Bubble Detection and Crash Prediction

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Preface

Thanks everyone who was there with me from Feb-2010 to July-2010 in my life. This thesis is a record of this part of my life and I dedicate it to all my colleagues at Robeco Asset Management, my teachers at the university, my parents, my friends, and those strangers who woke me up on the train every morning. Thanks for your kindness and love that keep my life warm and worth living.
Abstract

The main objective of the thesis is to test if the financial bubble detection method based on the Stochastic Critical Time model developed by Lin and Sornette in 2010 can be used to provide predictions of crash/deflation in the financial market. The thesis is divided into two parts: replication of paper results and test of statistical significance of crash signals translated from bubble warnings.

In the first part, we identified several problems with the test procedure of Lin & Sornette (which make it impossible to exactly replicate their results), presented solution to these problems, and on top of that, proposed a revised test procedure by fixing three major problems – parameter estimation, fixed time window length and forward moving predictions.

In the second part, we designed a statistical significance test (which has not been designed by Lin and Sornette) of crash signals translated from bubble warnings and tested the revised model on 93 asset price series, which belong to five different asset classes: sector-to-market, G10 currencies, Emerging Market currencies, Commodities and Emerging Equity Markets.

Test results showed that the ratio of successful crash signals is as low as 0.25, which is lower than the ratio of successful crash signals (around 0.3) translated from warnings generated by the naïve bubble model which always gives bubble warnings. What’s more, there is no obvious evidence of significant losses after crash signals.

From test result, we concluded that, the financial bubble warnings generated by the detection method has no statistical significance in predicting a forthcoming financial crash or deflation. Given the test result of statistical significance, it won’t be necessary to investigate the economic significance of the method.
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Chapter 1

Introduction

The possibility of predicting a forthcoming crash in the financial market by identifying a developing financial bubble was pointed out by Professor Sornette in the 1990s. In his book ‘Why Stock Markets Crash, 2003’ he made a summary of the development of his model up till that time, and his successful prediction of the Aug-2009 crash in the Chinese equity market brought his crash prediction model into public attention. This internship at Robeco Asset Management is about testing Sornette’s bubble detection model to see if it can provide reliable predictions for forthcoming crashes/deflations and useful information on forming trading strategies.

In the introduction, the relationship between crashes and bubbles is discussed in Section 1.1; then in Section 1.2, a short summary of the original crash prediction model – the Log-Periodic Power Law model is introduced; in Section 1.3, two recent bubble detection methods developed in 2009 based on the original crash prediction model is presented; and in Section 1.4, the research question of the project is raised with the structure of the rest of the thesis.

1.1 Crashes and bubbles

A financial bubble is usually recognized as a long-lived positive deviation of the actual asset price from its fundamental value. Financial crises, as a consequence of bubbles, may cause economic recession and damage on peoples’ well being. The recent worldwide crash in the stock market, for instance, has developed into a global economic recession during 2008. Obviously, it’s not the first severe crash in the most recent 100 years; going back in history, we can recall the 2000 crash, the 1987 and 1929 crash. Due to the catastrophic effect a crash can have on our well-being and their relatively high frequency compared to that under the random walk hypothesis of stock price, we can’t help wondering whether such a hazardous event happens all of a sudden or that it has left its footprints somewhere before its advent. If the former is true, we can almost do nothing about it; while for the latter, certain precautious measures can be taken.

For a long time, crashes were thought to be caused by an external shock. This is the truth, but not the whole truth. The internet bubble, which started from 1998 and ended in 2000 in a crash by losing about 70% from its peak, brought the other part of the truth into public attention – crashes can also act as the termination of a developing bubble.

What is the major cause of crashes – an exogenous shock or an endogenous bubble? Although it is still not clear to us, crashes that followed an endogenous bubble may be foreseeable. If we can recognize the price pattern of a bubble as a precursor, we can
foresee a probable crash. Something that needs clarification is that, in presence of a developing bubble, the advent of a crash is still stochastic instead of deterministic. Crash is only one of the two ways that a bubble disappears – the bubble can end in a crash or die a gradual death via much milder corrections. The positive probability for a bubble to end without a crash keeps rational investors remaining in the market as long as the risk (of a crash) they are taking is compensated by the high premium.

