hit is selected. The results achieved with the PPM-DU can be seen in the Figure 35 and Figure 36.
Results of MC/Data ; Comparison

Figure 35: Azimuth residuals calculated with the mean photon zenith/azimuth of every DOM and zenith from the track reconstruction. The Gaussian fit has revealed $\mu=37.8^\circ$ and $\sigma=69.0^\circ$.

Figure 36: Azimuth residuals calculated with the first photon zenith/azimuth of every DOM and zenith from the track reconstruction. The Gaussian fit has revealed $\mu=26.5^\circ$ and $\sigma=78.2^\circ$. 

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Both azimuth resolutions are not accurate. The azimuth reconstructed with the mean photon zenith/azimuth (Figure 35) has a resolution of 69.0°. For the azimuth reconstructed with the zenith/azimuth of the first photon, the resolution decreases to 78.2°. Furthermore, their mean are shifted from the center (37.81° and 26.46° respectively). It was anticipated to have a lower resolution for the azimuth due the large field of view and the PMT’s large angular separation. However, their resolutions are lower than expected. This may be attributed to the fact that the zenith/azimuth of the photons are not correlated with each other (despite being causally correlated). This means that the fit selection was not entirely correct. The track reconstruction is independent of the azimuth. As a possible solution eqn.23 can be combined with eqn.32 providing a new reconstruction algorithm. This algorithm still fits 4 parameters but the input variables have increased from 2 (t & z) to 4 (t, z, photon-zenith & photo-azimuth).

6.8. Analysis applied on PPM-DU Data

10 data files (each with a runtime of 30min) have been used. 58,080,140 events have been observed. The majority were events on a single DOM, 16331 events were 2-DOM coincidences, whereas only 296 events have been 3-DOM coincidences. Applying the same code on the PPM-DU data has shown the following results.

Figure 37 The figure displays the result from the inverse-gamma function for a given $\chi^2$ and ndf. The inverse-gamma function calculates what the probability is for a larger $\chi^2$ if the event with the same ndf would occur.

The figure points out that most events have a probability of ~50%, i.e. their fit is considered good. Only 205 of 296 are within the 90% confidence level. This means that the code has an event selection
Results of MC/Data; Comparison

of 70%, which is on almost on par with the value obtained from MC-events (77%). The difference in event selection is understandable. The code is designed for a fixed PPM-DU setup. However, the real PPM-DU setup is subject to the currents, which move the DOMs from their location.

Examples of successfully reconstructed events can be seen in Figure 39, Figure 38 and Figure 40.

Figure 39: Successful fit of event 5487505 from “RUN-PPM_DU-00651-20140923-105348”. The plot shows the arrival time (x-axis) vs the vertical height (y-axis) of each string

Figure 38: Successful fit of event 3640208 from “RUN-PPM_DU-00651-20140923-105348”. The plot shows the arrival time (x-axis) vs the vertical height (y-axis) of each string
Results of MC/Data; Comparison

Track_hits_3640208.000000
Entries: 6
Mean x: 60.36
Mean y: 6.193
RMS x: 54.19
RMS y: 29.25
Note that the fits were performed with a constraint on the range of the zenith; 120° to 180°.

6.9. Analysis applied on 18DOM string MC

The code can be also applied for a string with 18DOMs. The only modifications are the starting parameters for MINUIT. The upper bound of \( z_0 \) has been increased from 300m to 1000m (due to the increased size of the setup). Other parameters remain unchanged.

The initial runs with the 18DOM MC files have revealed a design flaw in the algorithm. In paragraph 4.3 it was discussed that there could be multiple L1 combinations for an event. This concept performs flawlessly with the PPM-DU. However, with increased number of L1s the number of combinations increases and thus requires more memory. In other words, an event that occurs on 4 (or more DOMs) requires more memory and thus more computation time. Therefore, only events with no more than 10000 will be fitted combinations (irrespective of how many DOMs have an L1).

A total of 21976 MC-events were processed. About 25% of these events pass the 10000-combination limit, whereas only 21% (of all events) were 3-DOM (or more) coincidences and causally correlated. Due to the increased number of DOMs, the number of 3-DOM coincidences has increased. This can be seen in Figure 41.

Figure 40: Successful fit of event 5487505 from “RUN-PPM_DU-00651-20140923-105348”. The plot shows the arrival time (x-axis) vs the vertical height (y-axis) of each string.

