Do stock prices react fast to the market information?

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Abstract

How long does it take for stock prices to fully absorb market information, especially whether the opening market prices have already taken into account the over-night information remains arguable. We use novel estimators to measure the volatility, volume, and spread revealed in the high frequency stock prices. The observed highly volatile prices with small trading volumes upon the market opening implies market opening prices do not fully absorb the information. Besides, persistent increasing patterns in the holding returns of small capitalized stocks with good news, suggest a slow pace for prices to fully absorb the information. We further confirm our findings by introducing a low-volatility stage shown in prices, and empirical observations show small capitalized stocks take a longer time to reach the low-volatility period.
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Chapter 1
Introduction

Stock prices embed market information, and the information absorbing process has inspired a large literature. A particularly intriguing question is how long it takes for a newly released public available information to be totally absorbed into stock price. Hitherto, there is no consensus among researchers upon this question. Some believe that the information is absorbed very fast, as the classic efficient-market-hypothesis (EMH) claims that the prices of all traded assets already reflect all the public available information in an instant. However, some believe that it will take 5 to 90 minutes for information to be absorbed, see Siddiqui and Misra (2017), Louhichi (2008) and Muntermann and Guettler (2007). These papers utilize the intraday high frequency data. They use abnormal return and abnormal price reaction to demonstrate the information absorbing process. Siddiqui and Misra (2017) find most part of information is absorbed in the first 5 to 10 minutes by applying ARMA(1,1) model in India market. Louhichi (2008) finds the abnormal return will increase sharply in the first 15 minutes. Muntermann and Guettler (2007) get similar results. They find the abnormal price reaction will last 90 minutes. Another strand of literature employ daily data, the findings therein support a longer time for information to be absorbed into stock price. Jegadeesh and Titman (1993), Kothari and Warner (1997), Fama (1998), and Daniel et al. (1998) demonstrate that stock prices have persistent trends for 3 to 12 months after some import information is released.

There is a great diversity of information in the financial market, examples are quarterly financial report, annual financial report, earning announcement, profit warning, merger and acquisition announcement, etc. We will only focus on profit warning. Profit warning is a special information disclosure way: when a company finds its earnings does not match the analysts’ expectations, it will make a profit warning prior to the public announcement of its earnings, and the profit warning is usually announced before the market opening. We are interested to find out when the information will be absorbed into the stock prices.

Since companies release profit warning before the market opening, we need to examine whether the information has been well processed in overnight tradings such that stock prices at the market opening auctions already reflect the information. The view is diversified. Some believe that during overnight period, investors have more time to process the information. As a consequence, more market participants are better informed, informa-
tion asymmetry is reduced, and the information is well embedded into stock prices at the opening auction. Greene and Watts (1996) find that in NASDAQ, firm stock prices react similarly to earning announcement irrespective of the release time. Moreover, they find that for overnight announcement, the opening price gives most part of price change in that trading day. Hodge and Pronk (2006) detect similar pattern in NYSE-listed firms. Compared to intraday announcement, the overnight announcement presents smaller spread and higher depth. Thus both studies suggest that in NASDAQ and NYSE, information asymmetry is reduced and most part of information has been absorbed. Abad et al. (2009) show in SIBE, an order driven market\(^1\) in Spain, information asymmetry decreases at the market opening if the information is released during overnight non trading period. But information asymmetry increases in the post announcement period if the information is released during trading time. It indicates information is better processed during overnight period and then better absorbed into stock price at the market opening. Doyle and Magilke (2009) find similar results in a order driven market.

While the aforementioned studies suggest reduced information asymmetry, some other studies suggest otherwise, with Francis et al. (1992), Gennotte and Trueman (1996) and Libby et al. (2002) being notable exceptions. Some earlier researches show that specialists always manage the transaction cost carefully. They also try to protect themselves from uncertainty of the market. Francis et al. (1992) find that specialists prefer to submit partial order rather than full order onto order book after the information is released during overnight period. Being wary of the increased market uncertainty, they would like to observe the direction and momentum first and then gradually feed the planned orders into the market. This will shrink the market liquidity and worsen the information asymmetry. Gennotte and Trueman (1996) demonstrate that the information asymmetry not only depends on specialist ability to process the information, but market makers’ ability to discern the informed traders from noisy traders. They show that although there are specialists who can use overnight period to process the information better, the noise traders are also accumulating.\(^2\) Consequently, the presence of noise traders impede information diffusion. Libby et al. (2002) work support Gennotte and Trueman (1996) finding — they show that specialists believe that if the information is released during overnight period, market uncertainty will increase at the market opening.

We will use the changes of volatility, volume and spread at the market opening compared to normal period to show whether the information asymmetry increases or decreases. If the information asymmetry increases, then the information is not well absorbed into stock price. Otherwise, at least part of the information is absorbed. Next, we motive our choices of the three parameters.

Market price is determined by all participants’ quotes. Participants include informed traders, noise traders and market makers. They choose different strategies based on their informed level to set the quotes. On one hand, the increase of better informed market

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1. There are no market makers. The orders submitted from electronic terminals to the system are routed to the centralized limit-order book.
participants will reduce the uncertainty and mitigate the induced volatility. On the other hand, the presence of noise traders — who trade stocks as response to less information or just trade for the need of liquidity or portfolio re-balancing — sparks more price uncertainty, and further increases the volatility. Market makers also play a role in the price discovery process. Gennette and Trueman (1996) demonstrate that market makers try to discern the direction and magnitude of the informed trading from aggregated order flows. If market makers could disentangle the order flows in a less costly way, they would request less spread to compensate for the risk exposure, and the spread will shrink. With more informed traders involved in the trading, it is easier for market makers to discern the informed trading. As a result, the spread will shrink. Moreover, Libby et al. (2002) explain experts would put partial order when they confront increasing price uncertainty. The partial order will make the volume shrink.

Bamber (1987), Barron et al. (2009) find market reacts differently with respect to company sizes. Bamber (1987) finds small companies typically spur more surprising trading but Barron et al. (2009) give a quite opposite conclusion. Besides, Muntermann and Guetler (2007) document the non-index company’s stock exhibits a persistent price reaction after the information disclosure. At the beginning of our research, we also notice there exists asymmetry trading reaction at the market opening after the information disclosure between large cap stocks and small cap stocks. We then apply the Kolmogorov-Smirnov test to confirm the trading reaction difference. And by introducing the holding return, we will try to find whether the information absorbing process is longer for small cap stocks or not. If there exists a longer persistent trend for small cap stocks, then it takes longer for information to be absorbed into small cap stocks.

We are not only interested in the information absorbing process at the market opening, but also in the general information absorbing process. Specifically, we would like to study how long it needs for information to be absorbed into stock price for a general case, control for the stock cap sizes and information content. Inspired by Patell and Wolfson (1984), we introduce the equilibrium to further explore the information absorbing behaviour. The method is based on the estimation of spot volatility, and the equilibrium is defined as a consecutive low volatility period. Again, we will consider the information absorbing difference between the large cap stocks and small cap stocks.

The remaining part of the thesis will proceed as follows. Chapter 2 gives basic model settings and gives estimators of volatility, volume and spread as indicators of information asymmetry. Chapter 3 discusses empirical results whether information is absorbed at the market opening after the information is released. We show that the information asymmetry gets worse and information is not well absorbed into the stock prices. Chapter 3 also argues that small cap stocks have different trading reaction compared to large cap stocks. Moreover, we will show that there exists a persistent upgrade trend (on average) for small cap stocks.

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3Market makers are also called liquidity providers. In general, they are firms or individuals who are asked to trade regularly and on a continuous basis. In short, you do not have to sell the equity or other financial products directly to the one who are willing to buy it or visa versa. You can trade with market makers and they would hold it as inventory or sell it to others buy putting order on the order book. This would make the trading cost much lower especially when trading is quite sparse.
cap stocks given a positive profit warning. In chapter 4, we introduce the equilibrium
to give a general description of information absorbing process and discuss the differences
between small cap stocks and large cap stocks.
Chapter 2

Basic model settings and proposed estimators

We introduce information asymmetry\(^1\) to indicate whether information is absorbed into stock prices at the market opening after over-night information is released — the increase of information asymmetry suggests that information is not fully absorbed into stock price. According to Gennotte and Trueman (1996), noise traders increase the information asymmetry while informed traders decrease the information asymmetry. Following Greene and Watts (1996), Libby et al. (2002) and Abad et al. (2009), we employ the changes of volatility, volume and spread to measure the changes of information asymmetry.

We first give the univariate expression for the estimator of volatility from a very general geometric form of stock price:

\[
S_t = e^{\int_0^t \mu_s ds + \int_0^t \sigma_s dW_s}, \quad t \in [0, T] \quad X_t = \log(S_t),
\]

where \(S_t\) is the stock price at \(t\), \(\int_0^t \mu_s ds\) denotes the drift process and \(\int_0^t \sigma_s dW_s\) is the diffusion. \(W_t\) is the general Brownian motion. \(X_t\) is the log form of stock price, thus

\[
dX_t = \mu_t dt + \sigma_t dW_t, \quad t \in [0, T].
\]

The quadratic variation or integrated volatility of \(X\) measures the overall variation upon time \(t\), and it is given by

\[
[X, X]_t = \int_0^t \sigma_s^2 ds, \quad t \in [0, T].
\]

\(\sigma_t\), the spot volatility, is estimated by the method proposed by Bollerslev et al. (2016). With the return at \(j^{th}\) minute after the market opening on Day0 (the day upon which

\(^1\)The classic information asymmetry is mainly researched in contract theory and economics which states the situation that in a transaction activity, one of the parties has more or better information. Specifically, in trading activities, it is defined as different participants have different level of information. Generally, some can be called informed traders while others are noise traders.
the financial information is released) defined as $r_{\tau_0 + j\Delta t}^2$, the univariate expression for the volatility estimator is given

$$\hat{\sigma}_{\tau_i} = \left(\frac{1}{k_n \Delta_n} \sum_{j=1}^{k_n} r_{\tau_j + j\Delta t}^2\right)^{1/2}, \hat{\sigma}_{\tau_0} = \left(\frac{1}{k_n \Delta_n} \sum_{j=1}^{k_n} r_{\tau_0 + j\Delta t}^2\right) i = -1, -2, -3, -4, -5.$$ 

where $\tau_0$ denotes the market opening time right after the information is released on Day0. Similarly, $\tau_{-1}$ is the market opening time one day before the information disclosure and $\tau_1$ denotes the market opening time one day after the information disclosure. $\hat{\sigma}_{\tau_0}$ denotes the estimated volatility at the opening time right after the information is disclosed. Similarly, $\hat{\sigma}_{\tau_{-1}}$ denotes the estimated volatility at the opening time one day before the information is released.

