Effect of irradiation on the evolution of brown dwarfs

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Abstract

Brown dwarfs are faint, not hydrogen fusing starlike objects, that might appear to be part of binary systems all over the universe. These brown dwarf companions, are in some cases close enough to the primary star, to form an accretion disk due to mass transfer. This accretion disk, and other sources of radiation, might alter the physical properties of the brown dwarf companion due to the amount of radiation it captures. One of the binary systems that most likely contains a brown dwarf companion is SAXJ 1808.4-3658. This system is located in the Galactic bulge, so it is very likely that the system is old, at least in the order of Gigayears. The temperature and radius of this companion have been constrained with observations of the mass transfer rate $\langle \dot{M} \rangle$ and binary period $P$. The calculated temperature and radius indicate that the brown dwarf companion is young and hot (the radius and temperature are significantly larger than an equally old isolated, not irradiated brown dwarf), but since we know that the system is most likely much older, a question arises. Can (c)old brown dwarfs increase their radius and temperature due to irradiation such that they look like hot, young brown dwarfs? Computations performed by the modeling program MESA indicated that brown dwarfs do indeed increase their radius and temperature due to irradiation by an amount which might be sufficient to explain systems like SAXJ 1808.4-3658. We have obtained strong evidence that the luminosity $L$ of a 0.05 $M_\odot$ brown dwarf companion which is irradiated by a flux of $9 \times 10^{-11}$ erg cm$^{-2}$ s$^{-1}$, can be two orders of magnitude larger, compared with not irradiated ones.

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1 Introduction and motivation

Since the 1960’s the hunt on brown dwarfs has been ongoing. These faint, low-mass starlike objects, first considered as a generous contributor to dark matter, are present in isolated systems as well in binary systems. In 1990 the issue of expanding low mass companions due to external heating came to sharp focus with the discovery of an eclipsing optical counterpart of the millisecond pulsar PSR 1957+20. Although it was not until the companion of the millisecond pulsar SAXJ 1808.4-3658 was proposed to be a brown dwarf with $M_{bd} \approx 0.05 M_\odot$ and $R_{bd} \approx 0.13 R_\odot$, by L. Bildsten and D. Chakrabarty in 2001, that effects of irradiation on brown dwarf evolution in binaries were studied in much more detail. Because this binary system is located in the Galactic bulge, it evidently must be very old; at least in the order of Gigayears. However, the effective temperature and radius of the companion brown dwarf is greater than expected; comparable with that of a young, hot brown dwarf. Therefore, in order to explain why the observed companion looks hot and young, the question whether (c)old brown dwarfs can significantly increase their radius and temperature when they are irradiated by an external source, needs to be answered. Radiation, primarily coming from the accretion disk around the neutron star, penetrates the surface of the brown dwarf such that its temperature and radius increases.

2 Brown dwarf definition and characteristics

Results from evolution computations indicate that brown dwarf radii and temperatures increase they are exposed by a quiescence flux from an external source such as accreting discs. In order to understand the physical processes responsible for this to happen, an introduction to definitions and characteristics regarding brown dwarfs is given in this section.

2.1 Failed stars

As fragmented clouds contract, gravitational potential energy is released as radiation and heat. The temperature of the core of the forming star increases during this process. When a temperature of $3 \cdot 10^6$ K is reached, hydrogen fusion starts. The resulting pressure in the core prevents gravitational collapse. Hydrodynamic and thermal equilibria are established as the main sequence era starts. The heating of the star’s core depends on the total released potential energy, which itself depends on the star’s mass and the rate at which the star contracts. This implies that lower-mass stars should contract to higher densities than more massive stars, for hydrogen fusion to start. Stars with masses lower than approximately $0.1 M_\odot$, have sufficient high densities inside their cores, for the electrons in the plasma to become partly degenerate. The highest occupied momentum states of the resulting electron gas prevents gravitational contraction to continue once a minimal radius is reached of approximately $7 \cdot 10^7$ m. For objects with masses lower than about 0.072 $M_\odot$, gravitational contraction stops even before hydrogen fusion can start since in those cases degeneracy pressure balances the inward gravitational force whilst $T_{\text{nuc}} < 3 \cdot 10^6$ K. Hence, hydrostatic equilibrium, but not thermal equilibrium is achieved. These ‘stars’, not being able of doing what they should, that is, fusing hydrogen (continuously) in their cores, is a first, rough description of what a brown dwarf is. [9]
2.2 Classification: star, brown dwarf or planet

The difference between (hydrogen-fusing) stars and brown dwarfs is well defined. It has everything to do with the 0.072 $M_{\odot}$ hydrogen burning mass limit (HBML) which is explained in section 3.3. The difference between brown dwarfs and giant gaseous planets however, is much more vague, since their mass, size and atmospheric properties are in some cases very similar. The discussion on how one can distinguish the two is twofold:

1. a star arises out of a contracting molecular cloud whereas a planet forms in the circumstellar disk;
2. a brown dwarf must exceed the mass threshold of 0.012 $M_{\odot}$ ($\sim 13 M_{\text{Jup}}$) in order to be able to fuse at least a single element (since brown dwarfs do fuse a little deuterium during their evolution), whilst planets are constrained to have lower masses, since they fuse nothing. [4][12]

2.3 Formation

The origin of brown dwarfs and the way they form remain unclear until today, although it seems plausible that they form in the same molecular clouds as ordinary stars do (and thus not in the circumstellar disk like planets). To overcome thermal equilibrium, they must form in the most dense regions of those clouds. Many questions remain unanswered. For instance, what process halts the subsequent accretion in such a way, that stars do not become more massive than 0.072 $M_{\odot}$ and therefore preventing to fuse hydrogen? Several mechanisms have been proposed in order to give an explanation: dynamical injection of brown dwarf embryos, dispersion of gas by radiation, wind pressure from nearby massive stars, and formation in turbulent pressure waves. Models performing three-dimensional hydrodynamic and radiative calculations are developed to test these scenarios. Because lower-mass brown dwarfs start off with less thermal energy from gravitational contraction than brown dwarfs in the high mass end, an ambiguity exists between temperature and luminosity (which can be measured quite accurately), and its mass and age. [4][12]

2.4 Observing Brown Dwarfs

The radiation emitted from the Brown dwarf's photosphere can be observed directly. The photospheric temperature of brown dwarfs lays in the range $200 \text{ K} < T_{\text{photo}} < 3000 \text{ K}$, although really low temperature brown dwarfs have not yet been observed (since it will take much more time to cool brown dwarfs to such low temperatures than the time between today and the Big-bang) but is an evident outcome of brown dwarf theory. The pressure and density of a brown dwarf photosphere lays in the range $10^{-6} \text{ g/cm}^3 < \rho_{\text{photo}} < 10^{-4} \text{ g/cm}^3$, $10^4 \text{ Pa} < P_{\text{photo}} < 10^6 \text{ Pa}$ respectively. These conditions allow neutral atomic and molecular gas species to dominate. The energy absorbing molecules in the photosphere yield complex spectral distributions very sensitive to temperature fluctuations which can be measured quite accurately. For the human eye most brown dwarfs appear magenta.

Just like any other astrophysical object, brown dwarfs are categorized by their spectral properties. Brown dwarfs are distinct in four different spectral classes: M-dwarfs ($2100 \text{ K} < T < 3500 \text{ K}$), L-dwarfs ($1300 \text{ K} < T < 2100 \text{ K}$), T-dwarfs ($650 \text{ K} < T < 1300 \text{ K}$), and Y-dwarfs ($T < 650 \text{ K}$), although the latter is hypothetical only. Since brown dwarfs cool over time, these classes are evolutionary as well. [4]

2.5 The start of the search

The first search for brown dwarfs was motivated by the search for objects belonging to dark matter, a category brown dwarfs nowadays do not belong to. It was not until 1990, when the first brown dwarf, Gliese 229B, was discovered, that theoretical predictions could be confirmed experimentally. Not because brown dwarfs are rare, but rather faint objects that radiate in the near infrared ($10^{-6} \text{ m} < \lambda_{\text{bd}} < 5 \cdot 10^{-6} \text{ m}$). It was the lack of advanced infrared optics that formed the main cause of the 30 year gap between theory and experiment. Until today brown dwarfs have been discovered with masses less than 0.01 $M_{\odot}$ and temperatures as low as 625 K. [4]
3 Evolution of brown dwarfs