To conclude, external shocks and existence of bubbles are the two causes of crashes, and we are concerned with endogenous crashes acting as consequences of the latter, which have the potential of being predicted ex-ante. Also, when there is a bubble, it can’t last forever and will disappear in a way – via a severe correction like a crash or a much milder and gradual correction. When we talk about ‘crash prediction’, what we actually mean is detecting a bubble and estimating a critical time at which the bubble is most likely to burst/deflate. Figure 1.1 provides an illustration of relationship between bubbles and crashes.

![Figure 1.1 Illustration of relationship between bubbles and crashes](image)

1.2 The Log-Periodic Power Law model

It is astonishing to find how little investors have learnt from history. The belief – something is different than before and the increase in asset price comes from fundamental increase – blinded investors who helped build up the bubble. Also, there are usually investors who buy the stock of an underlying asset not because it is worth the price but that they believe that there will be a ‘bigger fool’ to whom they can sell the stock at a higher price.
Such beliefs gave rise to financial bubbles and made the identification of a developing bubble in real time difficult. Previous econometric approaches failed to identify a prevailing bubble in the sense that ‘for each paper that finds evidence of bubbles, there is another one that fits the data equally well without allowing for a bubble’, as described in [Gurkaynak, 2008].

Professor Sornette and his team have been developing an approach to unveil a universal price pattern for an in-the-making bubble by adopting concepts and tools from complex system theories and statistical physics. In this school of bubble diagnosis method, the financial market is conceived as a complex system and crashes in financial markets are related to critical points that have been studied in statistical physics for phenomena like magnetism and melting. The behaviour of such a complex system is the aggregation of the behaviour of its individual participants.

Take the financial market as an example: investors in the market are the participants who are organized in a hierarchical structure (different sizes of investors) and competing with each other. When the herding effect becomes so obvious in the system due to the nonlinear interacting among individual investors that a large proportion of investors choose to herd and follow the trend, the financial market can arrive at a large scale of collective behaviour which would develop into a self-organized and self-reinforcing bubble regime. As time goes by, the system becomes more and more unstable as the bubble price increases and imitation dominates idiosyncratic opinion. When the critical time is reached, the change of regime happens.

According to observation, the price almost always exhibits 2 features prior to a crash:
• Faster than exponential acceleration until the crash/deflation time;
• Oscillations with accelerating frequency.

The Log-periodic Power Law (LPPL) model developed in [Johansen, Sonette and Ledoit, 2000] captured the above-mentioned features. It can be written as in equation (1.1) and a summary of deduction of the model can be found in Appendix 1.

\[
\log[p(t)] = A - B(t_c - t)^\beta \{1 + C \cos[\omega \log(t_c - t) + \phi]\}
\]

(1.1)

where \( p(t) \) is the price of a specific asset at time \( t \); \( A \) is the logarithm of the price at the critical point; \( B > 0 \) and \( 0 < \beta < 1 \) to ensure a super-exponential growth; \( t_c \) is the critical time, which is the most probable time for a crash to happen; \( C \) is the amplitude of the log-periodic oscillations, \( \omega \) is the angular log-frequency; and \( \phi \) an arbitrary phase determining the unit of the time.

Input data for model (1.1) are the asset price \( p(t) \) and calendar time \( t \); parameters \( A, B, C, \beta, \omega, \phi \) and \( t_c \) can be estimated by minimizing the sum of Squared Residuals (difference between the model fit and observed asset price).
Figure 1.2 is a plot of the price trajectory of SP500 with LPPL fit before the 1987 crash generated by us.

![Graph of SP500 1987 Crash](image)

**Figure 1.2:** SP500 1987 Crash. The smooth line is the fit of the LPPL model and the other one is the real trajectory of the stock.

