Abstract

In 2005, Edlund and Jörice have proven an approximation result within the Fine Topology. The fine topology is the weakest topology for which all subharmonic functions are continuous. They prove their result for a certain set they construct. This set $S$ is a domain which contains a subset $\mathcal{E}$ of the unit circle of positive measure, a subset of the unit Disk $\mathbb{B}$ and a subset of $\mathbb{C}\setminus\mathbb{B}$ such that $\mathbb{C}\setminus S$ is the union of similar open rhombs.

They prove that if $f \in A(S)$ and Hölder continuous of order $\alpha$, $0 < \alpha \leq 1$, up to the boundary of $S$ and $\mathbb{C}\setminus S$ is thin at $p$ a point of $\mathcal{E}$, then $f$, restricted to $B_i = S \cap \mathbb{B}$, has a fine analytic continuation $F$ in a fine neighbourhood $V_p$ of $p$ such that $F|_{V_p \cap B_i} = f|_{V_p \cap B_i}$.

Their result, especially their proof, gives rise to some questions. One of these questions is if one can replace the condition of Hölder continuity with the condition of regular continuity. Another interesting question is if the function algebras $R(K)$ and $A(K)$ are the same.

In this thesis we are going to prove that the result is also true if one replaces Hölder continuity with regular continuity and that $R(K) = A(K)$ for a certain compact set $K$ within the set $\mathcal{S}$.

For the second result we are going to use Vitushkin’s theorem which states that $R(K) = A(K)$ iff $\limsup_{\delta \to 0} \frac{\alpha\left(\overline{B(z, \delta)} \setminus K^o\right)}{\alpha\left(\overline{B(z, r \delta)} \setminus K\right)} < \infty$, $z \in \partial K$, $r \geq 1$.

($\alpha(\cdot)$ is the continuous analytic capacity). To compute the fraction, we are going to use an important result by X.Tolsa (2004) which states that continuous analytic capacity is semi-additive.

For the first result we will use notions and theorems from (bounded) analytic capacity ($\gamma(\cdot)$), logarithmic capacity ($\text{Cap}(\cdot)$) and relations between them. The most important estimation lemma which is used, is a lemma by Lyons which we found in an article by Fuglede and adapted it to our benefits.

Important notions in this thesis:

- Subharmonic Functions, Green’s Function and Balayage;
- Fine Topology, Thinness of Sets, Fine Neighbourhoods, Finely Analytic Functions and Fine Analytic Continuation;
- Logarithmic Capacity and a Lemma due to Lyons;
- (Bounded) Analytic Capacity and Ahlfors Functions;
- Continuous Analytic Capacity, Vitushkin’s Theorem and the Result by Tolsa.