The internal structure and evolution of isolated stars can be studied by making use of the equilibria of the hydrostatic and thermodynamic equations, together with the concept that nuclear fuel, mainly hydrogen, is consumed in star cores and surrounding shells. This concept clearly does not fully apply to brown dwarfs. Hydrostatic equilibrium implies that the inward gravitational force is exactly balanced by an outward pressure gradient. In thermodynamic equilibrium, the heating or cooling of a spherical shell of matter, is determined by the balance of heat production in nuclear reactions, at temperatures of approximately $10^7$ K, and heat loss as it radiates into space from the outer atmospheric layer (photosphere) at temperatures observed of around 2000 to $10^5$ K. The heat flux is carried away either by radiation only, or by a combination of radiation and convection, depending on whether the temperature gradient that would be required to carry away the heat entirely by radiation, is less or greater than the adiabatic temperature gradient at which convective instability sets in. Most stars contain regions that are dominated by convective heat flows and regions that are fully radiative. As stated before, nuclear reactions inside a stellar core and surrounding shells are responsible for heat production. These nuclear reactions also cause the nuclear composition of the star to vary on slow time scales depending on the mass of the stars. The nuclear reactions primarily fuse hydrogen into helium. Subsidiary reactions modify the abundances of other species like $^3$He, deuterium, lithium, beryllium, carbon, nitrogen and oxygen. In later stages much of helium is itself burnt to form a mixture of carbon and oxygen. In the final stages of the stellar evolution a much larger number of nuclear reactions take place involving all the elements starting with carbon. Via stellar winds, planetary nebula, outbursts of novae and supernovae, the products of these nuclear reactions (beside hydrogen and helium, which are formed in the early stages of the universe), are returned to the interstellar medium. [6][7]

3.1 Observing star formation

Before a star can settle into the hydrostatic and thermal equilibrium state, it first has to condense out of a contracting gas cloud. One important reason why star formation processes are much less understood than the later evolution, is simply because of this absence of equilibria. Another reason is that during formation processes the dense gas surrounding the protostar obscures direct view of what is happening. During the last 25 years of the 20th century, developments in infrared optics and techniques have significantly enriched our knowledge of star formation. But even today many questions remain unanswered. [7]

3.2 Stellar equations

A set of partial differential equations can be written down to model the physical processes that occur in stellar interiors. These equations apply to stars that have settled into near (hydrostatic + thermodynamic) equilibrium. The stellar equations are outlined in Appendix (A). These stellar equations do often not yield elementary solutions. However, stars can be well approximated by polytropes: gas spheres in which pressure is proportional to a power of density: $P \propto \rho^{1+1/n}$. It surely makes the mathematics more friendly. Characteristics of polytropes are treated in Appendix (B). [5][7]

3.3 Interior physics of brown dwarfs

The temperature of a brown dwarf core is insufficient to sustain the fusion reactions common to all main sequence stars. Thus, brown dwarfs cool as they age, passing through the different elementary classes, as can be seen in the left image of Figure 1 and in the image on the front page. In the stellar core, the velocities of the protons obey a Maxwell-Boltzmann distribution with average energy $kT_{\text{mac}} = 8.6 \cdot 10^{-8} \cdot 3 \cdot 10^6 \sim 0.1 \text{ keV}$, with $k$ the Boltzmann constant. However, the Coulomb repulsion between these protons is several MeV. Nevertheless, fusion is possible because of quantum mechanical tunneling. The nuclear reaction rate is governed by the proton pair energy $E$, at the high energy tail of the Maxwell-Boltzmann distribution (which scales as $\exp(-E/kT)$), and the nuclear cross section due to quantum mechanical tunneling through the Coulomb repulsion (which scales as $-E^{-1/2}$). The product of these factors defines the Gamow peak (right image of Figure 1) at a critical energy $E_{\text{crit}}$, which is roughly 10 keV for the reactions in the proton-proton (pp) chain.

\[
p + p \rightarrow d + e^+ + \nu_e
\]
\[
p + d \rightarrow ^3\text{He} + \gamma
\]
\[
^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + 2p
\]
Because $E_{\text{crit}} \gg kT$, it is only a tiny minority of protons which contain enough energy to start fusing with one another. The reaction rate is proportional to $(T/T_{\text{nuc}})^n$, where $n \approx 10$ for temperatures near $T_{\text{nuc}}$ in the pp chain (the large value of $n$ ensures the temperature to be close to $T_{\text{nuc}}$).

For low-mass main sequence stars, the mass is approximately proportional to the radius, which can be demonstrated by making use of the virial theorem. In equilibrium, thermal and gravitational energy are in balance:

$$\frac{GM^2}{R} \sim \frac{M}{m_p} kT_{\text{nuc}}$$

(2)

where $m_p$ represents the mass of the proton. It follows that $R \propto M$. This implies that the density $\rho$ increases with decreasing mass:

$$\rho \propto MR^{-3} \propto M^{-2}$$

(3)

since $M = \frac{4}{3}\pi R^3 \rho$. When the density becomes sufficiently high, another pressure component, electron degeneracy pressure, becomes important. Due to the Pauli exclusion principle, electrons fill up the lowest available energy states and the electrons in the higher energy levels contribute significantly to degeneracy pressure when $\rho T \propto \rho^{5/3}$ in the non-relativistic case (since the ideal pressure scales as $P \propto \rho T$). Calculations show that degeneracy pressure dominates when $\rho > 200 \text{ g/cm}^3$, and $T < T_{\text{nuc}}$. Using these scaling relations, one can show that degeneracy pressure becomes important when $M < 0.1M_\odot$. The mass range from about $0.001M_\odot < M < 0.1M_\odot$ has approximately a constant radius, because at the high mass end the degeneracy pressure scales as $R \propto M^{-1/3}$ (this relation has been used to calculate the radius of the companion brown dwarf in SAXJ 1808.4-3658). In contrast, at the low mass end, the Coulomb pressure (which is characterized by constant density, $\rho \propto M/R^3$ and thus $R \propto M^{1/3}$) starts to dominate over degeneracy, so that approximately $R \propto M^0$ implying no mass dependency at all (as shown in Figure 2); although several models work with $R \propto M^{-1/8}$. \[5\][6][7]

In 1963 Kumar calculated the mass at which a low mass object could withstand the gravitational force by electron degeneracy pressure instead of ideal gas pressure. This mass, the lowest mass a star can have being able to fuse hydrogen, has been called the hydrogen-burning mass limit (HBML). Several calculations have constraint this limit to be $0.070M_\odot < M_{\text{HBML}} < 0.080M_\odot$ for objects of solar metallicity; comparable with 73 to 84 Jupiter masses, depending on which (numerical) equation of state was used. An equation of state for black and brown dwarfs is given and explained in Appendix (C). \[2\][5][6][7][10]

### 3.4 Definition of brown dwarfs and planets

The canonical definition of a brown dwarf is a compact object that has a core temperature insufficient to support continuous fusion reactions, analogous to the mass requirement for brown dwarfs to have masses lower than the HBML. This definition certainly does not distinguish planets from brown dwarfs. It is rather the formation process which makes the two separable: planets form in circumstellar disks whilst brown dwarfs form out of interstellar gas cloud collapse. Since it is this gas that makes direct observation extremely difficult, astronomers
Figure 2: *Plot of mass versus radius.* As shown, brown dwarfs appear to have a constant radius, independent of their masses. This property limits the effective temperature to be a function of the luminosity only. Hence, measuring $T$ enables classification of the object being a star or a brown dwarf. [5]

ought to find the solution in eccentricity in binaries: brown dwarfs might orbit in a more eccentric way around the center of mass than planets do. However, observations showed that both planets and brown dwarf can have similar eccentricities. This is the point where Oppenheimer and Kulkarni (1998) propose a new definition of planets: objects for which no nuclear fusion of any kind takes place during the entire history of the object.

<table>
<thead>
<tr>
<th>Object</th>
<th>Mass $^7$ (M$_\odot$)</th>
<th>Fusion H</th>
<th>Fusion D</th>
<th>Contains Li &amp; D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Star</td>
<td>0.1-0.075</td>
<td>Sustained</td>
<td>Evanescent</td>
<td>No</td>
</tr>
<tr>
<td>Brown dwarf</td>
<td>0.075-0.065</td>
<td>Evanescent</td>
<td>Evanescent</td>
<td>Yes$^8$</td>
</tr>
<tr>
<td>Brown dwarf</td>
<td>0.065-0.013</td>
<td>Never</td>
<td>Evanescent</td>
<td>Yes</td>
</tr>
<tr>
<td>Planet</td>
<td>$&lt;0.013$</td>
<td>Never</td>
<td>Never</td>
<td>Yes</td>
</tr>
</tbody>
</table>

$^7$ Masses given here assume that the objects have solar metallicity.

$^8$ Brown dwarfs in this mass range have lithium abundances that are age dependent.

Table 1: *Definition of star, planet and brown dwarf.* [12]

Objects between $0.013 M_\odot > M > 0.08 M_\odot$ do fuse a little deuterium when they are young, because deuterium can be fused at still lower temperatures than hydrogen, primarily caused by the faster rate at which the D(p,γ)$^3$He reaction takes place, governed by the electromagnetic force. The pp reaction is much slower since it is driven by the weak nuclear force, requiring higher temperatures to start. Cooling curves of low mass objects show a clear separation between objects with continuous decreasing temperature as a function of time (planets) and curves with a tiny bump due to the halt of deuterium fusion (brown dwarfs). Conclusively, planets, brown dwarfs and stars are classified by their internal physics. Stars fuse hydrogen in equilibrium, brown dwarfs do not fuse hydrogen in equilibrium but do fuse some deuterium during their evolution; and planets never fuse anything; as illustrated in Figure 3 (although some physicists claim that a little fusion may occur via LENR, low energy nuclear reactions, or LANR, lattice assisted nuclear reactions). [6][12]
3.5 Observational identification of brown dwarfs

In principle there are five methods available in order to confirm that an object is substellar: [6][12]
1. the $L \propto T_{\text{eff}}^4$ relation
2. measuring lithium abundances
3. measuring molecular abundances
4. measuring deuterium abundances
5. studying the effects of dust formation.