This model not only captures the two features well but also provides insights in the causes of the two features. The super-exponential acceleration is the result of positive feedback investing that can be captured by a log power law; the oscillations come from the competition between nonlinear positive feedbacks from trend followers and negative feedbacks from fundamental investors, and it can be captured by the cosine part which is log-periodic in time. The super-exponential growth makes the bubble a transient regime that has to end at a finite time singularity and thus can be distinguished from a fundamental exponential growth.

Empirical analysis [Zhou, Sornette, 2003] on the log-periodic property based on both parametric and non-parametric methods showed that the LPPL pattern is only significant when a bubble is prevailing (especially prior to a crash) and universal restrictions for parameters are found and documented in papers such as [Lin, Ren, Sornette, 2009].

### 1.3 Bubble detection methods

Further work has been done to utilize the model to identify bubble pattern in the asset price. There are two most recent development of the LPPL model, which are the Stochastic Critical Time model (mentioned as SCT model afterwards) and the confined volatility LPPL model (mentioned as LPPL+ model afterwards).
• **The SCT Model**

In [Lin, Sornette, 2009], the finite-time singularity in price momentum with stochastic critical time model (mentioned as SCT model afterwards) is reported. With insight from the LPPL model, SCT model models the bubble price dynamics using a log power law (equation 1.2) where the critical time of a bubble is assumed to follow a mean-reverting process instead of the constant critical time $t_c$ in the LPPL model. Equation (1.3) is the expression for critical time series $\tilde{T}_c(t)$ by solving (1.2) for $\tilde{T}_c(t)$.

\[
\ln p(t) = A - B(\tilde{T}_c(t) - t)^{1-\beta} \tag{1.2}
\]

\[
\tilde{T}_c(t) = t + \left( \frac{A - \ln p(t)}{B} \right)^{1/(1-\beta)} \tag{1.3}
\]

Here $\ln p(t)$ is the log asset price, $\tilde{T}_c(t)$ is the critical time series, and $A$, $B$ and $\beta$ are parameters to be estimated.

By assumption, the critical time series $\tilde{T}_c(t)$ is stationary, so that the critical time series would fluctuate around its mean without straying away. The bubble detection method based on the SCT model is designed by searching for a parameter combination of $A$, $B$ and $\beta$ which can generate a stationary critical time series $\tilde{T}_c(t)$. If such a combination exists, it is concluded that there is a prevailing financial bubble in the given asset price, otherwise, there is no bubble according to the model. More details on the bubble detection method are documented in Chapter 2.

• **The LPPL+ model**

In [Lin, Ren, Sornette, 2009] the LPPL+ model is developed based on the original LPPL model. In LPPL+, the bubble price is modeled to be consisting of a deterministic LPPL part and a mean-reverting stochastic part that captures the behavior of the residual from fitting the LPPL model to observed price series. If the following two features are recognized in a price series, a bubble is said to exist according to the LPPL+ model:

- The estimation of parameters fall in certain intervals obtained from previous empirical studies;
- The residual from model fitting are stationary.

There are 2 drawbacks with the bubble detection method based on the LPPL+ model:

- It suffers from the data snooping (over-fitting) danger. The criterion that it uses to distinguish a bubble regime from a non-bubble time period is the parameter intervals obtained from empirical studies based on the LPPL model.
• Due to the non-linearity and complexity of the model, estimated parameters may be sensitive to initial values when different local minima are selected.

Therefore, we selected the bubble detection method based on the SCT model, whose filtering criterion (the stationarity test on critical time series) does not depend on empirical studies of the LPPL model. However, after further investigation, the bubble detection method described in [Lin, Sornette, 2009] turned out to be problematic as well.

When we found difficulties in replicating the results reported in the 2009 version of [Lin, Sornette, 2009], we contacted the author for more details on the implementation and results of the bubble detection method. The author sent us the newest version (2010) of the paper, in which both test procedures and results changed substantially. However, it’s still impossible to replicate the results reported in the 2010 version of the paper following the implementation procedure described in it, and therefore, we analyzed the problem with the implementation proposed by Lin & Sornette, revised and improved the model by solving problems found by our analysis and then test the statistical significance of the improved bubble detection method.