Let $V_{\tau_0 + j\Delta t}$ denote the volume at $j^{th}$ minute after the market opening on Day0 and $s_{\tau_0 + j\Delta t}$ denote the spread at $j^{th}$ minute after the market opening on Day0, which is calculated as the difference of bid and ask price at time $\tau_0 + j\Delta t$: $s_{\tau_0 + j\Delta t} = P_{\tau_0 + j\Delta t}^{\text{ask}} - P_{\tau_0 + j\Delta t}^{\text{bid}}$. $\hat{D}_{\tau_0}$ and $\hat{s}_{\tau_0}$ denote the estimated volume $^3$ and spread at $\tau_0$ respectively. Suggested by Bollerslev et al. (2016), the estimator of volume $\hat{D}_{\tau_0}$ is given by

$$\hat{D}_{\tau_i} = \frac{1}{k_n} \sum_{j=1}^{k_n} V_{\tau_i + j\Delta t}, \quad \hat{D}_{\tau_0} = \frac{1}{k_n} \sum_{j=1}^{k_n} V_{\tau_0 + j\Delta t} i = -1, -2, -3, -4, -5.$$ 

Similarly, the estimator of spread $\hat{s}_{\tau_0}$ is given by

$$\hat{s}_{\tau_i} = \frac{1}{k_n} \sum_{j=1}^{k_n} s_{\tau_i + j\Delta t}, \quad \hat{s}_{\tau_0} = \frac{1}{k_n} \sum_{j=1}^{k_n} s_{\tau_0 + j\Delta t} i = -1, -2, -3, -4, -5.$$ 

(As will be discussed later, $\hat{\sigma}_{\tau_0}, \hat{D}_{\tau_0}, \hat{s}_{\tau_0}$ will be used as research group while $\hat{\sigma}_{\tau_i}, \hat{D}_{\tau_i}, \hat{s}_{\tau_i}; i = -1, -2, -3, -4, -5$ will be used as control group.)

Bollerslev et al. (2016) provide the limit distribution of the estimators of volatility and volume:

**Theorem 1 (Theorem 1 of Bollerslev et al. (2016))** Under Assumption 1, 2 and 3:$^4$

$$\sqrt{k_n}(\hat{D}_{\tau} - D_\tau) \overset{\mathcal{L}_F}{\rightarrow} \eta_{\tau}$$

$^2$r_{\tau_0 + j\Delta t} = X_{\tau_0 + j\Delta t} - X_{\tau_0 + (j-1)\Delta t}$. Noticing that $X_t = \log(S_t)$ and $X_{\tau_0 + j\Delta t} - X_{\tau_0 + (j-1)\Delta t} \approx (S_{\tau_0 + j\Delta t} - S_{\tau_0 + (j-1)\Delta t})/S_{\tau_0 + (j-1)\Delta t}$. Hence $r_{\tau_0 + j\Delta t}$ is the return at $\tau_0 + j\Delta t$. $\Delta t = 1\text{minute}$.

$^3$For the volume, with respect to positive profit warning and negative profit warning, we will use bid volume and ask volume respectively. As common sense follows, if good news arrives, market participants would be more willing to buy rather than sell. If we use ask volume here, we would not be able to distinguish whether the abnormal shrink of volume comes from the effect of using ask volume or information asymmetry, then it will bring up a serious endogenous problem.

$^4$Assumptions and related notations are given in Appendix A
And
\[ \sqrt{k_n} (\hat{\sigma}_{\tau,n} - \sigma_{\tau,n}) \xrightarrow{L} \eta_{\tau} \]
where \( \eta_{\tau}, \eta'_{\tau} \) are centered Gaussian with variance \( v_{\tau}, \sigma_{\tau}^2/2 \) respectively.

Now we argue the validity of the technical assumptions. Assumption 1 is fairly standard in the study of high-frequency data. For Assumption 1 (i) Bollerslev et al. (2016) consider a more general form of stock price. Compared to our geometric form of stock price, they add a jump process \( J_t = \int_0^t \epsilon_s dN_s + \int_0^t \int_\mathbb{R} \delta(s,z) \mu(ds,dz) \). Since it is more general, all our cases are included in their research framework. Besides, volume is described as \( V_{i\Delta_n} = V_{\zeta_{i\Delta_n}, \epsilon_{i\Delta_n}} \) and \( D_t = \int \mathcal{V}(\zeta_t, \epsilon) F_\epsilon(de) \), where the latent state process \( \zeta \) captures time-varying conditioning information such as the intensity of order arrival and the shape of the order size distribution, and \( \epsilon_{i\Delta_n} \) are i.i.d shocks with distribution \( F_\epsilon \) that capture the random behaviour of order arrival. \( \mathcal{V}(\cdot) \) is a possibly unknown transform. This setting gives a very general description of volume. Therefore, we can also use this form to describe our volume. This explains Assumption 1 (ii). Assumption 1 (iii) imposes a mild smoothness condition on \( \sigma \) and \( \zeta \); such assumptions facilitate the derivation of the asymptotic results. The practical validity seems reasonable as well. For \( \sigma \), Fig 4.4 shows the averaged \( \sigma \) curve is smooth. For \( \zeta \), it is a nonobservable process, but as common sense follows, every stock has market liquidity limit (the maximum volume is the total market cap size). When choosing a large enough \( K_n \), Assumption 1 (iii) is valid.

Assumption 2 is again a standard condition in the study of high-frequency data. \( k_n \) — a “tuning parameter” that often appears in (almost all) nonparametric estimators — captures the size of local window, which presumably goes to infinity as \( n \to \infty \). Specifically, we need \( k_n^2 \Delta_n \to 0 \) to obtain some limit results. It, however, can not be “too large” compared to the number of observations. For example, we can set \( k_n = \sqrt[3]{n} \) if \( \Delta_n = 1/n \). With \( n = 1440 \) (the number of minutes per day), \( k_n = 11.3 \). Following Louhichi (2008), Abad et al. (2009), Siddiqui and Misra (2017) and Bollerslev et al. (2016), we set \( k_n = 15,^5 \) in the later chapter of empirical results, corresponding to a 15-minute window. Assumption 3 (ii) imposes some smoothness conditions that are very mild. More details and proof can be referred to Bollerslev et al. (2016) appendix part.

The implemented research method is an adjusted event study method proposed by Fama et al. (1969). By setting research group and control group, the change of market liquidity in research group compared to control group can be calculated. The market liquidity in research group is represented by \( \hat{\sigma}_{m}, \hat{D}_{m}, \hat{s}_{m} \), and the estimators in control group are defined as:

\[ \hat{D} = \frac{1}{n} \sum_{|i|}^{n} \hat{D}_{\tau_i}, \quad n = 5, i \in \{-1, -2, -3, -4, -5\}, \]

\[ ^5 \text{Bollerslev et al. (2016) uses } k_n = 30 \text{ for S&P ETF. Although our choice is } k_n = 15, \text{ the result is not sensitive to the choice of } k_n. \]
\[
\hat{\sigma} = \frac{1}{n} \sum_{i}^{n} \hat{\sigma}_{1}, \quad n = 5, i \in \{-1, -2, -3, -4, -5\}, \\
\hat{s} = \frac{1}{n} \sum_{i}^{n} \hat{s}_{1}, \quad n = 5, i \in \{-1, -2, -3, -4, -5\},
\]

where these estimators are just the average value of \( D_{\tau_{i}}, \sigma_{\tau_{i}}, s_{\tau_{i}} \), the volume, volatility and spread in those five days before the information is released. \( \Delta D_{\tau_{0}}, \Delta \sigma_{\tau_{0}}, \Delta s_{\tau_{0}} \) denote the change of volume, volatility and spread at market opening on Day0, and they are estimated by

\[
\Delta D_{\tau_{0}} = \log \hat{D}_{\tau_{0}} - \log \bar{D}, \quad \Delta \sigma_{\tau_{0}} = \log \hat{\sigma}_{\tau_{0}} - \log \bar{\sigma}, \quad \Delta s_{\tau_{0}} = \log \hat{s}_{\tau_{0}} - \log \bar{s}.
\]

Note that the log form of differences represent percentage changes of market liquidity in our research group compared to control group. In the next chapter, we apply the aforementioned non-parametric estimators to the empirical date set, and the observed patterns suggest the increase of information asymmetry.
Chapter 3

Empirical study

3.1 Data description

We are interested in whether information asymmetry increases at market opening on Day0 (the day upon which the financial information is released). To answer the question, we employ the announcement events of profit warnings to analyze empirical data. Our data comes from Deep Blue Capital’s database. It covers 2,027 profit warning cases from 758 companies in Europe market. All these profit warnings are released during the period from October 2015 to May 2017. For each case, we have intraday data at 1-minute scale for 10 days. And each case contains data from 5 days before the profit warning and 5 days after the profit warning. Specifically, every 1-minute data consist of ask, bid and mid price (all are in euro), ask and bid volumes. Besides the intraday data, we also have data records price, bid and ask volumes at market opening and closing for all days. We perform some simple data-clean procedure. For example, we clean case whose opening price is 5 euro, but the intraday prices are all above 15 euro. We also clean cases which are empty or partially empty.

The severity of the profit warning is hard to quantify, for which the overnight return (OR) after the information disclosure serves as a proper proxy. We follow the rationale that greater overnight return is exerted by stronger profit warning. For example, suppose we have recorded two overnight returns, 2% and 5%. Then we will regard the profit warning which brings up 5% overnight return has stronger impact on stock price compared to the one which brings up 2% overnight return. To see whether the market liquidity changes differently corresponding to different overnight returns, the research group is divided into 4 sub-groups. Inspired by Patell and Wolfson (1984) and Louhichi (2008), we select $[-5\%, -2\%, 2\%, 5\%]$ as the criterion to separate all the cases in terms of overnight return.

We also consider the impact from stock’s market capitalization size. Normally, large cap stocks (greater than ten billion) have more impact on the whole financial market than medium and small cap stocks. A good example is blue chips that constitute market index as a measure of the general behaviour of financial market. As a consequence, such stocks are under close inspection of financial analysts, and any news related to the associated
corporations will spread into the market very rapidly. However, medium and small cap stocks, being watched by less analysts, do not possess such efficient information diffusion channel.

### 3.2 Estimation results

Table 3.1 shows the result of market liquidity changes measured by $\Delta D_{\tau_0}$, $\Delta \sigma_{\tau_0}$, $\Delta s_{\tau_0}$. In Table 3.1, we divide all the cases into 2 parts and 8 groups to illustrate whether the overnight return and companies capitalized size affect the information absorbing process. The upper part of Table 3.1 shows results for all small cap stocks while the lower part gives results for large cap stocks. In each part, they are further separated into 4 groups by overnight returns. For example, the data in the row 2 column 2, $-0.0920^{***}$ means the average change of volume is $-0.0920$ at 1% significance level for small cap stocks having lower than $-2\%$ overnight return. This value indicates volume $D_{\tau_0}$ decreases 9.2% at market opening on Day0 compared to control group. The result in row 7 column 4 denotes volatility increases 73% at 1% significance level for large cap stocks with more than 5% overnight return.