3.5.1 The $L \propto T_{\text{eff}}^4$ relation

Since objects with masses below the HBML have roughly speaking a constant radius, the effective temperature of those objects are easily related to their radii by

$$T_{\text{eff}} = \left( \frac{L}{4\pi R^2 \sigma} \right)^{\frac{1}{4}} \quad (4)$$

where $\sigma$ is the Stefan-Boltzmann constant. Spectral synthesis models constrain $T_{\text{eff}}$ better than 10% compared with observations in most cases. However, a $0.013 \, M_{\odot}$ brown dwarf reaches this temperature only after approximately 100 Myr. Therefore, this technique works for very old and cool objects only. [6][12]

3.5.2 Lithium abundances

Distinguishing hot, young brown dwarfs from stars and giant gas planets, is most efficient by using the lithium depletion test. Objects that do not fuse hydrogen, objects with masses below the HBML limit, retain their initial lithium abundances forever.

![Figure 3: The color-magnitude diagram for Pleiades containing very low-mass star and brown dwarf members. The dotted line is an empirical main sequence at Pleiades distance. The location of the lithium depletion boundary is indicated by the solid line and is used to determine a precise age for the cluster of 125 ± 8 Myr. [15]](image)

This a direct implication of the nuclear reaction $\text{Li}^7(p,\alpha)\text{He}^4$. For very low mass stars, the lithium abundance inside the core will be depleted by this reaction in a period of approximately 50 Myr. For brown dwarfs in the mass range $0.065 M_{\odot} < M < 0.08 M_{\odot}$ however, this period takes 50 to 250 Myr. Below $0.065 \, M_{\odot}$, brown dwarfs keep their lithium abundances forever, since no hydrogen fusion reaction will ever take place. From theoretical models we know that very low mass stars and young, cool brown dwarfs are fully convective.
This remarkable feature implies that abundances in the core are reflected on the convective timescale in their observable atmospheres. Brown dwarfs typically have convective time scales of the order of decades, and scales proportional to \( L^{1/3} \). The evolutionary timescale however, is 6 to 8 magnitudes larger, so that atmospheric abundances can be considered identical to those inside the core. The Lithium test is very useful for determining the age of open clusters by finding the ‘lithium depletion boundary’, an imaginary line which separates faint objects without lithium from even fainter objects with lithium. In Figure 3 such boundary is presented for the case of the Pleiades. [6][12]

3.5.3 Molecular abundances

Once a brown dwarf has cooled down to about \( T_{\text{eff}} = 1500 \) K, lithium atoms begin to form molecules followed by an intensity drop of the Li I spectral line. Below 1500 K, chemical equilibrium between CO and \( \text{CH}_4 \) strongly favors \( \text{CH}_4 \). Strong absorption lines of methane appear in the range of 1 to 5 \( \mu \)m. Therefore, a spectroscopic detection of methane implies an effective temperature below 1500 K, requiring the object to be less massive than the HBML. Below 1000 K, ammonia can form and at still lower temperatures more exotic molecules are formed. [6][12]

3.5.4 Deuterium abundances

As stated before, when an object with deuterium is detected, it can not be a brown dwarf, since all initial deuterium abundances should be depleted due to fusion (no measurable abundances in the atmosphere means no measurable abundances in their interior either, due to the fully convective properties). Spectroscopic signatures of deuterium in the region of 1 \( \mu \)m to 8 \( \mu \)m include absorption lines of HDO and possibly \( \text{CH}_3\text{D} \). [6][12]

3.5.5 Dust Formation

Brown dwarf theory predicts the formation of dust (molecules up to 0.1 \( \mu \)m in size) in the atmosphere as brown dwarfs cool. Dust formation is also predicted for low mass main sequence stars. Dust formation occurs at approximately \( 10^{-4} L_\odot \), 1800 K. The luminosity drops as dust forms. [6][10]

3.6 Dark matter brown dwarfs

The search for brown dwarfs was driven by the search for dark matter. Applying the Salpeter initial mass function (IMF) in which \( dN/dM \propto M^{-2.35} \) to brown dwarfs, suggests that brown dwarfs ought to outnumber stars by two to three orders of magnitude. Whether Brown dwarfs or planets are a significant percentage of dark matter is still the subject of some debate. The majority of researches agree they are not; based on the microlensing experiments of the MaCHO (Massive Compact Halo Object) Collaboration. In the late 90’s the number of refereed papers in brown dwarfs increased rapidly due to the discovery of Gliese 229B, the first cool brown dwarf ever detected. [6][12]

3.7 Isolated brown dwarfs

The main reason why brown dwarfs in isolated systems are studied, is to acquire more knowledge of objects with masses below the HBML. The relative number of brown dwarfs compared to the number of higher mass objects, constrains star formation theory such that star formation out of interstellar gas clouds appears to be independent of the HBML. Observations of star formation regions in which very low-mass gas clouds are present, have revealed that brown dwarfs can form out of this process. Therefore, measuring the mass function would determine whether there is a lower limit to the mass of an object formed like a star, and not in a circumstellar disk. Because brown dwarfs cool as time passes, and can have an enormous range of luminosities over their lifetime, it is not easy to catalogue them. However, this issue is greatly simplified by examining two or more different brown dwarfs with the same age. In this case, mass can be considered to be a function of luminosity only, which can be observed directly. In order to find such a population of brown dwarfs with the same age, it is easiest to search for low-luminosity members of a well-studied open cluster in which, in principle, the age, distance and metallicity are measured accurately. However, since brown dwarfs are such faint objects, finding them outside star clusters requires intensive sky surveys, and can only be successful relatively close to Earth. For example, a 1000 K brown dwarf is detectable out to a distance of about 6 pc only, making use of the most advanced infrared optics. [5][6][12]
3.8 Brown dwarfs in open clusters and star-forming regions

Brown dwarfs can be identified by spectroscopic measurements, based on their lying above the ZAMS (Zero Age Main Sequence). In order to confirm that they are cluster members indeed, and not doppler-shifted background stars, accurate proper motion and radial velocity measurements need to be done. The Pleiades for instance, the richest nearby open cluster, containing objects with masses at the HBML and a nominal age of about 100 Myr, have effective temperatures of about 2500 K, corresponding to spectral class M6 V on the main sequence. Since the Pleiades have a fair proximity to the sun, browns dwarfs members should be detectable with modern optical CCD’s.

<table>
<thead>
<tr>
<th>Cluster Name</th>
<th>Distance (pc)</th>
<th>Age (Myr)</th>
<th>No. of Known Members</th>
<th>Area on Sky Sq. Deg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ursa Major</td>
<td>25</td>
<td>300</td>
<td>25</td>
<td>20</td>
</tr>
<tr>
<td>Hyades</td>
<td>46</td>
<td>600</td>
<td>550</td>
<td>100</td>
</tr>
<tr>
<td>Coma</td>
<td>80</td>
<td>500</td>
<td>50</td>
<td>20</td>
</tr>
<tr>
<td>Pleiades</td>
<td>130</td>
<td>125</td>
<td>800</td>
<td>25</td>
</tr>
<tr>
<td>IC 2602</td>
<td>155</td>
<td>30</td>
<td>125</td>
<td>10</td>
</tr>
<tr>
<td>IC 2391</td>
<td>160</td>
<td>30</td>
<td>100</td>
<td>8</td>
</tr>
<tr>
<td>Praesepe</td>
<td>170</td>
<td>600</td>
<td>800</td>
<td>25</td>
</tr>
<tr>
<td>Alpha Persei</td>
<td>175</td>
<td>75</td>
<td>350</td>
<td>30</td>
</tr>
</tbody>
</table>

Table 2: An overview of properties of the nearest open clusters (d < 200 pc). The Pleiades age is based on the lithium depletion boundary, which is 25 to 60% older than the age determined from the upper main-sequence turn off, but is ought to be more accurate. [12]

Half of the mass of the Pleiades is concentrated within a radius of approximately 2 pc, although the tidal radius is about 16 pc. The areas in the sky corresponding to circles with these radii are 2.5 and 250 square degrees, respectively. Therefore, a large area have to be observed in order to sample a significant part of the cluster. The lithium depletion boundary was used to estimate the age of the Pleiades at about 120 Myr, with the faintest Pleiad without lithium, PPL 15, defining the boundary. Spectra analysis of several brown dwarf candidates resulted in a lithium depletion boundary in the Pleiades of ± 1 mag, corresponding to an age for the cluster of \( \tau \sim 125 \pm 8 \) Myr. It is found to be a coincidence that at this age, the lithium depletion boundary yields masses of \( 0.075 \pm 0.005 \, M_\odot \), implying that all Pleiades members fainter than the lithium depletion boundary must be brown dwarfs. The faintest Pleiades found so far have masses of about \( 0.035 \, M_\odot \).