1.4 Research question and thesis structure

The objective of the ‘bubble detection and crash prediction’ project is to answer the following research question:

• Is it possible to make crash predictions based on the bubble detection method in real-time?

To answer the question, we need to generate bubble warnings using the bubble detection method, design a procedure that simulates the way bubble warnings would be used to give crash signals in real time and test the statistical significance of crash signals on a number of price series.

The rest of the thesis is organized in this way: Chapter 2 introduces the bubble detection method based on SCT model as described in the 2010 version of the paper [Lin, Sornette, 2009], our critical analysis on the method and the enhanced replication of the results. Chapter 3 contains the revision of the bubble detection method based on SCT model and the in-sample bubble warning results of the revised method. Chapter 4 contains the test procedure which we use to test the statistical significance of the crash signals translated from model bubble warning. Chapter 5 is the description of the out-of-sample test data and analysis of out-of-sample results. Chapter 6 concludes.
Chapter 2

Bubble Detection Method based on the SCT Model

With insight from the original LPPL model, the SCT model captures the log price dynamics by the power law model and the critical time of the end of a bubble is assumed to be a mean-reverting stochastic process instead of a constant as in the LPPL model. The rest of the chapter is divided into four parts: in Section 2.1, the bubble test based on the SCT model designed in [Lin, Sornette, 2010] is introduced, differences between the 2010 version and 2009 version of the paper are documented with possible reasons for the change and results found by Sornette are presented; in Section 2.2, two critical comments are presented with supporting evidence or arguments; in Section 2.3, results of our replication are compared with Sornette’s and further evidence supporting the critical comments on the 2010 procedure are reported; and Section 2.4 concludes.

2.1 Replication

In this section, a short introduction on the SCT model is included in subsection 2.1.1, and in 2.1.2 the bubble detection method based on the SCT model designed in [Lin, Sornette, 2010] is introduced with the difference of test procedures from the 2009 version of the paper.

2.1.1 Description of the SCT model

As introduced in Chapter 1, in the SCT model, the log price dynamics are modeled as

$$\ln p(t) = A - B(T_c(t) - t)^{-\beta} \quad (2.1)$$

By inverting equation (2.1), we can arrive at an expression for the critical time series:

$$\tilde{T}_c(t) = t + \left(\frac{A - \ln p(t)}{B}\right)^{\frac{1}{\beta}} \quad (2.2)$$

Let \( T_c \) denote the mean of \( \tilde{T}_c(t) \) and \( \tilde{T}_c(t) = \tilde{T}_c(t) - T_c \) denote the demeaned critical time series. By assumption, the demeaned critical time series (therefore the critical time series) is a mean-reverting process, which is an Auto-Regressive process of order 1 (AR(1)) following (2.3):

$$\tilde{T}_c(t) = \alpha \tilde{T}_c(t-1) + z_t \quad (2.3)$$

Here \( 0 < \alpha < 1 \) and \( z_t \) is a white noise process. This is the crucial assumption of the model and the bubble detection method based on the SCT model is designed by searching for parameter combinations which can generate stationary critical time series \( \tilde{T}_c(t) \).
2.1.2 Bubble Detection Method

In the 2010 version of the paper [Lin, Sornette, 2009], the bubble detection method based on the SCT model changed significantly from that in the 2009 version. Here I first introduce the most recent one reported in 2010 in detail and then document the differences in 2010 version from that in 2009 version.

Given historical price trajectory of a certain asset, the bubble detection method is applied to rolling time windows with length of N trading days that with move forward by a step of 25 trading days. Within each time window, search for a combination of parameters A, B, and β, with which, the critical time series \( \tilde{T}_i(t) \) constructed using (2.2) passes the stationarity test (Unit-Root Test, e.g. Dickey-Fuller Test). A bubble warning is flagged at the end of the time window if there is at least one parameter combination that can generate stationary critical time series.