All $\Delta \sigma_{\tau_0}$ results indicate at least 70% increase of volatility. According to Gennotte and Trueman (1996), Hodge and Pronk (2006) and Abad et al. (2009), if the information asymmetry is reduced, then the volatility would be at least unchanged. With sharply increased volatility, information asymmetry is getting worse rather than being improved. There is no obvious difference in volatility change between large cap and small cap stocks. However, volatility changes differently with different overnight returns. From results of $\Delta \sigma_{\tau_0}$, higher absolute overnight return brings up higher $\Delta \sigma_{\tau_0}$. Specifically, $\Delta \sigma_{\tau_0}$ in $OR < -5\%$ group is higher than in $OR < -2\%$ group and $\Delta \sigma_{\tau_0}$ in $OR > 5\%$ group is higher than in $OR > 2\%$ group. Besides, negative overnight return leads to higher $\Delta \sigma_{\tau_0}$. This is generally called leverage effect in volatility. It says decreased stock price brings up higher volatility than increased stock price. The leverage effect is just used to explain one interesting result in our research, more details about leverage effect of volatility can be found in Choi and Richardson (2016). The leverage effect can be detected by GARCH model, similar result got from our nonparametric method proves the estimator we apply here is simple but powerful. Besides the increased volatility, volume decrease is also observed. For $\Delta D_{\tau_0}$, although there are values which have no statistical significance, most of them show statistically significant decrease, especially for large cap stocks. For large cap stocks, except for the value in $OR < -5\%$ group, all the others record $\Delta D_{\tau_0} \approx -20\%$ indicating around 20% of volume shrink. The result is quite similar to Francis et al. (1992) that the price uncertainty will make specialists put only partial order on the order book and wait for the clear direction of the price. Libby et al. (2002) also offer similar results.

There is another interesting observation, the large cap stocks’ volumes shrink more than small cap stocks’ irrespective of overnight return. The intuition is as follows. As discussed before, compared to small cap stocks, large cap stocks have more specialists followers with sophisticated skills to process the information and make themselves informed traders.
Table 3.1: This table shows results for information asymmetry. Specifically, it shows the changes of volatility ($\Delta \sigma_{\tau_0}$), volume ($\Delta D_{\tau_0}$) and spread ($\Delta s_{\tau_0}$) at market opening on Day 0. Positive $\Delta \sigma_{\tau_0}$, $\Delta s_{\tau_0}$ and negative $\Delta D_{\tau_0}$ indicate that volatility and spread increase while volume decreases compared to control group. These changes indicate that information asymmetry increases. The increased information asymmetry shows that information is not fully absorbed into stock price at market opening after its disclosure. (*, **, *** denote the significant level at 10%, 5% and 1%).

<table>
<thead>
<tr>
<th></th>
<th>$OR &lt; -5%$</th>
<th>$OR &lt; -2%$</th>
<th>$OR &gt; 2%$</th>
<th>$OR &gt; 5%$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Small Cap</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta D_{\tau_0}$</td>
<td>-0.0367</td>
<td>-0.0920***</td>
<td>-0.0773***</td>
<td>-0.0241</td>
</tr>
<tr>
<td>$\Delta \sigma_{\tau_0}$</td>
<td>1.1345***</td>
<td>0.9859***</td>
<td>0.8172***</td>
<td>0.8587***</td>
</tr>
<tr>
<td>$\Delta s_{\tau_0}$</td>
<td>0.0979**</td>
<td>0.1284***</td>
<td>0.0997***</td>
<td>0.1051***</td>
</tr>
<tr>
<td><strong>Large Cap</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta D_{\tau_0}$</td>
<td>-0.1383</td>
<td>-0.2158***</td>
<td>-0.1885***</td>
<td>-0.2082**</td>
</tr>
<tr>
<td>$\Delta \sigma_{\tau_0}$</td>
<td>1.3978***</td>
<td>0.9176***</td>
<td>0.6516***</td>
<td>0.7300***</td>
</tr>
<tr>
<td>$\Delta s_{\tau_0}$</td>
<td>0.1035</td>
<td>0.1484***</td>
<td>0.0855***</td>
<td>0.0216***</td>
</tr>
</tbody>
</table>

Table 3.2: Data size for different groups

<table>
<thead>
<tr>
<th></th>
<th>$OR &lt; -5%$</th>
<th>$OR &lt; -2%$</th>
<th>$OR &gt; 2%$</th>
<th>$OR &gt; 5%$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Small Cap</strong></td>
<td>63</td>
<td>206</td>
<td>300</td>
<td>59</td>
</tr>
<tr>
<td><strong>Big Cap</strong></td>
<td>28</td>
<td>131</td>
<td>182</td>
<td>30</td>
</tr>
</tbody>
</table>
When they anticipate the greater uncertainty of price and higher trading cost, they would put partial orders onto the order book. Easley et al. (2001) show the worsening market liquidity will make informed traders split orders. The split action will not only further damage the liquidity, but also affect informed traders next action to put less orders onto the order book. Therefore, the higher proportion of informed traders, the less orders will be put on the order book. The last market liquidity indicator that should be discussed is $\Delta s_{\tau_0}$, the spread change. It is recorded that $\Delta s_{\tau_0} \approx 10\%$. As mentioned above, spread is the main source of market markers’ profit. Since market making is highly competitive, market makers have no incentive to increase the spread unless there is a higher risk premium to compensate their risk exposure. Hence the increased spread can also verify that the price uncertainty increases at the market opening after the information disclosure on Day0.

Given the significance level, the volume shrinks while the volatility and spread increase irrespective of company capitalized size and overnight return. Although there are 2 $\Delta D_{\tau_0}$ results that are not statistically significant, others are. Besides, all $\Delta \sigma_{\tau_0}$ and $\Delta s_{\tau_0}$ are statistically significant. The decrease of volume and increase of volatility and spread indicate the increased information asymmetry. Together with theory proposed by Greene and Watts (1996), we conclude that the information is not fully absorbed into stock price at the market opening on Day0.

In Table 3.1, we observe that $\Delta D_{\tau_0}$ differs between large cap stocks and small cap stocks. Since $\Delta D_{\tau_0}$ in Table 3.1 are just the mean values of $\Delta D_{\tau_0}$ in each group, to make sure small cap stocks have better volume situation than large cap stocks, we need to verify the distribution of $\Delta D_{\tau_0}$ for small cap stocks locate a bit more right than the distribution of $\Delta D_{\tau_0}$ for large cap stocks. Therefore, we introduce Kolmogorov-Smirnov test and histogram of $\Delta D_{\tau_0}$ to clarify it. Table 3.3 shows the statistics result of Kolmogorov-Smirnov test for the distribution of $\Delta D_{\tau_0}$ from large cap stocks group and small cap stocks group. By following the same principle to separate samples in Table 3.1, all the cases are divided into 4 groups based on the overnight return. The KS-pvalues denote P values from Kolomgorov-Smirnov two-sample test with respect to $\Delta D_{\tau_0}$ from large cap stocks and small cap stocks. We want to know whether $\Delta D_{\tau_0}$ from large cap stocks distributes differently with $\Delta D_{\tau_0}$ from small cap stocks.

<table>
<thead>
<tr>
<th>$OR &lt; -5%$</th>
<th>$OR &lt; -2%$</th>
<th>$OR &gt; 2%$</th>
<th>$OR &gt; 5%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>KS – Pvalues</td>
<td>0.7640</td>
<td>0.2398</td>
<td>0.1167</td>
</tr>
</tbody>
</table>

Table 3.3: The figure shows Kolmogorov-Smirnov test results for the distribution of $\Delta D_{\tau_0}$ w.r.t large and small cap stocks. The last two KS values indicate that in $OR > 2\%$ group and $OR > 5\%$ group, distributions of $\Delta D_{\tau_0}$ are different between large cap stocks and small cap stocks at approximately 10\% significance level.

A higher p-value for K-S test rejects the null hypothesis that the two samples come from the same distribution. From results in Table 3.3, we find for $OR > 2\%$ group and for $OR > 5\%$ group, we can almost reject the null hypothesis at 10\% level. However, the KS-p
value for $OR < -2\%$ and $OR < -5\%$ could not allow us to reject the null hypothesis. For $OR > 2\%$ group and for $OR > 5\%$ group, the KS test shows the distribution of $\Delta D_{\tau_0}$ for large and small cap stocks are statistically different. To make sure the result is robust to extreme values, we also plot a histogram of $\Delta D_{\tau_0}$ in Figure 3.1 for large cap stocks and small cap stocks in $OR > 2\%$ group to clarify it.

![Histogram of Volume Change](image)

**Figure 3.1:** The figure shows $\Delta D_{\tau_0}$ distribution for large cap stocks and small cap stocks having $OR > 2\%$. Specifically, it gives normalized distribution of $\Delta D_{\tau_0}$. It shows distribution of $\Delta D_{\tau_0}$ for small cap stocks locates a bit more right than the distribution of $\Delta D_{\tau_0}$ for large cap stocks. It indicates small cap stocks have less shrink of volume than large cap stocks.

Figure 3.1 depicts the distribution of $\Delta D_{\tau_0}$ for large cap stocks (red) and small cap stocks (blue) having $OR > 2\%$. On closer inspection, the distribution of $\Delta D_{\tau_0}$ from small cap stocks locates more right than the distribution of $\Delta D_{\tau_0}$ from large cap stocks. Together with higher mean values of $\Delta D_{\tau_0}$ from Table 3.1 and around 10% significant level from KS test, for cases having positive profit warning ($OR > 2\%$ and $OR > 5\%$), the $\Delta D_{\tau_0}$ from small cap stocks is larger than $\Delta D_{\tau_0}$ from large cap stocks. It indicates the shrink of volume for small cap stocks is smaller than that for large cap stocks under positive profit warning shock.

Francis et al. (1992), Easley et al. (2001) and Libby et al. (2002) document that when facing great market uncertainty, the specialists will not put all orders onto the order book at one time. Instead, they will put partial orders or split the orders into small ones and put them gradually onto the order book. Since there are more specialists following large cap stocks, higher shrink of volume may come from their carefulness as a response to the increasing price uncertainty. By contrast, with less specialists following the small cap
stocks, the partial order effect and order splitting effect are not that strong, hence less volume shrink is observed. Since there is less specialists following small cap stocks, then the information will be absorbed slower than large cap stocks. If this speculation is correct, stock price would show a persistent upgrade trend for small cap stocks. In order to verify this speculation, we introduce holding return (HR) defined as $HR = \log(P_T/P_0)$. $P_T$ is the closing auction price at Day $i \in \{1, 2, 3\}$. $P_0$ is the opening auction price at Day 0.