Brown dwarfs in star-forming regions (age 1 Myr) are much more intrinsically luminous than those in the open clusters like the Pleiades, and therefore should be easier to detect. However, on the other hand it is much more difficult to gain evidence that any object inside a star forming region is in fact substellar. For instance, the lithium test is of much less value, because all low-mass stars should still have their initial lithium abundances, so no lithium depletion boundary can be computed. Also, the theoretical isochrones for young, low-mass objects are uncertain and thus the ages are difficult to determine in this stage. Based on existing models, any object with spectral type M7 or lower, must be a brown dwarf if it contains lithium. To see why, stars more massive than the HBML deplete all off their lithium before they can cool to the M7 corresponding effective temperature whilst browns dwarfs can cool to M7 when they are much younger, and still retain their lithium. [1][6][12][15][16]
3.9 Companion brown dwarfs

Searching for brown dwarf companions of nearby stars is interesting mainly because it is the most effective way to detect very cool objects (objects with lowest luminosity). However, a difficulty that arises is that scattered light of the primary star disables direct detection of the brown dwarf. Fortunately, there are several techniques to circumvent this difficulty. First, if one searches at longer wavelengths, the contrast between the brown dwarf and the primary star is lowest. Second, if the search for companions is aimed at white dwarfs only, the resulting contrast between the brown dwarf and the primary star is very small due to the intrinsic faintness of these stars. Third, the usage of a coronagraph will make the observation of the fainter companion possible. In 1995 Nakajima successfully applied the third technique on the Palomar 60-inch telescope by showing that the star Gliese 229 has a companion with a luminosity of less than $10^{-5} \, L_\odot$ (see Figure 4). [6][11][12]
4 Physics of binary systems

When two different stars are close enough together such that they sense each others gravitational field, and hence orbit a communal center of mass, the two stars form a binary system (Figure 5). Binary systems are of great importance in several fields of astrophysics and are responsible for many challenging physical problems to occur in the current research field. Observing binary parameters such as the (binary) distance $a$ between the two stars, or the period $P$ in which the center of mass is circumvented by one of the stars, (which allow the masses of the stars to be directly determined making use of the mass function, equation 12), constrain other stellar parameters like radius and temperature, such that they can be indirectly estimated. From this the individual mass-luminosity relationship ($L/L_\odot \propto (M/M_\odot)^n$) of the stars can be obtained.

At some stage of the binary evolution, the two stars come close enough together such that the more massive star starts to consume its lower mass companion, resulting in the ‘ignition’ of several interesting, powerful physical processes such as accretion: the process in which matter under the influence of gravity is attracted to a massive body, and increases its mass. [8]

4.1 Accretion

The study of accretion as a power source in binaries is still the area where the greatest progress in the understanding of accretion in general is made. This is because of the fact that stars in binaries reveal more about themselves (spatial dimensions and mass in particular) than isolated ones. The advantage is greatest when eclipsing binaries are studied. This is because binaries with the orbital plane so near the line of sight of the observer, reveal information about their spatial relations due to their eclipses. The continuity of accretion processes is governed by the conservation of angular momentum. Nearly all transferred material is not able to land on the accreting star until it has lost a significant part of its angular momentum. This material forms accretion disks, which in turn are efficient machines with the capability to drain gravitational potential energy and convert it into radiation. [8]

4.2 Mass transfer between binary members

Two explicit arguments apply to binaries that transfer matter at some stage of their evolution. Namely,

1. the binary separation decreases, or one of the stars may increase its radius, until the moment where one companion is able to remove the outer layers of the atmosphere of the other star (called Roche lobe overflow, see section 4.6);
2. a significant part of mass in the form of stellar wind will be ejected by one of the two stars, which is captured gravitationally by the companion (stellar wind accretion).

The first argument does not necessary imply Roche lobe overflow (section 4.7), the binary separation may decrease also because orbital angular momentum is not conserved due to stellar wind mass loss, or gravitational radiation for example. [8]

4.3 Keplerian orbits

Problems regarding mass transfer via Roche lobes, paths that mass will follow during accretion, were first studied by the French mathematician Edouard Roche. He considered the orbit of a test particle (a particle that does not influence the system such that the three-body problem have to be considered) in a gravitational potential formed by two massive objects orbiting their center of mass driven by their common gravitational attractions. The Roche approach assumes that the stars move in Keplerian, circular orbits, which in many cases is a fair approximation. In some cases however, such assumptions, as well that the stars are regarded as point masses, can not be made.

For convenience, it is recommendable to write the masses of the two objects as $M_1 = m_1 M_\odot$ and $M_2 = m_2 M_\odot$ so that Kepler’s law conveniently can be written as

$$a^3 = \frac{GMP^2}{4\pi^2}$$

(5)
A binary system with a compact star of mass $M_1$ and a companion of mass $M_2$ orbiting their common center of mass with binary separation $a$.

where $M = M_1 + M_2 = m M_\odot$. For binary periods of the order of years, days or hours, $a$ can be expressed as a function of the period $P$ and mass ratio $q = M_2/M_1$:

$$a = \begin{cases} 1.5 \cdot 10^{13} m_1^{1/3}(1 + q)^{1/3} P_{yr}^{2/3} \text{ cm} \\ 2.8 \cdot 10^{11} m_1^{1/3}(1 + q)^{1/3} P_{day}^{2/3} \text{ cm} \\ 3.5 \cdot 10^{10} m_1^{1/3}(1 + q)^{1/3} P_{hr}^{2/3} \text{ cm} \end{cases} \quad (6)$$

where $P_{yr}$, $P_{day}$ and $P_{hr}$ are values given in years, days and hours, respectively. [8]

### 4.4 Gas dynamics

All accreting matter, like most of the material in the universe, appears in gaseous form. This means that the particles that form the gas interact only by collisions, rather than by short-range forces. On average, a gas particle will travel a mean distance before its momentum will change due to collisions. This distance is called the mean free path $\lambda$. If the gas can be considered to be uniform at length scales a few times $\lambda$, the particles will end up to have a mean velocity $v$, due to their randomized collisions. This velocity $v$, is therefore equivalent with the velocity of the gas as a whole. In a reference frame moving with this velocity $v$, the particles obey a Maxwell-Boltzmann distribution of velocities, described by a temperature $T$. If the gas is viewed at macroscopical length scales only ($L \gg \lambda$), the gas can be considered as a continuous fluid, with velocity $v$, temperature $T$ and density $\rho$ defined at each point. Together with the laws of conservation of mass, momentum and energy, the behavior of the gas can be described reasonably accurate.

Given a gas with velocity $v$, density $\rho$ and temperature $T$, which are all functions of position $r$ and time $t$, the mass conservation is ensured by the continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0 \quad (7)$$

where in this case the dot $\cdot$ represents the dot product. Since the gas has a pressure $P$ at each point due to the thermal motion of the particles, the pressure can be related to density and temperature by a specific EoS (usually the perfect gas law yields a fair approximation in these cases). A pressure gradient ($\nabla P$) of the gas has the same dimensions as force, since momentum is transferred this way. Other forces acting on gasses are often expressed in terms of force density $f$. Important contributors to this force density are the local gravitational acceleration $f = -\rho g$, and viscosity, which is the transfer of momentum along velocity gradients, by random motions of the gas. Mathematically, a gas flow between two stars is governed by the Euler equation

$$\rho \frac{\partial v}{\partial t} + \rho (v \cdot \nabla) v = -\nabla P + f \quad (8)$$

where the term $\rho (v \cdot \nabla) v$ represents the convection of momentum through the fluid by velocity gradients. If the center of mass is the reference frame, such that it is rotating with angular velocity $\omega$, an extra term in the Euler equation is introduced to take account for the centrifugal and Coriolis forces:

$$\frac{\partial v}{\partial t} + (v \cdot \nabla) v = -\nabla \Psi_R(r) - 2(\omega \times v) - \frac{1}{\rho} \nabla P \quad (9)$$

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where $\times$ represents the cross product, $\omega = (\frac{GM}{a^3})^{1/2} e$, with $e$ the unit vector normal to the orbital plane, and the term $-2(\omega \times v)$ is the Coriolis force per unit mass, and the quantity $-\nabla \Psi_R(r)$ accounts for both the gravitational and the centrifugal force. Macroscopic gas dynamics are further governed by the concepts of adiabatic flows, isothermal flows and hydrostatic equilibrium. Whenever it is necessary to consider gas behavior on lengthscales comparable to the mean free path between collisions, the concepts regarding plasma physics needs to be taken into account. [8]

4.5 Roche potential

The term $\Psi_R(r)$ is known as the Roche potential (left image of Figure 6) and is given by

$$\Phi_R(r) = \frac{GM_1}{|r-r_1|} - \frac{GM_2}{|r-r_2|} - \frac{1}{2} (\omega \times r)^2$$

(10)

where $r_1$ and $r_2$ are the position vectors of the centers of the two stars. Equipotential surfaces of $\Psi_R(r)$ and especially their cross sections in the orbital plane, as can be seen in the right image of Figure 6, visualize the paths that accreted mass will follow. [8]

Figure 6: Left: A surface representing the Roche potential. The larger pit is around the compact (more massive) star. The downward curvature near the edges is due to the centrifugal force; a test particle attempting to rotate about the binary at these distances experiences a net outward force. Right: Sections in the orbital plane of the Roche equipotentials $\Psi_R(r) = \text{constant}$, for a specific value for $q$. The center of mass, two stars and the Lagrangian points $L_1$-$L_5$ are shown. The saddle point $L_1$ forms a pass between the two Roche lobes. $L_2$ and $L_3$ are the points where mass can escape from the system. $L_4$ and $L_5$ (the so called Trojan asteroid points) are local maxima of $\Psi_R(r)$, but Coriolis forces stabilize synchronous orbits of test particles at these points.