- **Procedure in the 2010 version**

  Detailed procedure of the test reported in 2010 version of the paper is summarized in the four steps below: **time window construction; critical time series construction; parameter estimation** and **flag a warning**.

1. **Time window construction:**

   Construct time windows of N trading days that slide with a time step of 25 trading days from the beginning to the end of the available financial time series. The parameter combination \((A_i, B_i, \beta_i)\) within the \(i\)th time window is denoted by \((A_i, B_i, \beta_i)\), with subscript \(i\) referring to the time window.

2. **Critical time series construction:**

   In this step, we first introduce the way to construct the (demeaned) critical time series within each time window using a certain parameter combination \((A_i, B_i, \beta_i)\) . In the next step, we introduce the parameter estimation method adopted in the paper based on tests on (demeaned) critical time series constructed in this step.

   Within each time window \([t_{i\text{end}}^{\text{end}} - N + 1,t_{i\text{end}}^{\text{end}}]\) ending at \(t_{i\text{end}}^{\text{end}}\), consider a combination of parameters \((A_i, B_i, \beta_i)\) and define the critical time series:

   \[
   T_{c,i}(t) = t + \left( \frac{A_i - \ln p(t)}{B_i} \right) \left( \frac{1}{1 - \beta_i} \right), t = t_{i\text{end}}^{\text{end}} - N + 1, \ldots, t_{i\text{end}}^{\text{end}}
   \]  

   (2.4)

   Calculate the mean of \( \tilde{T}_{c,i}(t) \) using (2.5) and construct \( \tilde{\bar{c}}_{i}(t) \) by (2.6)

   \[
   T_{c,i} = \frac{1}{N} \sum_{t_i=N+1}^{t_{i\text{end}}} \tilde{T}_{c,i}(t)
   \]  

   (2.5)

   \[
   \tilde{\bar{c}}_{i}(t) = \tilde{T}_{c,i}(t) - T_{c,i}
   \]  

   (2.6)
3. Parameter Estimation

The parameter estimation procedure adopted can be summarized as the solving procedure to a constrained optimization problem.

The optimization problem to be solved is summarized as:

**Objective:** \( \max(p_i) \)

**Restrictions:**

i) \(-25 < T_{t,i} - t_{end}^i < 250; \)

ii) \(\tilde{T}_{t,i} \) passes **DF Test**\(^1\) at significance level 95%;

iii) \(\tilde{Y}_{t,i} \) passes **Variance Homogeneity Test** at significance level 95%;

Where \(p_i\) is the p-value in the **Trend Test**.

**Test Description**

The three tests mentioned above are the DF Test, Variance Homogeneity Test and Trend Test. The purpose of these tests is to test the stationarity of constructed time series. Since the power of DF Test is low (same other similar unit-root tests), the Variance Homogeneity Test and Trend Test serve as complementary tests to exclude non-stationary time series that would pass the DF Test.

Descriptions of the three tests are mentioned below:

**DF Test**

Here, the demeaned critical time series is modelled to be an AR (1) process (see equation 2.3) with no linear trend and no intercept – it has mean zero because it is the demeaned process. The null Hypothesis is that \(\alpha = 1\) for the \(\alpha\) in equation (2.3), which means that there is a unit-root in the demeaned critical time series. If the null hypothesis is rejected at the significance level of 95%, the time series is considered as stationary.

**Variance Homogeneity Test**

The Variance Homogeneity Test is to ensure that the variance of the demeaned critical time series doesn’ change over time. In the paper, it is not clearly described which the Variance Homogeneity Test is used. The most well-known versions of this test are: the Levene's test, Bartlett's test, and the Brown–Forsythe test. They are modifications of the F-test which test if two populations have the same variance but reduce the possibility of giving false conclusions when the populations are not drawn from normal distribution. The Variance Homogeneity Test we use to replicate Sornette’s result is the Levene’s Test\(^2\).

---

\(^1\) The Dickey Fuller Test is to test the existence of a unit-root in an Auto-Regressive (AR) model.