Table 3.4 only shows the result for small cap stocks. We apply two standards to divide all the cases into groups. The first is again the overnight return, and the second is $\Delta D_{\tau_0}$. Different cases have different volume change. We select 3 different $\Delta D_{\tau_0}$ as standards to separate these cases. The first standard *Allsample* includes all samples in the group separated by overnight return. The second standard $\Delta D_{\tau_0} > 0$ denotes cases which have positive volume change. The last standard is chosen to include cases having $\Delta D_{\tau_0}$ locating in the highest quantile of all $\Delta D_{\tau_0}$.

Table 3.4 shows the holding return (HR) values in different groups from Day0 to Day1, Day2 until Day3. For example, in row 6 column 3, we read the result 0.0123. It denotes the average holding return from Day0 to Day1 is 1.23% for cases in $OR > 2\%$ group. In row 7 column 2, we get −0.0031. It denotes the average holding return from Day0 to Day1 is −0.31% for cases which have $\Delta D_{\tau_0} > 0$ and $OR < −2\%$.

For small cap stocks in $OR > 2\%$ group and $OR > 5\%$ group, from Day0 to Day3, the average holding return (HR) shows an persistent upgrade trend. For example, average $HR$ for samples in $OR > 2\%$ group increases from 0.7% to 1.7% approximately. In $OR > 5\%$ group, the average $HR$ increases from 0.18% to 1.78%. The result implies that from Day0 to Day3, the good news is kept absorbed into stock price. However, for large cap stocks, there exists no such trend. Small cap stocks having negative overnight returns do not show this persistent trend, either. In economics analysis, there always exits such asymmetry phenomena. For instance, Louhichi (2008) finds there will be a slight reverse for a former decreased stock price.

Besides, the increase of $\Delta D_{\tau_0}$ brings up the increase of $HR$. For example, the $HR$ in Day3 increases from 1.73% to 2.98% with the increase of $\Delta D_{\tau_0}$. In addition, the $HR$ in Day2 increases from 1.45% to 2.72% with the increase of $\Delta D_{\tau_0}$. It indicates that on average level, the increased volume will bring up a greater upgrade trend.

We notice that the improvement of $\Delta D_{\tau_0}$ brings up a greater $HR$. Therefore, we want to know whether spread, another liquidity indicator, has similar indicating power as volume does. Specifically, we want to know whether the improvement of spread will also bring up a similar upgrade $HR$ trend. To achieve this, we separate all samples into different groups based on the overnight return $OR$. For samples in each group, they are further divided into sub-groups by different spread change. $\Delta s_{\tau_0} < 0$ denotes the spread shrinks compared to control group and $\Delta s_{\tau_0} < 0.25$ quantile denotes the spread change locates in the lowest quantile. All results shown in Table 3.6 are again the holding return $HR$ for small caps from the market opening at Day0 to market closing at Day0, Day1, Day2 until Day3. For example, the value at row 7 column 3 is read as 0.0188. It denotes the average $HR$ from Day0 to Day1 is 1.88% for cases which have $OR > 2\%$ and $\Delta s_{\tau_0} < 0$.

In Table 3.4, we find the $HR$ increases with the increase of $\Delta D_{\tau_0}$. In other words, the
<table>
<thead>
<tr>
<th>Day</th>
<th>All sample</th>
<th>$\Delta D_{\tau_0} &gt; 0$</th>
<th>$\Delta D_{\tau_0} &gt; 0.75$ quantile</th>
<th>OR $&lt; -5%$</th>
<th>OR $&lt; -2%$</th>
<th>OR $&gt; 2%$</th>
<th>OR $&gt; 5%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day0</td>
<td>0.0077</td>
<td>-0.0014</td>
<td>0.0077</td>
<td>0.0018</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0037</td>
<td>-0.0030</td>
<td>0.0108</td>
<td>0.0103</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0004</td>
<td>-0.0035</td>
<td>0.0134</td>
<td>0.0111</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Day1</td>
<td>0.0069</td>
<td>-0.0017</td>
<td>0.0123</td>
<td>0.0105</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0036</td>
<td>-0.0031</td>
<td>0.0157</td>
<td>0.0216</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.0037</td>
<td>-0.0010</td>
<td>0.0207</td>
<td>0.0188</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Day2</td>
<td>0.0090</td>
<td>-0.0006</td>
<td>0.0145</td>
<td>0.0128</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0052</td>
<td>-0.0038</td>
<td>0.0193</td>
<td>0.01844</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0004</td>
<td>-0.0047</td>
<td>0.0272</td>
<td>0.0215</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Day3</td>
<td>0.0142</td>
<td>-0.0006</td>
<td>0.0173</td>
<td>0.0178</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0050</td>
<td>-0.0040</td>
<td>0.0213</td>
<td>0.270</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.0008</td>
<td>-0.0026</td>
<td>0.0298</td>
<td>0.0291</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.4: The table shows averaged holding return ($HR, HR = \log(P_{T_i}/P_{T_0})$ from Day0 to Day1, Day2 and Day3 for small cap stocks. $P_{T_i}$ is closing price at Day $i$ ($i \in \{0, 1, 2, 3\}$) and $P_{T_0}$ is the opening price at Day0. For small cap stocks with positive overnight return ($OR > 2\%, OR > 5\%$), $HR$ shows a upgrade persistent drift (shown in red). For example, $HR$ in $OR > 2\%$ group increases from 0.77% to 1.73%. Besides, with the increase of $\Delta D_{\tau_0}$, $HR$ increases as well.(Shown in grey.)
\[
\begin{array}{cccc}
OR < -5\% & OR < -2\% & OR > 2\% & OR > 5\% \\
\hline
\text{Big cap} & & & \\
\hline
\text{Day0} & -0.0078 & 0.0024 & -0.0029 & -0.0065 \\
\text{Day1} & -0.0172 & 0.0047 & -0.0023 & -0.0004 \\
\text{Day2} & 0.0083 & 0.0112 & -0.0007 & 0.0081 \\
\text{Day3} & 0.0084 & 0.0126 & -0.0024 & 0.0053 \\
\end{array}
\]

Table 3.5: The figure shows holding return results for large cap stocks. HR results for large cap stocks do not indicate any persistent trend.

improvement of volume will bring up the increase of holding return HR. However, Table 3.6 reveals that a narrowing spread is not associated with an increase return. Taking an example from HR at Day3 in OR > 2% group, with the narrowing spread, the HR first increases from 1.73% to 2.27%, but then decrease to 1.74%. This implies the decrease of spread can not bring up the increase of average holding return HR. Therefore, only the improvement of volume can bring up a greater HR.
<table>
<thead>
<tr>
<th>Day</th>
<th>All sample</th>
<th>$OR &lt; -5%$</th>
<th>$OR &lt; -2%$</th>
<th>$OR &gt; 2%$</th>
<th>$OR &gt; 5%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All sample</td>
<td>0.0077</td>
<td>-0.0014</td>
<td>0.0077</td>
<td>0.0018</td>
<td></td>
</tr>
<tr>
<td>$\Delta s_{r_0} &lt; 0$</td>
<td>0.0129</td>
<td>-0.0034</td>
<td>0.0104</td>
<td>0.0008</td>
<td></td>
</tr>
<tr>
<td>$\Delta s_{r_0} &lt; 0.25$ quantile</td>
<td>0.0108</td>
<td>-0.0047</td>
<td>0.0092</td>
<td>-0.0023</td>
<td></td>
</tr>
<tr>
<td>Day1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All sample</td>
<td>0.0069</td>
<td>-0.0017</td>
<td>0.0123</td>
<td>0.0105</td>
<td></td>
</tr>
<tr>
<td>$\Delta s_{r_0} &lt; 0$</td>
<td>0.0089</td>
<td>-0.0061</td>
<td>0.0188</td>
<td>0.0171</td>
<td></td>
</tr>
<tr>
<td>$\Delta s_{r_0} &lt; 0.25$ quantile</td>
<td>0.0027</td>
<td>-0.0079</td>
<td>0.0179</td>
<td>0.0183</td>
<td></td>
</tr>
<tr>
<td>Day2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All sample</td>
<td>0.0090</td>
<td>-0.0006</td>
<td>0.0145</td>
<td>0.0128</td>
<td></td>
</tr>
<tr>
<td>$\Delta s_{r_0} &lt; 0$</td>
<td>0.0042</td>
<td>-0.0079</td>
<td>0.0201</td>
<td>0.0159</td>
<td></td>
</tr>
<tr>
<td>$\Delta s_{r_0} &lt; 0.25$ quantile</td>
<td>0.0007</td>
<td>-0.0082</td>
<td>0.015</td>
<td>0.0135</td>
<td></td>
</tr>
<tr>
<td>Day3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All sample</td>
<td>0.0142</td>
<td>-0.0006</td>
<td>0.0173</td>
<td>0.0178</td>
<td></td>
</tr>
<tr>
<td>$\Delta s_{r_0} &lt; 0$</td>
<td>0.0085</td>
<td>-0.0067</td>
<td>0.0227</td>
<td>0.0228</td>
<td></td>
</tr>
<tr>
<td>$\Delta s_{r_0} &lt; 0.25$ quantile</td>
<td>0.0051</td>
<td>-0.0068</td>
<td>0.174</td>
<td>0.0212</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.6: The figure shows holding return for small cap stocks w.r.t $\Delta s_{r_0}$. Not like $\Delta D_{r_0}$, $\Delta s_{r_0}$ does not have similar indication power. Specifically, $HR$ does not increase with the decrease of $\Delta s_{r_0}$.
3.3 Robustness check

To check the robustness of HR results we have got from small cap stocks having positive overnight returns, we consider two problems: whether the HR results are greatly affected by extreme values? Are results mainly contributed by the market trend? To avoid the bias introduced by extreme values, the data description is given in the form of quantile values. Besides, we calculate the mean value only for data locating between the first quantile and the last quantile shown as Fmean. Fmean is calculated only by half of data which locate at the middle part of the data distribution. Since the persistent upgrade trend only exists among samples with positive overnight return (OR), we hence only give results of HR from Day0 to Day3 in OR > 2% group. Results are shown in Table 3.7:

<table>
<thead>
<tr>
<th></th>
<th>All sample</th>
<th>ΔD&lt;sub&gt;τ_0&lt;/sub&gt; &gt; 0</th>
<th>ΔD&lt;sub&gt;τ_0&lt;/sub&gt; &gt; 0.75 quantile</th>
</tr>
</thead>
<tbody>
<tr>
<td>min</td>
<td>-0.1368</td>
<td>-0.1332</td>
<td>-0.111069</td>
</tr>
<tr>
<td>25%</td>
<td>-0.0125</td>
<td>-0.0081</td>
<td>0.000683</td>
</tr>
<tr>
<td>50%</td>
<td>0.0192</td>
<td>0.0223</td>
<td>0.030183</td>
</tr>
<tr>
<td>75%</td>
<td>0.0530</td>
<td>0.0600</td>
<td>0.063591</td>
</tr>
<tr>
<td>max</td>
<td>0.238411</td>
<td>0.1585</td>
<td>0.158569</td>
</tr>
<tr>
<td>Fmean</td>
<td>0.0185</td>
<td>0.0234</td>
<td>0.0305</td>
</tr>
</tbody>
</table>

Table 3.7: This table shows the data description of HR at Day3 from small cap stocks with OR > 2%. We use data between 25% quantile and 75% quantile to calculate Fmean, aiming to avoid the affection from extreme values. Fmean results show that HR increases with the increase of ΔD<sub>τ_0</sub>. And these results are immune from extreme values.