4.6 Roche lobe

The shape of these equipotentials is driven entirely by the mass ratio $q$, but the overall scale depends on binary separation $a$. For matter orbiting at large distances ($r \gg a$) the binary is viewed as a point in the center of mass (CM). Thus, equipotentials at large distances appear to be just a point mass in a rotating frame. Equivalently, a point mass orbiting circular equipotentials around the center of one of the stars ($r_1$ or $r_2$), will not be significantly affected by the gravitation of the companion. Hence, the potential $\Psi_R(r)$ has two deep valleys centered on $r_1$ and $r_2$, respectively. The most interesting and important feature of Figure 6 is the eight shaped line, which shows how these two valleys are connected. This critical surface surrounding each star is known as a Roche lobe. The lobes join at the inner Lagrange point $L_1$, which is a saddle point of $\Psi_R(r)$, just like a high mountain pass splits two valleys. Material in one of the two lobes near $L_1$ is therefore much easier transferred into the other lobe than to escape the crucial surface. If initially both stars are assumed to be significantly smaller than their Roche lobes and that the rotation of each star on its axis is synchronous with...
the orbital motion, with axes orthogonal to the binary plane, then mass transfer can only occur via the wind mechanism. [8]

4.7 Roche lobe overflow

If for some kind of reason one of the stars swells up such that it fills its Roche lobe, the binary is then called semi-detached, and hence will efficiently transfer mass to the other star. This type of mass transfer is called Roche lobe overflow. A star cannot be significantly larger than its Roche lobe, since the mass more or less will be instantly removed from the star once its Roche lobe is filled. Once the lobe-filling condition is met, through either evolutionary expansion of the star or a decrease of the binary separation due to angular momentum loss, the Roche lobe filling star needs to adjust its structure such that its radius attunes with the lobe. In some circumstances both stars fill their Roche lobe simultaneously. Such binary systems are called contact binaries. [8]

4.8 Formation of accretion discs

During the binary evolution, gas from the donor star gains angular momentum due to the orbital motion of the donor around the center of mass. As stated before, the gas flows towards the compact star by first forming an accretion disk. The gravitational energy lost is then radiated in the form of X-rays (however, if the accretor is a white dwarf instead of a neutron star or black hole, the emitted radiation peaks in optical/UV, since white dwarfs have much weaker gravitational fields in the inner regions of the accretion disc). This emitted radiation may be captured by both the accreting and donor star, which might alter their evolutionary tracks.

In many cases, the so called thin accretion disc model applies in which \( h(r)/r \gg 1 \), with semi-thickness \( h(r) \) and radial coordinate \( r \). As the gas leaves the donor star via the Lagrangian point \( L_1 \), it has gained a substantial amount of angular momentum equal to the specific orbital angular momentum. If the orbit of a test particle, released from rest at \( L_1 \), lays in the gravitational field of the accretor alone, then the resulting orbits of the main stream in the binary plane will be elliptical. The gravitational field of the secondary star causes this process to run slowly. This is because the presence of the donor star alters the \( 1/r \) effective potential of the accretor, which is required for closed periodic orbits. Therefore, if a continuous stream of matter follows this trajectory, it will intersect with itself, with a dissipation of energy via shock waves as a consequence. As the gas is about to leave at \( L_1 \), it has not much possibilities of losing its angular momentum, resulting in circular orbits of lowest energy. With these assumptions, the gas is expected to orbit the accretor in the binary plane at the circularization radius \( R_{\text{circ}} \), such that the Keplerian orbit at \( R_{\text{circ}} \) has the same angular momentum as the transferring gas, as its passes through \( L_1 \). The circular velocity associated with this is given by

\[
\nu_\phi = \left( \frac{GM_d}{R_{\text{circ}}} \right)^{1/2}
\]

with \( M_d \) the mass of the donor star and \( \phi \) the azimuthal, cylindrical coordinate. The movement of the gas then occurs within a thin ring, which needs to expand radially both inwards and outwards, in order to form an accretion disc. The resulting inertial forces acting on the gas are then dictated by the product of \( R_{\text{circ}} \) and \( \nu_\phi \). Forming an accretion disc this way can only occur if particles within the ring collide with one another, such that viscosity starts to play a significant role. When this is the case, the movement of the gas is called hydrodynamic gas flow, which is a widely studied phenomenon. [8]
5 Binary system SAXJ 1808.4-3658

In 2000, King estimated that there are several neutron stars in the galactic bulge accreting from brown dwarfs with a time averaged transfer rate $\langle \dot{M} \rangle \approx 10^{-11} M_\odot \text{ yr}^{-1}$. Bildsten and Chakrabarty showed in 2001 that SAXJ 1808.4-3658 supports this hypothesis. The measured orbital parameters of this accreting millisecond pulsar, with frequency $\nu = 401 \text{ Hz}$, $P = 2.01 \text{ hr}$, projected semi-major axis $a_\times \sin i = 62.8$ light-milliseconds ($\sim 1.9 \cdot 10^4 \text{ km} \sim 0.03 R_\odot$) and a time averaged flux $\langle F \rangle = 9 \cdot 10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1}$, allows the calculation of the companion mass, $M_c$, and radius $R_c$. Bildsten and Chakrabarty claimed that the companion cannot be a main sequence donor, since it would require a nearly face on inclination and a higher $\langle \dot{M} \rangle$ than observed. The predicted $\langle \dot{M} \rangle$ is more consistent with the mass transfer rate observed if a hot 0.05 $M_\odot$ brown dwarf is the donor. The remaining problem is to find a solution of why and how this brown dwarf yields a required radius of 0.13 $R_\odot$, in order to fill the Roche lobe, since the age of the system implies that the brown dwarf must be old, in the order of Gigayears, and thus should have cooled down. Substituting these binary parameters into the mass function yields

$$f_x = \frac{(M_c \sin i)^3}{(M_c + M_x)^2} = \frac{4\pi^2(a_\times \sin i)^3}{GP_{\text{orb}}^2} = 3.8 \cdot 10^{-5} M_\odot \quad (12)$$

with $i$ the binary inclination (the angle between the line of sight and the orbital angular momentum vector), and $M_x$ the mass of the neutron star, leads to a value for $M_c$, for a certain value for the inclination $i$ and mass of neutron star $M_x$. It is most likely that the neutron star in this system has a mass within the range $1.4 M_\odot < M_x < 2.0 M_\odot$, which constrains the mass of the brown dwarf companion to the values $0.043 M_\odot < M_c < 0.054 M_\odot$, as can be seen in the left image of the Figure 7. The lower part of this image shows the mass transfer rate as a function of inclination.

Several supporting pieces of evidence have been presented that claim that SAXJ 1808.4-3658 is not viewed face on. For instance, a 2% modulation in X-ray intensity at the orbital period has been measured, with the minimum occurring when the neutron star is behind the companion, implying that face-on inclination is very unlikely. From RXTE observations of the April 1998 outburst, a time-averaged flux $\langle F \rangle$ was measured of $9 \cdot 10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1}$. Thus, the total irradiation flux received per second by the brown dwarf companion (only half of the companion’s surface is irradiated)

$$\frac{1}{2} \cdot 4\pi R^2 \langle F \rangle = \frac{1}{2} \cdot 4\pi (0.13 R_\odot)^2 \cdot \langle F \rangle = 2\pi \cdot 0.0169 \cdot (6.96 \cdot 10^{10} \text{ cm})^2 \cdot 9 \cdot 10^{-11} \approx 5 \cdot 10^{10} \text{ erg s}^{-1} \quad (13)$$

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The mean density of a Roche lobe filling companion (for the case where \( M_c \ll M_x \)) is set by the binary period. The radius of the companion can be expressed in terms of the companion mass which was first done by Faulkner et al. in 1972:

\[
R_c = 0.17 \left( \frac{M_c}{0.1 M_\odot} \right)^{1/3} R_\odot = 0.17 \left( \frac{0.05}{0.1} \right)^{1/3} R_\odot \approx 0.13 R_\odot
\]

(14)

It follows, as can be seen in the right image of figure 7, that old brown dwarfs, white dwarfs and main sequence stars can be excluded. [2][9]
6 Modules for Experiments in Stellar Astrophysics (MESA)

The one dimensional evolution module, MESA Star, has been used to find out if brown dwarfs can significantly increase their temperature and radius when they are irradiated by an external source. Mesa Star, combines many of the numerical and physics modules for simulations of a wide range of stellar evolution scenarios. These simulations apply to very low mass stars, brown dwarfs, giant gaseous planets, white dwarfs as well to very massive stars.