Trend Test

The Trend Test is used to test if there is an obvious linear trend in the demeaned critical time series \( \tilde{t}_c(t) \) in time. It is carried out by testing if the slope coefficient in the linear regression (2.7) is zero. The null hypothesis for the Trend Test is \( \gamma_1 = 0 \), which means there is no obvious linear trend.

\[
\tilde{t}_c(t) = \gamma_0 + \gamma_1 t, \quad t = t_i - N + 1, \ldots, t_i. \tag{2.7}
\]

The test statistics in the Trend Test is the t-statistics in the linear regression for coefficient \( \gamma_1 \).

Solving Procedure in the paper:

The procedure taken to solve the above-mentioned optimization problem is as follows.

i) Set the p-value in the Trend Test as the optimization objective and use the Tabu algorithm, which is a search method based on a local escape with cycle-avoidance techniques, to choose the first 10 combinations of \( \beta_1, \beta_2, \beta_3 \) that maximize the p-value. Combinations that give smaller value of the objective will be abandoned and will be avoided in future iterations. How the first 10 combinations are chosen is not stated in the paper.

ii) For the demeaned critical time series \( \tilde{t}_c(t) \) generated by the 10 candidates chosen from step i), carry out the Dickey-Fuller (DF) test at significance level of 95%.

iii) For those demeaned critical time series \( \tilde{t}_c(t) \) passed i) and ii), carry out a Variance Homogeneity Test to make sure that the variance in the first half and the second half of \( \tilde{t}_c(t) \) are not significantly different (otherwise, there may be undesired spikes in the first or the second half of the time series).

iv) Check restriction \(-25 \leq T_{c,i} - t_i^{end} \leq 250\) to ensure that the estimated termination time of the bubble is not too far away from the time when the estimation is carried out.

4. Flag a warning

If at least one of the 10 candidates that pass the DF test, Variance Homogeneity Test and satisfy the restriction \(-25 \leq T_{c,i} - t_i^{end} \leq 250\), a bubble warning is flagged at the end of the time window. Three levels of warnings significance are quantified by \(T_{c,i} - t_i^{end}\):

- level1: \(T_{c,i} - t_i^{end} \leq 250\)
- level 2: \(T_{c,i} - t_i^{end} \leq 90\)
- level 3: \(T_{c,i} - t_i^{end} \leq 50\)

Here, \( T_{c,i} \) is the mean of the critical time series within the \( i \)th estimation time window, which corresponds to the estimated crash time; \( t_i^{end} \) is the end of the \( i \)th estimation time window. The smaller \( T_{c,i} - t_i^{end} \) (when positive), the closer is the estimated crash time to the end of the estimation time window.
Changes in the procedure and why

The new procedure reported in the 2010 version of the paper has 3 main differences from that in the 2009 version:

(i) The tests in step 3 are different. In the 2009 one, a DF test with significance level 99.5\% is used to select \((A^*, B^*, \beta^*)\), if more than one combination can generate stationary critical time series, the one with minimum variance is chosen;

(ii) The restriction \(-25 \leq T_{c,j} - c_{\text{end}} \leq 250\) in step 4 is different: in 2009 version, the upper bound is 50 instead of 250;

(iii) The way to quantify different levels of warnings is different: in 2009 version, it is quantified by the value of beta: (level 1, beta<1; level 2, beta<2/3; level 3, beta<0.5).

The first difference – taking the 10 combinations that maximize the p-value in the t-test for coefficient of the trend term, is to exclude \(\widetilde{I}_{c,j}(t)\) which passes the DF test but exhibits an obvious trend (see figure 2.1-a); the variance homogeneity test is to exclude \(\widetilde{I}_{c,j}(t)\) which passes the DF test but has high spike in the first or second part of the time series (see figure 2.1-b). These series are identified as stationary time series because of the low power (high false positive rate) of the DF test.

As for the second and third, we see no intuitive reasons.