Fmean results in Table 3.7 imply that the average HR is still significantly positive after deleting extreme values from calculation. Besides, the increase of ΔD<sub>τ_0</sub> still brings up the increase of Fmean HR. Table 3.8 shows the average index return (in the Stoxx50 return column) and the corrected holding return (Corrected HR) (Corrected HR is calculated by minusing the index return<sup>1</sup> from the original HR). The selected index is STOXX 50 index in Europe.<sup>2</sup> Similar to results shown in Table 3.7, only results of HR from Day0 to Day3 in OR > 2% group is shown.

---

<sup>1</sup>The index return is calculated from the same time period that HR is calculated

<sup>2</sup>STOXX 50 is the leading blue-chip index in Eurozone stock market.
Table 3.8: This table shows corrected HR at Day3 in small cap stocks group with OR > 2% and index return. When we minus the averaged Stoxx50 return from averaged HR, we get CorrectedHR. CorrectedHR results are robust to market trend. After controlling the market trend, we still observe that Corrected HR increases with the increase of $\Delta D_{\tau}$. 

The CorrectedHR results in Table 3.8 imply that the market trend does not seriously affect HR results. And the increase of HR with the increase of $\Delta D_{\tau}$ has not been affected. 

Now we implement a simple trading strategy to further check the robustness of the persistent upgrading trend. For all small cap stocks with positive overnight return, we buy 5% of shares available at the opening auction on Day0. If the total value exceeds 100k euro, we only buy 100k euros of this stock. Then we hold them until the closing time at the Day3 and sell them at that moment. Figure 3.2 shows aggregated profit and loss graph from Day0 to Day3. To get the aggregated profit and loss graph, we add all profits and losses together for all cases mentioned above. 

Results in Table 3.4 show that holding return HR keeps increasing from Day0 to Day3 for small cap stocks in OR > 2% group. However, the aggregated profit shown in Fig 3.2 tells otherwise. Sub-fig (a) in Fig 3.2 shows an obvious upgrade trend indicating the information is kept being absorbed into stock price at Day0. This is in line with results in Table 3.4. Sub-fig (b) indicates the aggregated profit jumps to 9500 euro at the market opening, then it keeps fluctuating around 90000 euro. Most part of sub-fig (c) and sub-fig (d) imply that profit is fluctuating around 85000 euro. This can be regarded as noise behavior. Therefore, we get no proof from sub-fig (c) and sub-fig (d) that information is absorbed into stock prices on Day2 and Day3. This contradicts the result we get in Table 3.4. We believe the difference of results comes from different calculation methods. For the calculation of HR in Table 3.4, every stock has the same weight. However, when calculate the profit, purchase cost is treated as potential weight. Stocks having higher purchase cost have higher weight. 

Table 3.9 shows the statistics description of purchase cost at Day0 (value of 5% shares available at the opening auction or 100k) and statistics description of profit at Day0, Day1, Day2 and Day3 closing auction. Results in Table 3.9 show the aggregated profit increases sharply until the closing auction at Day1. This is similar to results we get in Fig 3.2. It again shows the persistent trend only lasts 2 days from aggregated profit perspective. If we follow the above mentioned strategy and hold the position until Day1 closing auction,
Figure 3.2: These four figures show profit and loss graph from Day0 to Day3 for small cap stocks having positive overnight return. Sub-fig (a) shows an obvious upgrade trend of profit. The aggregated profit increases from 10000 euro to around 60000 euro. Sub-fig (b) shows at market opening auction on Day1, the aggregated profit jumps to around 95000 euro. For the rest of Day1, the aggregated profit only fluctuates around 90000 euro. This fluctuation behavior can be regarded as noise. For most part in sub-fig (c) and sub-fig (d), since the aggregated profit keeps fluctuating around 85000, they are regarded as noise as well. Therefore, from profit angle, the information is kept being absorbed into stock price on Day0 and market opening on Day1.
we would have on average 1% holding return ($HR$) (312/32161) and the total profit would be 92728.93 euro. The 1% profit ratio is similar to the 1.23% $HR$ in Table 3.4. For a 2-day holding period, 1% is a good return rate. Especially a on average 32161 euro investment can bring 927729 euro return. It is a quite worthy business.

<table>
<thead>
<tr>
<th>Purchase cost</th>
<th>Day0 profits</th>
<th>Day1 profits</th>
<th>Day2 profits</th>
<th>Day3 profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>minimum</td>
<td>5.42</td>
<td>-9181.06</td>
<td>-6905.24</td>
<td>-7563.05</td>
</tr>
<tr>
<td>25%</td>
<td>7038.62</td>
<td>144.14</td>
<td>-130.77</td>
<td>-117.11</td>
</tr>
<tr>
<td>50%</td>
<td>19140.61</td>
<td>63.86</td>
<td>131.83</td>
<td>155.25</td>
</tr>
<tr>
<td>75%</td>
<td>48185.98</td>
<td>581.66</td>
<td>843.10</td>
<td>817.90</td>
</tr>
<tr>
<td>maximum</td>
<td>100000</td>
<td>8208.38</td>
<td>11379.31</td>
<td>11689.65</td>
</tr>
<tr>
<td>mean</td>
<td>32161.26</td>
<td>151.07</td>
<td>312.21</td>
<td>311.62</td>
</tr>
<tr>
<td>profit</td>
<td>44868.02</td>
<td>92728.93</td>
<td>92552.01</td>
<td>94977.79</td>
</tr>
</tbody>
</table>

Table 3.9: This table shows the description of purchase cost at Day0 and description of profits at Dayi. (i = 0,1,2,3). We observe that profit increases sharply at Day0 and Day1. At Day2 and Day3, profit does not increase a lot.

### 3.4 Empirical implication

Our empirical findings imply that the information asymmetry increases at market opening after the profit warning disclosure, thus we conclude that information is not fully absorbed into stock price at market opening. From Table 3.1, we find the liquidity change is different between large cap stocks and small cap stocks. A Kolmogorov-Smirnov test further confirms that the volume changes are statistically different for large cap stocks and small cap stocks having positive overnight return. Using holding return ($HR$), we find for a small cap stock, a persistent upgrading trend for stock price is attributed to a positive overnight return brought by corresponding information. Other indicators of an increase of $HR$ include the volume increases at the market opening, but not a decrease in spread.

In short, for small cap stocks having positive overnight return, the information is kept being absorbed into stock price in the coming days. For large cap stocks and small cap stocks with negative overnight return, such evidence have not been found to support the
argument.
Chapter 4

Refined volatility estimator and equilibrium

In chapter 3, we have argued two interesting results related to information absorbing. We find that information is not fully absorbed into stock price at market opening on Day0. And for small cap stocks whose overnight returns at Day0 are positive, it takes days for information to be absorbed. Specifically, this information absorbing process is shown as a persistent upgrading trend. However, there does not exist consensus on a general method to describe the information absorbing process for a general case. Therefore, we introduce “equilibrium”\(^1\) to illustrate this problem.

The idea comes from a general observation: when most part of the information is absorbed into stocks price, participants of the stock would show consensus over the stock price. Intuitively, when the consensus is reached at a certain level, participants would have no willingness to trade more since most value of the information has already been integrated into stock’s price. Less trading willingness will induce sparser orders put onto the order book. Besides, the volatility will also decrease. We call the status with low volatility and low volume the equilibrium. Therefore, if we can detect such equilibrium, we can then treat the equilibrium period as the time when the information is absorbed into the stock price.

4.1 Econometric theory of volatility estimator

In the first chapter, a simple form of volatility estimator has been given

\[
\hat{\sigma}_{\tau_0} = \sqrt{\frac{1}{k_n \Delta_n} \sum_{j=1}^{k_n} r_{\tau_0+j \Delta t}^2}
\]

where \(k_n = 15\) and \(\Delta t = 1\) minute.

\(^1\)We will show later that this equilibrium is a empirical method to catch a consecutive low volatility period. It is not the same equilibrium used in theoretical research.
We introduce this volatility from a very basic spot volatility definition. Details for bandwidth and time frequency selection have not been discussed. In the later discussion, all these details will be covered.

We start with introducing a general background of volatility estimation. It aims to give readers a first impression of volatility estimator. Volatility estimation has evolved a lot, and the main attention is focused on spot volatility and integrated volatility over a certain period of time. The popular methods for univariate integrated volatility include realized volatility (RV) by Andersen et al. (2003), two-time scale realized volatility (TSRV) by Zhang et al. (2005), (MSRV) multiple-time scale realized volatility (MSRV) by Zhang et al. (2006), wavelet realized volatility and kernel realized volatility (KRV) by Fan and Wang (2008), Barndorff-Nielsen and Shephard (2004). Details for these estimators can be referred to a well organized literature review by Pigorsch et al. (2012). For spot volatility, since Foster and Nelson (1994) proposed a rolling and block sampling filters method to estimate, there follows a lot others work to extend this area, namely Fan and Wang (2008), Andreou and Ghysels (2002) and Kristensen (2010). Malliavin et al. (2009) start another way based on Fourier-based method for spot volatility. The above works also consider the situation with or without noise.

As discussed before, the quadratic variation of $X_t$ has the expression

$$[X, X]_t = \int_0^t \sigma^2_s ds, \quad t \in [0, T]$$

Suppose the time interval $[0, T]$ is divided by $n$ discrete points such that the $i^{th}$ time points is denoted as $t_i = \frac{it}{n}, \quad i = 1, \ldots, n$. Our goal is to estimate

$$\sigma^2_t = \frac{d[X, X]_t}{dt}$$

To achieve this, we first explain parameters and variables for the estimation. Suppose $K(x)$ is the kernel with support on the interval $[-1, 1]$ which has the property that $\int_{-1}^1 K(x)dx = 1$, $[t - b, t + b]$ is the time interval with bandwidth $b$, and $X_{ti}$ denotes the log form of stock price with $t_i \in [t - b, t + b]$. Besides, $\delta$ denotes the time length between two grid points with the form $\delta = \frac{T}{n}$, $\Delta X_t$ is defined as

$$\Delta X_t = X_t - X_{t-\delta}$$

and

$$[\hat{X}, X]_t = \sum_{t_i \leq t} \Delta X^2_{ti}$$

then the kernel weighted volatility estimator is:
\[ \hat{\sigma}_t^2 = \frac{1}{b} \sum_{t_i = t-b}^{t+b} K\left(\frac{t_i - t}{b}\right)(X_{t_i} - X_{t_{i-1}})^2 \]
\[ = \frac{1}{b} \sum_{t_i = t-b}^{t+b} K\left(\frac{t_i - t}{b}\right) \Delta X_t^2 \]
\[ = \frac{1}{b} \int_{t-b}^{t+b} K\left(\frac{s - t}{b}\right) d\hat{[X, X]}_s \]

In Fan and Wang (2008), the convergence and CLT properties for a more general d-dimensional volatility have been proved. We then reproduce the CLT property for 1-dimensional volatility estimator in Theorem 2.