MESA star builds 1D spherically symmetric models by dividing the stellar structure into cells, with a total amount of hundreds to thousands, depending on the complexity of the object considered. Thus, the number of time steps depends on stellar structure parameters like the nuclear burning rate, gradients of state variables, composition, and several other values as shown in Figure 8. These cells are numbered with one at the surface and increasing inwards to the center. MESA star is able to solve the stellar equations simultaneously for all cells from the surface to the center. Each cell contains several mass averaged variables, whilst others are defined at the outer face, like for instance the irradiation flux parameter, such that external heating only affects the outer layers. The inner boundary of the innermost cell is usually the center of the star and, therefore, has radius, luminosity, and velocity equal to zero. The cell mass-averaged variables are density $\rho_k$, temperature $T_k$, and mass fraction vector $X_{i,k}$. The boundary variables are mass interior to the face $m_k$, radius $r_k$, luminosity $L_k$, and velocity $v_k$. From this other values are obtained such as $\epsilon_{nuc}$, $\kappa$, $\sigma_k$, and $F_k$. The variables are then calculated at time $t + \delta t$.

\[
\rho_k = \frac{dm_k}{4/3\pi(r_k^3 - r_{k+1}^3)}
\]

or, solving for $\log r_k$,

\[
\log r_k = \frac{1}{3} \log \left( r_{k+1}^3 + \frac{3}{4\pi} \frac{dm_k}{\rho_k} \right)
\]

Similar equations, both analytically and numerically, can be written for the remaining structure variables, which are stated in Appendix A (section 10.1). [13]
7 Results

The first successful evolution computation performed in MESA regarding brown dwarfs is shown in Figure 9. The upper plot, where effective temperature is plotted as a function of time, shows curves of objects with several masses. The curves belonging to objects within the mass range $0.02 \, M_\odot < M \leq 0.07 \, M_\odot$ (lower than the HBML), show negative slopes as time passes whereas objects with $M > 0.07 \, M_\odot$ appear to converge to constant temperatures. The radii (expressed in solar units) of these objects show similar behavior: radii of objects within the mass range $0.02 \, M_\odot < M \leq 0.07 \, M_\odot$ show negative slopes as times passes, whilst objects with masses greater than $0.07 \, M_\odot$ show constant behavior.

Figure 9: Upper graph: Cooling curves of several isolated astrophysical objects in the mass range $0.02 \, M_\odot < M < 0.1 \, M_\odot$. As can be seen clearly from the upper three curves, thermal equilibrium is established due to the fact these objects have masses above the HBML, and therefore fuse hydrogen continuously, and thus are main sequence stars. The lower six curves do not show constant temperature behavior, since their masses are below the HBML, and therefore are brown dwarfs. Lower graph: Radius time dependence.
7.1 HR-diagram of irradiated 0.05 $M_\odot$ brown dwarfs

Visualizing evolutionary tracks is done best by HR-diagrams. The HR-diagram in Figure 10 has been obtained with the total irradiation flux parameter fixed at $5 \cdot 10^{10}$ erg s$^{-1}$ and column depth $3$ g cm$^{-2}$, where column depth is the parameter which accounts for how deep the radiation can penetrate through the surface of the brown dwarf. (One remark has to be made explicitly here: since mass transfer, mass loss, binary periodicity and magnetic flux are not incorporated into the MESA code, and also the irradiation flux is set constant, this model is theoretical only and does not apply to the real physical system SAXJ 1808.4-3658). The HR-diagram shows that this final temperature is similar for all tracks: the time when irradiation sets in does not seem to have influence on this final temperature (a little less than 4000 K). It does affect the luminosity however: the later irradiation sets in, the larger the luminosity will end up to be.

Figure 10: This HR-diagram shows the evolutionary tracks of several irradiated 0.05 $M_\odot$ brown dwarfs, with a none-irradiated 0.05 $M_\odot$ brown dwarf and a 0.1 $M_\odot$ main sequence star as a reference.
7.2 Effect of irradiation on a 0.05 $M_\odot$ brown dwarf radius

In Figure 11 the radius (in solar units) of a 0.05 $M_\odot$ brown dwarf is plotted against time. These graphs show the effect of irradiation on the radius-time relation. The 0.13 $R_\odot$ radius of the brown dwarf companion of the system SAXJ 1808.4-3685 is indicated by the green line. As can be seen in the left upper graph, the case where irradiation starts when the brown dwarf is still in the formation process, the slopes are much steeper, than when irradiation starts later. This trend seems to be continuous. The later irradiation sets in, the slower the radius will increase.

![Figure 11](image)

Figure 11: All four curves represent radii-time dependencies for a 0.05 $M_\odot$ brown dwarf. The upper left graph represents the situation in which irradiation started after 0.1 Gyr, whereas the lower right graph indicates the situation where irradiation started after 5 Gyr. These situations are clearly insufficient to approximate the 0.13 $R_\odot$ green line. On the contrary, the upper right graph shows the result for irradiation setting in after 0.5 Gyr, the lower left after 1 Gyr. As can be seen, the 0.5 Gyr curve fits best. Intersection takes place at approximately $t = 9$ Gyr.

The column depth values are expressed in g cm$^{-2}$. As stated before, this value expresses the density of the outer atmospheric layers in order to govern the penetration depth of the X-rays. The reason why the column depth set on 10 g cm$^{-2}$ is not shown for all tracks is because in those cases the MESA-code did not converge. As can be seen, the exact column depth value does not seem to alter the tracks. The curves belonging to $t_{\text{irrad}} = 0.5$ Gyr intersect the 0.13 $R_\odot$ green line at $t \approx 9$ Gyr.
7.3 Effect of irradiation on a 0.05 $M_\odot$ brown dwarf its temperature

Figure 12 indicates that the later irradiation takes place, the greater the resulting temperature difference will end up to be. The final temperature for all situations (about 3900 K) is equal, independent of the moment that irradiation starts. Again, the exact value for the column depth does not appear to be of significant importance.

Figure 12: These four curves represent effective temperature-time dependencies for a 0.05 $M_\odot$ brown dwarf. These images show that all cases converge to a constant temperature a little lower than 4000 K.
8 Discussion

The plots in the previous section clearly show that irradiation indeed alters the evolutionary tracks of brown dwarfs. Brown dwarf radii and effective temperatures increase when irradiation sets in after a certain time in such a way, that systems like SAXJ 1808.4-3658 might be explained. The part of irradiation flux emitted primarily by the accretion disk, which is captured by the brown dwarf companion, analogously speaking upgrades the brown dwarf to some higher temperature class, a little higher than the M-dwarf regime. The final temperature of all four values of $t_{\text{irrad}}$, approximately 3900 K, fall into the temperature range that were constrained by Bildsten and Chakrabarty in 2001. They found that the temperature of the side of the companion facing the neutron star, lays within the range $3800 \text{ K} < T_H < 4800 \text{ K}$, making use of the relation $4\pi(a - R)^2 \sigma_b T_H^4 = L_{\text{th}}$ (equation 4), with $i$ the inclination, and the quiescent luminosity $L_{\text{th}}$ approximated by the relation of Brown et al. (1998)

$$L_{\text{th}} = 6 \cdot 10^{32} \text{ erg s}^{-1} \left[ \frac{\langle \dot{M} \rangle}{10^{-11} M_{\odot} \text{ yr}^{-1}} \right]$$

(17)

The evolutionary tracks of irradiated and not irradiated brown dwarfs show significant differences in luminosity $L$ as can be seen directly in the HR-diagram (Figure 10) but also from equation 4 in relation with temperature-time dependencies shown in Figure 9 and Figure 12. If we take a close view to the value for the effective temperature of a not irradiated 0.05 $M_{\odot}$ brown dwarf at $t = 9$ Gyr (Figure 9), conveniently assuming the age of SAXJ 1808.4-3658 to be this old (value of intersection with the 0.13 $R_{\odot}$ green line, as shown in the right upper graph of Figure 11), we see that the effective temperature is approximately 800 K, (T-dwarf regime), whereas this value for an irradiated 0.05 $M_{\odot}$ brown dwarf, as can be seen in Figure 12, is approximately 3900 K. This leads to a temperature ratio $T_{\text{irrad}}/T_{\text{no irrad}} \approx 5$. If we also assume the radius in both situations to be constant at this time (which is a fair approximation on reasonably large timescales for the not irradiated case, but rather rough in the irradiated case), equation 4 yields a luminosity difference

$$\frac{L_{\text{irrad}}}{L_{\text{no irrad}}} = \left( \frac{T_{\text{irrad}}}{T_{\text{no irrad}}} \right)^4 = \left( \frac{3900}{800} \right)^4$$

(18)

resulting in a difference of at least two orders of magnitude.