Figure 2.1-a Figure 2.1-b

Figure 2.1 Non-stationary time series that would pass the DF test: 1-a time series with an obvious linear trend; 1-b time series with high spike.
2.1.3 Sornette’s results

Following the procedure in the 2010 version of the paper, results on daily close price of NASDAQ from 1 January 1980 to 31 October 2008 given by Sornette are presented in table 2.1.

Analysis of Sornette’s results:
- Warnings are flagged for the deflation in 1983 and the crash after the internet bubble in 2000.
- One false alarm in 1988;
- Bubble warnings for the crash in 2000 are too early with forward moving predicted crash time (indicated by the last column in the table);
- The estimated crash date 29-Nov-99 is earlier than the end of the estimation time window (22-Dec-1999) in which it was generated. Such estimation (prediction) caused by the negative lower bound in restriction \(-25 \leq T_{c,i} - t_{\text{end}} \leq 250\) is of no significance in real time.

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<tr>
<td>'16-Nov-1999'</td>
<td>10.424</td>
<td>1.2641</td>
<td>0.8592</td>
<td>'4-Feb-00'</td>
</tr>
<tr>
<td>'22-Dec-1999'</td>
<td>9.0118</td>
<td>0.38992</td>
<td>0.7670</td>
<td>'29-Nov-99'</td>
</tr>
<tr>
<td>'6-Mar-2000'</td>
<td>8.5147</td>
<td>0.024693</td>
<td>0.4056</td>
<td>'16-Jun-00'</td>
</tr>
</tbody>
</table>

Table 2.1: Sornette’s results on NASDAQ, the first column is the end of the time window in which a warning is flagged; last column Date_mean corresponds to the date indicated by the mean of the critical time series which can be considered as the predicted crash date given by the model.
2.2 Critical Comments on Parameter Estimation Procedure:

This section contains two critical comments on the procedure that Lin and Sornette adopted in the 2010 version of the paper. The first one is supported by sensitivity analysis on the parameters they report and the second is supported by analysis on the procedure.

My comments are:
1. The Tabu algorithm is not a good enough searching method solving this problem.
2. The optimization problem is ill-defined by taking one of the restrictions as the optimization objective; the solving procedure adopted is of a wrong order which makes the results search method dependent.

Below, I provide evidence that supports the 2 comments.

1. Tabu algorithm is not a good enough searching method solving this problem.

Sornette uses the Tabu algorithm in selecting 10 candidate parameter combinations \((A_i, B_i, \beta_i)\) that give the maximum p-value in the Trend Test within the \(i\)th time window. Evidence from sensitivity test shows that Tabu algorithm in this case is not looking at a dense enough parameter grid.

To see if the parameter they choose gives the maximum p-value (the objective of the optimization problem), we take the 10-Mar-1999 case to carry out the sensitivity test on A, B, and Beta.

Each time, the value of one parameter varies a little bit within its neighborhood, while the other 2 are kept unchanged. In this case, the parameters chosen by Sornette are A=9.8454; B=0.8867; Beta=0.8295. According to the result, the p-value given by the time series they selected is around 0.025. Figure 2.2 shows how p-value of the test changes with respect to the change in A, B and Beta.

From the results shown in figure 2.2, we can see that given A and B, Beta gives the largest p-value in its neighborhood; however, when A is increased by 0.015 or B reduced by 0.004 (with the other 2 fixed), the P-value can be increased greatly.

Further analysis shows that the newly generated time series by parameter combinations that give larger p-values (increase A to 9.862 or decrease B to 0.8827) satisfy other restrictions as well. Therefore, we can at least conclude that the Tabu algorithm is not looking at a dense enough grid; otherwise it should have chosen a combination of parameters that better optimizes its objective.

Therefore, in Chapter 4 when we improve the bubble detection method, we use another search method with a denser grid.
2. The optimization problem is ill-posed and the solving procedure is of a wrong order.

We first analyze problems with the optimization problem and the solving procedure; then, we provide possibly consequences of these problems.

(1) Problem with Sornette’s optimization problem and the solving procedure.