**Theorem 2** Under Assumptions 4-7, we have
\[ \sqrt{n}b\{\hat{\sigma}_t^2 - \sigma_t^2\} \rightarrow \sigma^2(t)Z, \]
where the convergence is in distribution, and Z is a normal distribution with mean zero and variance \(2\lambda(K)\), where
\[ \lambda(K) = \int_{-1}^{1} K^2(x)dx \]

We will argue conditions are met with our estimator later. In Fan and Wang (2008), they do not consider the problem of kernel and bandwidth selection. Kristensen (2010) and Figueroa-López and Li (2016) propose 2 different bandwidth selection methods based on different assumptions. But both bandwidth selection results are variables and sensitive to certain given assumptions. Quite different from above two works, Mykland and Zhang (2009) propose to choose a fixed bandwidth. However, the selected fixed bandwidth is always quite large. Boswijk and Zu (2013) also have a too long bandwidth problem.

For bandwidth selection, a too long bandwidth will make the estimator not be sensitive to the new added data. Besides, it needs too much data to give one value. This will make us do not have enough estimated volatility values at the boundary part.(time near the market opening and closing period). By contrast, a too short bandwidth will make estimator be too sensitive to extreme values. Since we will estimate the volatility at every minute in Day0 and try to detect the low volatility period, the bandwidth should not be too long or too short. We finally select bandwidth \(k_n = 15\) by referring to Bondarenko and Mastorakis (2005), Louhichi (2008), Abad et al. (2009), Lunde and Zebedee (2009) and Siddiqui and Misra (2017). And the kernel \(K(x)\) is selected as \(K(x) = \frac{1}{2} e^{-|x|}\) suggested by Figueroa-López and Li (2016). We put all the argument details for \(k_n, \Delta n, \text{and } K(x)\) selection into appendix part.\(^2\)

Then the volatility estimator is

\(^2\)See Appendix B
& &
\[ \hat{\sigma}_\tau = \sqrt{\frac{1}{k_n \Delta n} \sum_{i=\lfloor \tau/\Delta n \rfloor - k_n}^{\lfloor \tau/\Delta n \rfloor + k_n} K\left( \frac{\tau - i \Delta n}{k_n \Delta n} \right)(\Delta X_i)^2} \]

with \( k_n = 15, \Delta n = 1 \text{ minute} \) and \( K(x) = \frac{1}{2} e^{-|x|} \).

Similar to the argument given in Chapter 2, we will show that conditions are also met with above estimator. Assumption 4, 5 are very general conditions in study of high frequency data. Suggested by Fan and Wang (2008), they are satisfied for common volatility processes like Geometric OU model, Nelson GARCH diffusion model and CIR model. Since these models have strong power to describe the volatility process in reality. We believe these two conditions are also met with our empirical data. Assumption 3 imposes a constrain on the mean drift \( \mu_t \). This is a general setting and according to Fan and Wang (2008), assumption 6 is often met by price models. Similar to argument given for Theorem 1, the bandwidth is used to control the asymptotic property. It permits us to make feasible inference. Therefore we select bandwidth more from the empirical perspective, and consider more about the robustness and sensitivity of our estimator. The selected kernel is \( K(x) = \frac{1}{2} e^{-|x|} \). This kernel is twice almost surely differentiable (except at 0). Assumption 4 asks for \( K(\cdot) \) be twice differentiable, in the proof, we find it would be enough to assume \( K(\cdot) \) is a.s twice differentiable. For the kernel, although \( \int_{-1}^{1} \frac{1}{2} e^{-|x|} < 1 \), we can normalize the value to make it meet the assumption 4. This normalization will not affect the convergent property.

Since this is a two sides estimator, we actually choose a 30-minute interval and select 31 data points (data frequency is 1 minute) to estimate the volatility at the center point.

### 4.2 Definition of equilibrium and selection procedure

After clarifying the volatility estimator, we now introduce the method to define the equilibrium. For equilibrium, it can be considered from volume. Specifically, equilibrium can be defined as the period when there is very sparse volume. Normally, market participants will trade less when they believe the information has been mostly absorbed into stock price. However, the sparse trading can not only be induced by the reached equilibrium price. It can also come with the increasing market uncertainty as shown in chapter 3. At market opening, the price uncertainty increases. Then the volume decreases while the volatility increases. Hence the volume is not a robust indicator to indicate the equilibrium.

Compared to volume, volatility is a more direct indicator. Low volatility period implies that there is no great disagreement for the price at present. Otherwise, if participants hold very different opinions, they would set different execution prices on the order book.

\(^3\) Although we do not have the volatility value for the first 15 min after the market opening, we have shown at the market opening the information is not fully absorbed into stock price, together with the result shown in Figure 4.4 that volatility is normally quite high at the first moment of market opening, it will not affect our equilibrium detection results.
As a result, price will keep fluctuating and induce high volatility. This is the reason why volatility is a good indicator to indicate the level of disagreement. Therefore, it will be chosen as the indicator for the equilibrium. The volume will assist us to ascertain the result and make the result robust.

We propose the following algorithm to define the equilibrium. The first step is to estimate the volatility at Day0 for every minute point and then collect all the volatility to pick out the volatility value which locates at the least 10% quantile, or the first decile. And in second step, the equilibrium is defined as period during which the volatility locates 5 times consecutively in the first decile for large caps and 10 times for small caps. The details are put into appendix part.

For the selection of quantile and number of consecutive times, we consider it from robustness and sensitivity to set the criterion. For robustness, a lower quantile, for example a 5% quantile, will bring up the problem that less data will be available to determine the quantile value. This value is usually not stable and it will damage the robustness. The robustness will also be damaged once a small number of consecutive volatility is selected to determine the equilibrium. However, by contrast, a higher quantile and a larger number of consecutive volatility will damage the sensitivity of estimator. Finally, after several experiments, we select the mentioned quantile and numbers of consecutive low volatility to determine the equilibrium. And it turns out our estimation method works well. We have looked through all the detected equilibrium, for almost all the cases, the detected equilibrium locates inside a consecutive low volatility period. Besides, there is no sharp jump or plunge around the equilibrium. In addition, results in Table 4.1 show that at detected equilibrium, volume shrinks while spread and volatility keep non-increased.

Figure 4.1 below shows an example of selected equilibrium. To further show the equilibrium determined by the algorithm has good properties. We introduce the change of market liquidity to verify this. The results are shown in tale 4.1. LC stands for large cap while SC stands for small cap. $\Delta s$, $\Delta \sigma$ and $\Delta D$ denote the change of spread, volatility and volume at the equilibrium moment compared to control group.

For example, value in row 2 column 1 denotes that at the detected equilibrium for large caps with positive overnight return ($OR$), the volume decrease 35% compared to control group. With careful inspection, one obvious observation comes from $\Delta D$, except for small cap stocks in $OR < -2\%$ group, all the other $\Delta D$ have significant sharp declination. At the same time, there exists no significant spread increase. Besides, almost all groups have a negative symbol for $\Delta s$. This confirms that the sparse trading does not come from the carefulness for the increasing market uncertainty. As for $\Delta \sigma$, the positive overnight return groups show the declination of the volatility while the negative groups show the non significant positive results. With non-increased volatility, non-increased spread and decreased volume, we conclude that the equilibrium detected here is a fine indicator to indicate the moment such that the information is absorbed into the stock price and participants reach a consensus for the equilibrium price. However, there is no evidence to claim that the information has been thoroughly absorbed into the stock price. As shown in chapter 3, for

\footnote{More argument can be referred to appendix B}
Figure 4.1: The figure shows an example of detected equilibrium. The vertical dash line shows the location of detected equilibrium. Around the detected equilibrium, volatility keeps low and price keeps fluctuating around a certain level. 

small cap stocks with positive overnight return, there exists a persistent upgrade trend for days. It indicates a good news is kept being absorbed into stock price for longer time than one day.
\[ \Delta \sigma = 0.0277 \quad 0.0279 \quad -0.0764^{**} \quad -0.1228^{***} \]
\[ \Delta D = -0.3453^{***} \quad -0.0341 \quad -0.2562^{***} \quad -0.0823^{***} \]
\[ \Delta s = -0.008 \quad -0.0079 \quad 0.0078 \quad -0.0004 \]

Table 4.1: This table shows market liquidity change at equilibrium. Specifically, it shows the change of volatility, volume and spread at the detected equilibrium moment compared to the control group. Non-increased volatility, decreased volume and non-increased spread indicate that people trade less but market uncertainty does not increase. This is what we expect for the equilibrium.

4.3 Empirical results

We introduce Figure 4.2 to show the distribution of equilibrium time for both large cap stocks and small cap stocks. From the distribution of equilibrium time, we can not only know when the equilibrium is generally reached for both large and small cap stocks, but also observe that the distribution of equilibrium time is different between large cap stocks and small cap stocks. The difference is also observed in chapter 3.

From the histogram Figure 4.2, most of the equilibrium locates between 11 a.m. to 3 p.m. irrespective of market cap size. This to some degree clarifies the affection from lunch time. If a strong lunch time effect does exist, then most of equilibrium time would locate in a very narrow period. However, from 11 a.m. to 3 p.m., most part of the equilibrium time distributes almost evenly in this time interval. Besides, the first peak of equilibrium time appears after 10 a.m. It indicates that more than 1 hour is needed to reach the equilibrium. This result can also be verified in Figure 4.4, we will show it later.

Besides, the distribution of equilibrium is different between large cap stocks and small cap stocks. A direct observation is that it takes more time for small cap stocks to reach the equilibrium. This makes sense because there are more specialists following large cap stocks and it will make the information be absorbed faster. To prove the difference is also statistically significant, we introduce the Kolmogorov-Smirnov test again. The Kolmogorov-Smirnov test shows that the null hypothesis can be rejected at 1% level. It implies that the distribution of equilibrium time for large cap stocks is significantly different from small cap stocks. However, it recalls that for different cap groups, we apply different criterion to determine the equilibrium. The large caps utilize 5 consecutive low volatility to determine the equilibrium while small caps use 10. The request for more consecutive low volatility will probably make it take longer time to reach the equilibrium. And it will introduce bias to make us believe it takes longer time for small cap stocks to reach the equilibrium. To avoid the bias, we then take the 10 consecutive times for both groups and get the histogram.
Figure 4.2: The figure shows distribution of equilibrium time. We can observe that small cap stocks’ equilibrium time distribution locates more right than large cap stocks’. This indicates that information needs more time to be absorbed into small cap stocks. Recall that we choose different numbers of volatility for small cap stocks and large cap stocks to define the equilibrium. This may introduce bias. We will solve this problem in Fig 4.3 by choosing the same criterion.

shown in Figure 4.3. Luckily, similar conclusion is reached. From Figure 4.3, the small caps’ distribution locates more right than large caps’. The Kolmogorov-Smirnov test result rejects the null hypothesis at 5%. Therefore, even with the same criterion, it takes longer for small cap stocks to reach its equilibrium. And we can claim that it takes longer for information from small cap companies to be absorbed.