8.1 Physical interpretation of results

Figures 9-12 contain interesting information regarding the behavior of brown dwarfs, which in principal, can be considered to be good approximations of real physical systems like SAXJ 1808.4-3658. However, some limitations of the MESA-code might have simplified realistic scenarios such that constraints like equation 18, cannot be used to constrain brown dwarfs companions in general. [3]

8.1.1 Evolution of temperature and radius of not irradiated low mass objects (Figure 9)

The curves of the upper graph of Figure 9, where effective temperature is plotted against time, show a clear distinction in the evolutionary tracks of brown dwarfs and main sequence stars. As can be seen, the curves belonging to objects with mass $0.08 M_{\odot} < M < 0.1 M_{\odot}$ show constant temperatures as time passes, as expected for objects above the HBML, which are thus main sequence stars, since thermal equilibrium governs this constancy. On the contrary, objects within the mass range $0.02 M_{\odot} < M < 0.07 M_{\odot}$, objects with masses all lower than the HBML, continue to cool down after the formation process, simply because thermal equilibrium is absent in those cases. Of course low-mass main sequence stars will also cool down eventually, as they leave the ZAMS, but this takes much more time than the current age of the universe. Similarly, the radius-time relation presented in the lower graph of Figure 9, shows constant radii for main sequence stars and decreasing radii for brown dwarfs.

8.1.2 Evolutionary tracks

The evolutionary tracks described in the HR-diagram (Figure 10) show a direct relation between luminosity and the moment where irradiation sets in. As can be seen, the slopes describing the increase of luminosity due to irradiation, are very similar. This implies that the sooner irradiation sets in, the greater the luminosity difference with not irradiated 0.05 $M_{\odot}$ brown dwarfs will be, after a certain time. From this diagram also follows that effective temperature is not depending on the moment where irradiation sets in, due to the fact
that $T \propto (L/R^2)^{1/4}$. This property probably can be extended to brown dwarfs with masses other then $0.05 \, M_\odot$, especially in the low mass end, due to the mass-radius relation $R \propto M^{0.5}$. 

### 8.1.3 Effect of irradiation on radius and temperature dependencies

The fact that the decrease of a $0.05 \, M_\odot$ brown dwarf radius halts once it is suddenly irradiated by a certain flux, is of course due to the fact that there is no tendency to shrink because the brown dwarf can not cool down. Radiation, coming primarily from the accretion disc around the neutron star, prevents heat to continue to flow outwards, since it is very likely that continuous heating fixes the entropy of the brown dwarf at a value higher than would occur in the absence of heating, probably causing the minimum orbital period of binaries like SAXJ 1808.4-3658 to increase. The sooner a brown dwarf is irradiated after formation, the sooner the shrink of the radius will be halted, and the faster the radius will increase due to the halt of outward heat flow. The situation in which irradiation sets in after 5 Gyr, as is shown in the upper right graph of Figure 11, fits the physical properties of SAXJ 1808.4-3658 best, although it seems plausible that if the irradiation flux of $5 \cdot 10^{10} \, \text{erg s}^{-1}$ is assumed to be continuous, the brown dwarf would have been a main sequence star initially, taking in account the time-averaged mass transfer rate $\langle \dot{M} \rangle = 10^{-11} \, M_\odot \, \text{yr}^{-1}$. If this mass transfer rate is assumed to be typical during the time $\tau$ the brown dwarf has been irradiated, the time between the start of irradiation $t_{\text{irrad}} = 0.5 \, \text{Gyr}$ and the time at which the curve of Figure 11 intersects with the 0.13 $R_\odot$ green line, approximately 9 Gyr, then the total mass that has been transferred during its evolution is given by

$$\tau \cdot \langle \dot{M} \rangle = 8.5 \cdot 10^9 \, \text{yr} \cdot 10^{-11} M_\odot \, \text{yr}^{-1} = 0.085 \, M_\odot$$

yielding an initial companion mass above the HBML.

### 8.2 Limitations of the model

As stated before, computations by MESA could be performed by making several assumptions and simplifications. Firstly, the MESA-code assumes spherical symmetry in order to solve the much easier 1D stellar structure equations. Since only half of the surface of the brown dwarf companion is irradiated, it might not be realistic to assume spherical symmetry by adding a factor $1/2$ to equation 13. Secondly, the irradiation flux parameter is just a single parameter, added to the MESA-code, and is not governed by binary parameters like the period $P$, binary separation $a$, frequency $\nu$ or time-averaged transfer rate $\langle \dot{M} \rangle$. Also, the companion mass was set fixed at $0.05 \, M_\odot$ from the start. In the first stages of the mass transfer process the companion mass was higher than the HBML, yielding different interior physical properties than objects below the HBML, which would result in different evolutionary tracks, especially during the first few billions of years. Efforts to incorporate these binary parameters within the MESA-code resulted in convergence problems, but is seems plausible that it can be done if the code is slightly adjusted at the time steps where the stellar equations do not converge. Thirdly, the value for the quiescence flux, $5 \cdot 10^{10} \, \text{erg s}^{-1}$, is a little to high since it is very likely that during the early stages of the accretion process, the radiation emitted by the accretion disc was a little smaller. Hence, assuming continuity overestimates a more realistic value. On the other hand, the transient external heating during outbursts has been neglected, and perhaps does contribute significantly to the time-averaged irradiation flux. If so, then the overestimation of the quiescence flux due to the fixed companion mass probably will be sufficiently small. Also, the column depth parameter has not been modelled, but rather a reasonable value was picked in accordance with the measured energy of the emitted X-rays.

### 9 Conclusions and future work

Brown dwarf radii and temperatures increase if they are irradiated by an external source such as quiescence flux emitted by accretion discs. Evolutionary tracks of brown dwarfs in binaries clearly show differences compared with brown dwarfs in isolation. Observables, like effective temperature for instance, can possibly be used to constrain the properties of brown dwarfs in binaries. It might even be possible to clarify why expected luminosities differ from observed ones. To extend this work, mass transfer, mass loss, variable irradiation flux and other binary parameters like the period or binary separation should be incorporated into the MESA-code, in order to model more realistic physical systems. This implies that the occurrence of convergence problems should be circumvented by adjusting the boundary conditions at the point where these problems arise.
10 Appendix

10.1 (A) Stellar equations

A set of partial differential equations model the physical processes that occur inside stars that are into ‘near-equilibrium’. These equations can have different forms and difficulties, depending on which components are taken into account, and whether situations like convection processes are ignored, simplified, or fully treated. It all comes down to incorporate all processes into the equation of state (EoS). Fortunately, in most stars, at most stages of evolution, the main contribution to outward pressure comes just from radiation pressure, the perfect gas law and degenerate electron pressure; where the latter even can be ignored during most stages of the main sequence. These partial differential equations, describing stellar evolution, are traditionally written for the four structure variables \( P \) (pressure), \( T \) (effective temperature), \( r \) (radius) and \( L \) (luminosity), with Lagrangian mass-coordinate \( m \) as the independent variable. The stellar equations are:

\[
\frac{\partial \ln(P)}{\partial m} = -\frac{Gm}{4\pi r^4 P} \quad (20)
\]

\[
\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \quad (21)
\]

\[
\frac{\partial \ln(T)}{\partial m} = \frac{\partial P}{\partial m} \min(\nabla_r, \nabla_a) \quad (22)
\]

\[
\frac{\partial L}{\partial m} = \epsilon - \epsilon_\nu - C_p T \left( \frac{\partial \ln(T)}{\partial t} - \nabla_a \frac{\partial \ln(P)}{\partial t} \right) \quad (23)
\]

where \( \nabla_r \) and \( \nabla_a \) are respectively given by

\[
\nabla_r \equiv \frac{3 \kappa PL}{16 \pi c G m T^4} \quad (24)
\]

\[
\nabla_a \equiv \left( \frac{\partial \ln(T)}{\partial \ln(P)} \right)_S \quad (25)
\]

with density \( \rho \), opacity \( \kappa \), specific heat \( C_p \), adiabatic gradient \( \nabla_a \), radiative gradient \( \nabla_r \), nuclear energy generation rate \( \epsilon \), neutrino loss rate \( \epsilon_\nu \), being known functions of pressure \( P \), temperature \( T \) and the abundances \( X_i \) \((\sum X_i = 1)\) of the various nuclear species. Equations (2), (3), (4) and (5) can be solved for a given distribution of \( X_i(m) \), and these abundances are then updated according to the prescription that, at a radiative meshpoint \( (\nabla_r < \nabla_a) \), is

\[
\frac{dX_i}{dt} = A_i \sum_j \alpha_{ij} R_j, \quad \sum_i A_i \alpha_{ij} = 0 \quad (26)
\]

where \( A_i \) is the atomic number, \( R_j \) is the local rate of the \( j \)th nuclear reaction, and \( \alpha_{ij} \) are stoichiometric integers giving the number of particles created or destroyed per reaction. The number of solved compositions can be quite large, although in practise only a modest number of composition variables have an important influence on the structure of the star. In a convection zone however, a more complicated recipe is needed, based on the fact that the composition is uniform in the zone as a result of convective mixing. In a semi-convection zone an even more complicated method should be applied, usually based on the concept that the composition is determined by the so called neutral condition for convection: \( \nabla_r = \nabla_a \).