Figure 41: Coincidence distribution for the 18DOM setup. All hits in an inter-DOM coincidence are causally correlated.
The most remarkable aspect in Figure 41 is the extremely low number of hits on a single DOM (compared to Figure 20). This is most likely due to the increased size of the setup. As a result, it is less likely to only have hits on a single DOM.

The 18DOM string has been tested with and without the use of the residuals (see 4.5). In both cases the majority of the events did not pass the goodness-of-fit criteria (90% confidence level).

![Probability distribution](image)

Figure 42 The figure displays the result from the inverse-gamma function for a given $\chi^2$ and ndf. The inverse-gamma function calculates what the probability is for a larger $\chi^2$ if the event with the same ndf would occur. Only the probabilities of 3-DOM coincidences or more (4613 events) are shown in this figure. The x-axis is from 1% to 100%. Red: fit without residuals. Blue: fit with residuals

Figure 42 shows that the difference between the fit with and without the residuals is insignificant. Their event selection lies at 1028 and 1031 events (or 22%) respectively. The zenith resolution has not improved either. In both cases it is about 20° (3° lower compared to the zenith resolution of the PPM-DU).
As a result, it can be concluded that the residuals are also not very useful for the MC events. Their improvement is minimal and adds significant computation time.

The zenith distribution seen in Figure 43 is primarily dominated with 3DOM coincidences (as can be seen in Figure 41). As a result, the zenith resolution is similar to the PPM-DU. The zenith resolution should improve if 4 or more DOM coincidences are taken into account. Unfortunately, most 4 or more DOM coincidences do not pass the 10000 combinations constraint (explained earlier in this paragraph). As a result, the statistics is very low. The events that did pass the 10000 combinations constraint did have an improved zenith resolution of 6.9°.

The reconstruction of the azimuth angle has improved. The mean is now at 10.4deg. However, σ remains in the same order of magnitude as those obtained from the PPM-DU.
Comparing Figure 44 with Figure 36 shows that including more DOMs only improves the mean value. However, the resolution remains around 70°, which is probably the best possible resolution with the current DOM design.

Figure 44: Azimuth residuals calculated with the first photon zenith/azimuth of every DOM and zenith from the track reconstruction. The Gaussian fit has revealed $\sigma=73.8^\circ$. 
Figure 45 and Figure 46 show several examples of successful events.

Note that the fits were also performed with a constraint on the range of the zenith; 120° to 180°.
7. Conclusion

The algorithm for a single string can only reconstruct 4 of the 5 parameters. In addition to this bottleneck, a single string is degenerate within a certain phase-space. Both are fundamental drawbacks and hinder a correct track reconstruction. There is no solution to the latter bottleneck. However, the multi-PMT design of the DOMs offered an opportunity to calculate the 5th parameter (the azimuth angle). However, the calculation of the azimuth is sensitive to the zenith reconstruction. Furthermore, the PMTs have a relative angle of 60°. As a result, the azimuth resolution is around 70° and thus not sufficiently accurate for the 5th parameter.

Two possible reconstruction methods were used; the time-based and angle-based reconstruction. The angle-based track reconstruction was initially intended to complement the time-based reconstruction (or offer an improvement if possible). However, the results have shown that the angle-based reconstruction has a bias for the zenith angle at 120°. This is most likely due to only reconstructing 3 parameters.

The time-based reconstruction was performed and compared with L1 and L2 hits. 95% of all L1 hits are also L2 hits. As a result, no significant change was observed in the resolution. On the contrary, an improvement in the quality of the events was observed. Therefore, the analysis on the data was performed with L2s.

The reconstruction initially used all L2 hits with no bias. However, including a bias on the first arrival hits has improved the quality. The inclusion of the 3σ cut should also improve the quality (by rejecting outliers). However, this proved not to be the case and is therefore excluded in the final algorithm.

Finally, the goodness of fit technique selected events with a confidence level of 90%. With this criterion, the final algorithm had an event selection of 77% and 70% for the MC-files and data-files respectively. Furthermore, the zenith resolution is about 17°. The zenith resolution for an 18DOM string is about 20°. This is a significant decrease in performance for a full string. It is therefore advised to reevaluate the code for a full string. Preliminary analysis has shown that with 18DOMs you have more L2s and thus more combinations for an event. This requires a large amount of memory and thus large computation time. To prevent this, a cut was introduced that rejects about 75% of all events. Furthermore, the algorithm is designed to reconstruct only single muons in an event (i.e. it cannot reconstruct two or more muons in an event).
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