This is a reasonable result. There are larger numbers of specialists following large cap stocks, the information related to large cap stocks can be processed more efficiently than the information related to small cap stocks. Then the equilibrium could be reached earlier compared to small cap stocks.

We introduce Figure 4.4 to further clarify the above claims. It shows the averaged volatility until equilibrium for both large cap stocks and small cap stocks at Day0. In Figure 4.2 and Figure 4.3, we have found the first peak of equilibrium is reached after 10 a.m. Both curves in Figure 4.4 shows the averaged volatility decreases very fast in the first hour after market opening, then after 10 a.m., the declination rate becomes much smaller compared to the first hour. And at this moment, the first group of equilibrium begins to appear, shown as the first peak in equilibrium distribution.

Besides, the averaged volatility for large cap stocks decreases faster than small cap stocks. This can support the claim that large cap stocks reaches equilibrium faster than
small cap stocks. The green line denotes the averaged volatility curve for STOXX 50 index\textsuperscript{5}. Compared to averaged volatility curve for large cap stocks and small cap stocks, index volatility is much lower. Besides, index volatility curve shows more obvious volatility smile pattern. To illustrate this, we show the index volatility curve independently in Fig 4.5 (a). Fig 4.5 (a) shows a volatility-smile curve. It implies index volatility first decreases, then after 3 p.m., affected by US stock market opening, the volatility increases sharply in a very short time. Fig 4.5 (b) shows the averaged volatility at Day-5 (5 days before the information disclosure)\textsuperscript{6}. We observe the volatility on Day-5 is much lower than volatility on Day0 for both large cap stocks and small cap stocks. From this difference, we argue that information shock does increase the stock price volatility. Besides, in Fig 4.5 (b), we can also observe the volatility smile pattern that volatility increases sharply after 3 p.m.

In this chapter, we have argued that a consecutive low volatility period can be regarded as a equilibrium. This equilibrium has good properties such as decreased volatility, volume and non-changed spread. And it then can be used to indicate the moment when the information is absorbed and price reach a consensus level with which participants have no willingness to trade more. A further research finds that it needs more time for information related to small cap stock to be absorbed into stock price.

\textsuperscript{5}We randomly choose 150 trading days in 2016 to calculate the STOXX 50 index volatility curve

\textsuperscript{6}We need this fig to show the differences of volatility curve with or without information shock. We choose the furthest day before Day0 to make result immune from information affection
Figure 4.4: The figure shows averaged volatility curve for small cap stocks, large cap stocks at Day0 and STOXX 50 index volatility curve. For both small cap stocks and large cap stocks, volatility decreases very fast in the first hour after market opening. And volatility curve of large cap stocks is lower than volatility curve of small cap stocks.
Figure 4.5: (a) is the averaged volatility calculated from STOXX 50 index. It shows volatility smile curve. We observe index volatility decreases after market opening. Then affected by US stock market opening, index volatility increases sharply after 3 p.m. (b) is the averaged volatility on Day-5 for large cap stocks and small cap stocks. Both curves are much lower than averaged volatility curves shown in Fig 4.4. From this, we know information shock can increase volatility.
Chapter 5

Conclusions

This thesis studies how the financial information is absorbed into stock price after a corporate makes a profit warning public. We find at the market opening on Day0 (the day upon which the financial information is released), volatility and spread increase while volume shrinks. Such changes suggest increased information asymmetry, which indicates that the information is not fully absorbed into stock price at the market opening on Day0. These results are in line with the theoretical prediction from Gennotte and Trueman (1996), and resonate with existing empirical results, see, for example, Libby et al. (2002) among others.

We also find that small cap stocks with positive profit warning have a persistent positive trend, such trend, however, disappears when the profit warning is negative. Moreover, large cap stocks do not exhibit such trend, neither.

Another interesting finding is that the volume change $\Delta D_{\tau_0}$ for small cap stocks at market opening shows good prediction power of stock returns: an increasing $\Delta D_{\tau_0}$ is accompanied by an increasing average holding return. Specifically, with higher volume $D_{\tau_0}$ compared to normal period at market opening on Day0, the average holding returns from Day0 to Day3 are also higher. However, the spread — another market liquidity indicator — does not possess this prediction power.

The last contribution of this thesis is the introduction of the consecutive low volatility period defined as equilibrium, which serves as a general description of information absorbing process. We demonstrate that the distribution of equilibrium times are dependent on the company capitalized size. There is moderate variation among small cap and large cap stocks: information needs more time to be absorbed into small cap stock price than large cap stock price. One possible explanation is that the large cap stocks are watched closely by analysts. In the event of news release, the related information will propagate via various channels more efficiently.
Chapter 6

Popular Summary

In this thesis, we study how long it takes for a newly released public available information to be totally absorbed into stock price. There is a great diversity of information in the financial market. We only focus on profit warning. Profit warning is a special information disclosure way: when a company finds its earnings does not match the analysts expectations, it will make a profit warning prior to the public announcement of its earnings. Since profit warning is usually announced before market opening, we first study whether a newly released profit warning is well processed during overnight non-trading period, and the opening market prices take into account information. We use the changes of volatility, volume and spread at the market opening compared to normal period to show whether the information asymmetry increases or decreases. If the information asymmetry increases, then the information is not well absorbed into stock price. Otherwise, at least part of the information is absorbed. The observed highly volatile prices with small trading volumes and increased spread upon the market opening implies market opening prices do not fully absorb the information.

At the beginning of our research, we notice there exists asymmetry trading reaction at the market opening after the information disclosure between large cap stocks and small cap stocks. We then apply the Kolmogorov-Smirnov test to confirm the trading reaction difference. And by introducing the holding return, we find that small cap stocks with positive profit warning will show a persistent upgrade trend. However, this trend disappears when the profit warning is negative. Moreover, large cap stocks do not exhibit such trend irrespective of its profit warning. We also find the volume change $\Delta D_{\tau_0}$ for small cap stocks at market opening shows good prediction power of stock returns: with higher volume $D_{\tau_0}$ compared to normal period at market opening on Day0, the average holding returns are also higher.

The last contribution of this thesis is the introduction of the consecutive low volatility period defined as equilibrium. At the detected equilibrium, compared to normal period, the decreased trading volume, the non-increased volatility and spread indicate that market participants trade less but market uncertainty does not increase. With this good property, we then believe this consecutive low volatility period can be regarded as the moment that market participants reach some consensus over stock price and have no willingness to trade.
more. Then this moment can be taken as the time that information is absorbed into stock price. Moreover, by plotting the equilibrium time distribution, we find that there exists moderate variation among small cap and large cap stocks: information needs more time to be absorbed into small cap stock price than large cap stock price.
Bibliography


Appendix A

A.1 Assumption and notation for Theorem 1

This section gives conditions and assumptions for Theorem 1 proposed by Bollerslev et al. (2016). They assume $P$ is a jump-diffusion process of the form

$$dP_t = b_t dt + \sigma_t dW_t + dJ_t$$  \hspace{1cm} (A.1)

where $b$ is an instantaneous drift process, $\sigma$ is stochastic spot volatility process, $W$ is a Brownian motion, and $J$ is a pure jump process. The price is sampled at discrete times $\{i\Delta_n : 0 \leq i \leq [T/\Delta_n]\}$, where $T$ denotes the sample span and $n$ denotes the sampling interval of the high-frequency data. They denote the high-frequency asset returns by $r_i = P_i\Delta_n - P_{(i-1)}\Delta_n$. All these denotations are similar to ours. Their empirical analysis as well as ours are using an infill econometric theory with $\Delta_n \to 0$ and $T$ fixed. This setting is standard for analyzing high-frequency data. It allows us to nonparametrically identify processes of interest in a general setting with essentially unrestricted nonstationary and persistence. They denote the trading volume within the high-frequency interval $[(i-1)\Delta_n, i\Delta_n]$ by $V_{i\Delta_n}$. This denotation is also similar to our volume denotation. To describe the trading volume, they introduce a general state-space model

$$V_{i\Delta_n} = \mathcal{V}(\zeta_{i\Delta_n}, \epsilon_{i\Delta_n})$$  \hspace{1cm} (A.2)

where $\zeta$ is a latent state process, $\epsilon_{i\Delta_n}$ are i.i.d. transitory shock with distribution $F_\epsilon$, and $\mathcal{V}()$ is an unknown transform. Let

$$D_t = \int \mathcal{V}(\zeta_t, \epsilon)F_\epsilon(\text{d}\epsilon)$$  \hspace{1cm} (A.3)

**Assumption 1** (i) The price process $P$ is given by (A.1) for $J_t = \int_0^t \epsilon_s dN_s + \int_0^t \int_{\mathbb{R}} \delta(s,z)\mu(ds,dz)$, where the processes $b$ and $\sigma$ are c\'{a}dl\'{a}g (i.e., right continuous with left limit) and adapted; $\sigma$ is positive for $t \in [0,T]$ almost surely; the process $\epsilon$ is predictable and locally bounded; $N$ is a counting process that jumps at the scheduled announcement times which are specified by the set $A$; $\sigma$ is a predictable function; $\mu$ is a Poisson random measure with compensator $\nu(ds,dz) = ds \otimes \lambda(dz)$ for some finite measure $\lambda$. 


(ii) The volume process \( V \) satisfies (A.2). The process \( \zeta \) is càdlàg and adapted. The error terms \( (\epsilon_i) \) take values in some Polish space, are defined on an extension of \((\Omega, \mathcal{F})\), i.i.d. and independent of \( \mathcal{F} \).

(iii) For a sequence of stopping times \((T_m)_{m \geq 1}\) increasing to infinity and constants \((K_m)_{m \geq 1}\) we have \( E|\sigma_{i\wedge T_m} - \sigma_{s\wedge T_m}| + E|\zeta_{i\wedge T_m} - \zeta_{s\wedge T_m}| \leq K_m|t - s| \) for all \( t, s \) such that \( |s, t|\mathcal{A} = \emptyset \).

In addition, they need the following conditions for the nonparametric analysis, where they denote \( M_p(\cdot) = \int V(\cdot, \epsilon) p F_\epsilon(\epsilon) \).

**Assumption 2** \( k_n \to \infty \) and \( k_n^2 \Delta_n \to 0 \)

**Assumption 3** The function \( M_2(\cdot) \) is Lipschitz on compact sets and the functions \( M_2(\cdot) \) and \( M_4(\cdot) \) are continuous.