Numerically, as is done in MESA, the stellar equations can be written as:

\[
\log r_k = \frac{1}{3} \log \left( r_{k+1}^{3} + \frac{3}{4\pi} \frac{dm_k}{\rho_k} \right) \quad (27)
\]

The Langrangian time derivative of the radius at face \( k \) is given by

\[
v_k = r_k \frac{d \log r_k}{dt} \quad (28)
\]

The pressure \( P_k \) is set by momentum conservation at interior cell boundaries:

\[
P_{k-1} - P_k = \frac{d m_k}{dm} \left[ \frac{dP}{dm} \right] \text{hydrostatic} = \frac{d m_k}{dm} \left[ \frac{dP}{dm} \right] \text{hydrodynamic} = -\frac{d m_k}{4\pi r_k^4} \frac{G m_k}{4\pi r_k^4} + \frac{a_k}{4\pi r_k^4} \quad (29)
\]
where $\overline{dm_k} = 0.5(dm_{k-1} + dm_k)$ and $a_k$ the Lagrangian acceleration at face $k$, evaluated by the change in $v_k$ over the timestep $\delta t$.

The temperature of the inner cells $T_k$ is governed by the energy transport across inner cell boundaries:

$$T_{k-1} - T_k = \overline{dm_k} \left[ \nabla_{a_k} \left( \frac{dP}{dm} \right)_{\text{hydrostatic}} \frac{T_k}{\overline{T}_k} \right]$$

(30)

where $\nabla_{a_k}$ is the adiabatic gradient at face $k$, $\overline{T}_k = (T_{k-1}dm_k + T_kdm_{k-1})(dm_k + dm_{k-1})$, the temperature interpolated by mass at face $k$, and $\overline{P}_k = (P_{k-1}dm_k + P_kdm_{k-1})(dm_k + dm_{k-1})$, the pressure interpolated by mass at face $k$. [7][13]
10.2 (B) Polytropic approximations

As we have seen in section 9.1, solving the stellar equations is not straightforward. Remarkably, stars in the main sequence band are very well approximated by polytropes: gas spheres in which the pressure is proportional to a power of density. This relation can be written parametrically by

\[ \rho = \rho_c \theta^n \]
\[ p = p_c \theta^{n+1} \]

so that

\[ p \propto \rho^{1+1/n}, \]

where \( n \) is a constant and \( \rho_c \) and \( p_c \) are the critical density and pressure, respectively. When this relation is combined with hydrostatic equilibrium and self-gravity, the so called Lane-Emden relation is obtained:

\[ \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d\theta}{dr} = -\theta^n \]

(31)

Provided that the radius \( r \) is scaled by a factor \( \left( \frac{(n+1)p_c}{4\pi G \rho_c^2} \right)^{1/2} \). This equation has the so called 'Emden' solutions which start from \( \theta = 1 \) at the center and reach \( \theta = 0 \) at a dimensionless radius which can readily be obtained, provided that \( n \leq 5 \). For a still larger value for \( n \), radius and mass are unbounded. For brown dwarfs the parameter \( n = 1 \) often is chosen, and in the low mass end \( n = 1 \) yielding no dependence between the mass and radius. The polytropic index \( n \) is in practice not dictated by the equation of state of the gas (except for white dwarfs), but instead by the temperature distribution and heat transport processes. The heat transport processes itself are governed by the relation between radiative opacity of the material, and the temperature and density. Radiative equilibrium is comparable with the value \( n \sim 3 \), for stars on the main sequence above \( M = 0.5 \, M_\odot \). To see why, the following assumptions are made.

10.2.1 Pressure

Pressure is assumed to be a combination of the ideal gas law and radiation only

\[ p = \frac{\mathcal{R} \rho T}{\mu} + \frac{1}{3} aT^4 \equiv \frac{\mathcal{R} \rho T}{\mu} (1 + \xi) \]

(32)

where \( \mathcal{R} \) is the gas constant and \( \xi \) the radiation to gas pressure ratio.

10.2.2 Opacity

The opacity \( \kappa \) is written in the form

\[ \kappa = \kappa_{\text{Th}} + \kappa_{\text{Kr}} \frac{3\mathcal{R} \rho}{aT^3} = \kappa_{\text{Th}} + \frac{\mu \kappa_{\text{Kr}}}{\eta} \]

(33)

where \( \kappa_{\text{Th}} \) is the Thomson-scattering opacity (a constant), and \( \kappa_{\text{Kr}} \) a constant which is a rough replacement for Kramer’s opacity law \( k \propto \rho/T^3 \): an approximation for the absorption of photons by bound-free electronic transitions in a highly ionized gas.

10.2.3 Uniform nuclear energy generation

The rate at which energy is generated by nuclear reactions is assumed to be uniform throughout the entire stellar interior. This assumption results in an exact \( n = 3 \) polytrope, with \( \rho \propto T^4 \) and \( \rho \propto T^3 \). In this case, Eddington’s quartic equation applies, which in terms of \( \xi \) is given by

\[ \eta(1 + \eta)^3 = \frac{\mu^4 M^2}{M_{\text{Edd}}^2} \]

(34)

where \( \xi \) is constant due to the fact that the star is an exactly \( n = 3 \) polytrope and by the definition in equation ().

The Eddington mass \( M_{\text{Edd}} \) is a mass whose value is determined by fundamental constants and the dimensionless mass \( (m = 2.01824) \) of the \( n = 3 \) polytrope only:

\[ M_{\text{Edd}} \equiv 2.01824 \frac{\mathcal{R}^2}{G} \left( \frac{48}{\pi aG} \right)^{1/2} \approx 18.3 M_\odot \]

(35)

10.2.4 Luminosity

With these assumptions, the luminosity can be written in the form

\[ L = \frac{4\pi aG M T^4}{3k\rho} = \frac{4\pi G M \xi^2}{(\xi \kappa_{\text{Th}} + \mu \kappa_{\text{Kr}}) (1 + \xi)} \]

(36)

The last three equations imply that the luminosity only depends on the mass \( M \). \cite{7}
10.3 (C) Equation of state for brown and black dwarfs

Main sequence stars a little above the HBML, contain stellar interiors with substantially lower temperatures and higher densities than more massive main sequence stars like the Sun. Consequently, the equation of state (EoS) becomes much more complicated. High densities instead of high temperatures cause the materials near the center to be ionized (pressure ionization). Also, the electron gas may be substantially degenerate. This electron degeneracy pressure allows stars to support themselves in hydrostatic equilibrium without a temperature gradient due to nuclear reactions. Once this is the case, the objects should be called brown dwarfs, as we have seen in previous sections. Theoretically, brown dwarfs cool to zero temperature, since either degenerate electron pressure, or else the pressure of the liquid or solid state (as in planets), can support the star against gravity or arbitrarily low temperature. If the temperature is so low as to contribute negligibly to pressure support, the object is sometimes called a black dwarf. This means that the radius of a black dwarf is determined simply by its mass and chemical composition. A brown dwarf on the contrary, can be somewhat larger, since the internal temperature does contribute significantly to the outward pressure. Zapolsky and Salpeter (1969) constructed black dwarf models using a relative simple equation of state. They obtained a radius mass relation which can be approximated for all $M \leq 10^{-5} M_{\odot}$:

$$R = R_{\text{ch}} \left[ \left( \frac{M_{\text{ch}}}{M} \right)^{2/3} - \left( \frac{M}{M_{\text{ch}}} \right)^{2/3} \right]^{1/2} \left[ 1 + 3.5 \left( \frac{M_{\text{pl}}}{M} \right)^{2/3} + \frac{M_{\text{pl}}}{M} \right]^{-2/3}$$

(37)

where the characteristic radius and masses are respectively defined as $R_{\text{ch}} = 0.0228(Z/A)R_{\odot}$, $M_{\text{ch}} = 5.83(Z/A^2)M_{\odot}$, and $M_{\text{pl}} = 0.0016(Z/A)^{3/2}(Z^2/A)^{3/4}M_{\odot}$, where $Z$ and $A$ are the atomic number and atomic weight of the chemical constituents. The angular brackets are used because the quantities are averaged by weight over the different constituents. This equation gives a $R \propto M^{1/3}$ relation for planetary masses. For higher masses, where electron degeneracy pressure dominates, equation (18) gives $R \propto M^{-1/3}$. As stated in section 3, the combination of the two leads to $R \propto M^0$, except that the radius goes to zero as $M \rightarrow M_{\text{ch}}$ (the Chandrasekhar limit). A maximum radius of approximately 0.1 $R_{\odot}$ can be reached for an object with the mass of Jupiter ($\sim 0.001M_{\odot}$). Brown dwarfs however, being hot enough such that the central temperature contributes to outward pressure, will have somewhat larger radii than is given by equation (18), like the brown dwarf companion of SAXJ 1808.4-3685 with $R_{\text{bd}} = 0.13 R_{\odot}$. [5][7]
References