They also list other notations. \( v_t = M_2(\zeta_t) - M_2(\zeta_1) \). Variables \( (\eta_\tau, \eta_\tau', \eta_\tau^{'-}, \eta_\tau') \) are conditionally on \( \mathcal{F} \) and are mutually independent, centered Gaussian with variances \( (v_\tau, v_\tau', \sigma_\tau^2 / 2, \sigma_\tau^2 / 2) \). For sequence \( Y_n \) of random variables, they write \( Y_n \overset{L_s}{\to} Y \) if \( Y_n \) converges stably in law towards \( Y \), meaning that \( (Y_n, U) \) converges in distribution to \( (Y, U) \) for any bounded \( \mathcal{F} \)-measurable random variable \( U \). For a generic random sequence \( Y_n \), write \( Y_n \overset{L|\mathcal{F}}{\to} Y \) if the \( \mathcal{F} \)-conditional distribution function of \( Y_n \) converges in probability to that of \( Y \) under the uniform metric.

### A.2 Assumption and notation for Theorem 2

The following are assumptions given by Fan and Wang (2008) for theorem proof.

**Assumption 4** \( \sup \{||\sigma_s - \sigma_t||, s, t \in [0, T], |s - t| \leq a\} = Op(a^{1/2} \log a^{1/2}) \), \( \sup_{0 \leq t \leq T} ||\sigma_t^2|| = Op(1) \)

**Assumption 5**

\[
\sup \left\{ \left\| \int_{t_{i-1}}^{t_i} \{\sigma(s) - \sigma(t_{i-1})\} dW_s \right\|^2, i = 1, \ldots, n \right\} = Op(n^{-2+\eta})
\]

where \( \eta > 0 \) is an arbitrarily small number. The drift \( \mu_t \) satisfies

**Assumption 6**

\[ \sup \{||\mu_t - \mu_s||, |t - s| \leq a\} = Op(a^{1/2} \log a^{1/2}) \]

Bandwidth \( b \) and kernel \( K \) satisfy

**Assumption 7**

\[ b \sim n^{-1/2} / \log n, K(\cdot) \text{is twice differentiable with support } [-1, 1] \text{and } \int_{-1}^{1} K(x)dx = 1 \]
Appendix B

B.1 Bandwidth and kernel selection

We give details for bandwidth and kernel selection. Kristensen (2010) and Figueroa-López and Li (2016) propose methods to select the optimal bandwidth and kernel. Both of the work achieve the selection by minimize the MSE (mean squared error). The MSE of the kernel estimator is defined as

\[ \text{MSE}_n(b) = \mathbb{E}[(\hat{\sigma}_\tau^2 - \sigma^2_\tau)^2] = \mathbb{E}[\sum_{i=1}^{n} K_b(t_i - \tau)(\Delta X_i^2 - \sigma^2_\tau)] \]

where

\[ K_b(t_i - \tau) = \frac{K(\frac{t_i - \tau}{b})}{b} \]

The core purpose is to choose \( b = b_n \) to minimize \( \text{MSE}_n \). Under different assumptions, the Kristensen (2010) and Figueroa-López and Li (2016) proposed 2 different bandwidth result. But notice both choices of bandwidth are variables and quite sensitive to different assumptions.

Quite opposite to above two works, Mykland and Zhang (2009) propose to choose a fixed bandwidth. They also prove the CLT and asymptotic properties of such a fixed bandwidth choice. However, the fixed bandwidth selected here is always quite large, Boswijk and Zu (2013) also find a large fixed bandwidth.

The reasoning of \( \Delta_n \), bandwidth, kernel selection would be given in the following work. First the \( \Delta_n \), as discussed in the data description part, we have 1 minute data, to decide the optimal \( \Delta_n \), we could only select lower frequency data, namely 2 minutes, 3 minutes, 5 minutes data or even longer time interval. However, from theoretical research work w.r.t high frequency data, \( \Delta_n = \frac{T}{n} \). The convergence and asymptotic properties are always reached when \( n \to \infty \) which means \( \Delta_n \to 0 \). Besides, in Kristensen (2010) and Boswijk and Zu (2013) simulation work, with the increasing of \( \Delta_n \) the MSE would increase sharply. Especially when \( \Delta_n = 15 \) minutes, MSE almost explodes. Another reasoning comes from the fact the popular researches tend to select the least \( \Delta_n \) they could get which is in accordance with theoretical work, like Abad et al. (2009) and Bollerslev et al. (2016). Therefore, in
our research work, we would also use the least \( \Delta_n \) we could get, namely \( \Delta_n = 1 \) minute.\(^1\)

To illustrate the bandwidth selection, we first discretize the estimator of volatility to make it be in line with the estimator we proposed in the chapter 2:

\[
\hat{\sigma}_T = \sqrt{\frac{1}{k_n \Delta_n} \sum_{i = \lfloor \tau / \Delta_n \rfloor}^{\lfloor \tau / \Delta_n \rfloor + k_n} K\left( \frac{\tau - i \Delta_n}{k_n \Delta_n} \right) (\Delta X_i)^2}
\]

The na"ıve estimator is just the special case when the kernel is taken as \( K(x) = 1_{(0,1]}(x) \), now the bandwidth mentioned above become \( k_n \Delta_n \). Selection of bandwidth becomes selection of \( k_n \). We will choose the fixed \( k_n \) with two reasons. First is for the purpose of this research work. We aim to define the equilibrium via detecting low volatility period. Since normally the equilibrium does not last long, hence our volatility estimator should not only be a good estimator in general theory, but sensitive to real world price change as well. The variable bandwidth is got under quite strong assumption and so far these estimators are not popular in empirical estimation, for example, Boswijk and Zu (2013) only apply the estimator for EURO FX future which has 1 second data for 24 hours everyday. They do not extend it to general stock volatility estimation.

We will not choose the \( k_n \) proposed by Mykland and Zhang (2009), either. As in its simulation work, it indicates the convergence rate is relatively slow. Besides, bandwidth in their work is also too large to be implemented in empirical research. Recall our original research purpose. We want to detect the equilibrium from a consecutive low volatility period perspective. Therefore, the detected equilibrium should be quite sensitive to price change. Then a too long bandwidth would bring up several problems. Firstly, it will consume too much data at the opening and closing period no matter we take the one-sided kernel or two-sided kernel. Secondly, it will not be sensitive enough to new added price change if it is divided by a large bandwidth. This will filter out the information we caring most. Lastly, we only have around 500 data points per day, too long bandwidth will make us get few estimated volatility. Therefore, we will consider more from estimator’s sensitivity and robustness to choose another fixed \( k_n \).

In many empirical research work which focus on intraday stock’s price behavior, 5 minutes, 15 minutes and 30 minutes are the most popular intervals to choose, we will choose 15 minutes interval, in other words, \( k_n = 15 \). The selection is made backed by several research work Bondarenko and Mastorakis (2005), Louhichi (2008), Abad et al. (2009), Lunde and Zebedee (2009) and Siddiqui and Misra (2017). We also consider \( k_n = 15 \) by considering estimator’s sensitivity and robustness. Bondarenko and Mastorakis (2005) simulation work find that compared to 30 minutes and 5 minutes interval, 15 minutes interval choice will show higher efficiency. Other empirical works find the volatility, trading volumes, abnormal return increase sharply in the first 15 minutes of post-announcement period. It means the choice of bandwidth \( k_n = 15 \) will catch most of the price change we are interested in. Moreover, the volatility calculated from a shorter interval would be

\(^1\)Notice here in the estimation of spot volatility, the independence of two neighbor returns are not needed
affected by extreme values and the result would not be robust to general cases. Oppositely, a longer interval would not be sensitive to a new coming price change $\Delta_{n}X^{2}$. Therefore, by making a trade off and refer to mentioned literature, we select $k_{n} = 15$ as our research bandwidth. Suggested by Figueroa-López and Li (2016), $K(x)$ is chosen as $K(x) = \frac{1}{2}e^{-|x|}$. Figueroa-López and Li (2016) show it has better efficiency behaviour. Besides, compared to flat weighted kernel, this kernel give higher weight at the time pot the volatility is estimated.

### B.2 Selection criterion of equilibrium

For small cap companies, the criterion is 10 consecutive times of volatility locating in the least 10% quantile. While for large cap companies, only 5 is consecutive times is selected. Compared to large cap companies, the trading activities density are much sparser for small cap stocks. Therefore, if a smaller number of consecutive low volatility is selected, the detected equilibrium would not be the one that we expect. And the results in table B.1 verify the claim.

In table B.1, $\Delta s$, $\Delta \sigma$, $\Delta D$ are defined similarly as defined in the chapter 2. They are the change of spread, volatility and volume at the detected equilibrium compared to control group. In chapter 2, the volume is estimated by

$$D_{\tau_{i}} = \frac{1}{k_{n}} \sum_{j=1}^{k_{n}} V_{\tau_{i}+j\Delta t}$$

But here the estimator of volume at time $t_{i}$ is defined as

$$D_{t_{i}} = \frac{1}{k_{n}} \sum_{j=t_{i}-[k_{n}/2]}^{t_{i}+[k_{n}/2]} V_{t_{i}+j\Delta t}$$

$k_{n} = 15$

Table B.1 shows the results of market liquidity change for small cap companies when selecting different consecutive times. Values in column $OR > 2\%$, 10 denote the market liquidity change at the detected equilibrium by selecting 10 as consecutive low volatility numbers for cases having positive overnight return. And values in column $OR < -2\%$, 5 denote the market liquidity change of the equilibrium determined by selecting 5 as consecutive low volatility numbers for cases having negative overnight return. Easy to find for the group with criterion selected as $num = 5$, there are statistically significant positive spread. Then it is hard to illustrate whether the low volume and volatility come from peoples consensus on a certain price level or carefulness for the uncertainty of the market price. Therefore, different from large cap companies, we will select the criterion as $num = 10$ for small cap companies. Contrary to small cap companies, the large cap companies do not show such difference with different $num$ choice. With consideration of sensitivity and
<table>
<thead>
<tr>
<th></th>
<th>OR &gt; 2%, 10</th>
<th>OR &gt; 2%, 5</th>
<th>OR &lt; -2%, 10</th>
<th>OR &lt; -2%, 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δσ</td>
<td>-0.1228***</td>
<td>-0.1170****</td>
<td>0.027</td>
<td>0.058</td>
</tr>
<tr>
<td>ΔD</td>
<td>-0.0823***</td>
<td>-0.804***</td>
<td>-0.0341</td>
<td>-0.0439</td>
</tr>
<tr>
<td>Δs</td>
<td>-0.0004</td>
<td>0.0423**</td>
<td>-0.0079</td>
<td>0.0377*</td>
</tr>
</tbody>
</table>

Table B.1: Results of market liquidity for small cap companies with different choice of consecutive times.

practical meaning, the num = 5 will be selected for the large cap companies equilibrium detection.