- Problem with the optimization problem:
  
  First of all, the objective of the optimization problem is not relevant.

  What we are looking for is stationary time series and the Trend Test should function as a complementary test to exclude time series that exhibit linear trend but can still pass the DF test. Setting the p-value of Trend Test to be the objective of the optimization problem is like putting the cart before the horse.
Also, the objective is maximized without a lower bound.

Let: B represents the set of parameter combinations which generates critical time series that passes the DF test;
C represents the set of parameter combinations which generates critical time series that passes the Variance Homogeneity test;
D represents the set of parameter combinations which generates critical time series that satisfies restriction $-25 \leq T_{c,i} - t_{i,\text{end}} \leq 250$;

The solution to the optimization problem defined by Sornette should fall in the feasible set, which is the intersection of B, C, and D (see figure 2.3).

*Figure 2.3. B, C, and D represent the set of parameter combinations which generate critical time series that passes the DF test, Variance Homogeneity test and satisfies the restriction $-25 < T_{c,i} - t_{i,\text{end}} < 250$ respectively. Shaded shape is the intersection of B, C, and D and is called the feasible set of the optimization problem.*

However, setting the p-value of the Trend Test to be the optimization objective instead of using it as a restriction, the Trend Test can fail to exclude time series with linear trend when all of the time series that the search algorithm has looked at have extremely small p-values. The method will always choose ten candidates no matter if the time series pass the test or not, such candidates are chosen not because they pass the Trend Test, but only because their linear trend is not as obvious as their other competitors.

In other words, **one more restriction should be added to the optimization problem:**

- **Restriction iv**: the demeaned critical time series $\tilde{T}_{c,i}$ passes Trend Test at the significance level of 95%.
Let A denote the set of parameter combinations which generate critical time series that pass the Trend Test at the significance level of 95%, then, the solution to the optimization problem should fall in the intersection of A, B, C and D, rather than that of B, C, and D.

Figure 2.4 illustrates the case that the chosen candidates fall out of A but in the intersection of B, C, and D.

Figure 2.4. O is the set chosen by the Tabu Algorithm, which falls out of A but in the intersection of B, C and D.

- **Problem with the Solving Procedure**

  The problem with the solving procedure is that Sornette solves the problem by first optimizing objective without constraint and then checking restrictions.

  Assuming the optimization problem defined by Sornette is well-posed, the procedure adopted to solve it is still not correct. The optimization problem in this case is a constrained one and its solution should not only maximize its objective but also satisfy the three restrictions.

  What Sornette does to solve the problem is:
  - First solve the optimization problem without constraint by choosing a limited number of candidates.
  - Check if they fall in the feasible set. If not, he claims that there is no solution to the problem.

  Making such a conclusion is presumptuous not only because of that 10 candidates is far fewer than enough to make the conclusion, but also that which candidates are chosen highly depends on which of them are looked at by the search algorithm. The logic is similar to the situation that you can never conclude that there is no one in the room only because no one answers your knock on the door, especially when you are not waiting long enough or when you are knocking on the wrong door.

  The right way to solve such a problem should be: specify the intersection of B, C and D as the feasible set and within the feasible set, search for the one that maximizes the optimization objective using an as dense as possible grid. However, Sornette’s procedure not only has a wrong order but also suffers from the fact that it only looks at a limited number of candidates.
(2) Possible consequences of the problems:

From the previous analysis, we can see that, if the feasible set, defined as the intersection of parameter combinations generating time series that satisfy all restrictions, is non-empty, a warning is flagged in the time window under inspection. Therefore, restrictions of the optimization problem determine whether there is a bubble warning in the time window. The optimization objective, however, helps to pick the parameter combination which generates the time series and thus gives the prediction. Therefore, the objective of the optimization problem determines the predicted critical time.

The first problem with the optimization problem – irrelevant optimization objective – makes the method fail to give results that are most close to stationary time series and thus influence the predicted critical time;

The second problem with the optimization problem – not setting Trend Test as a restriction – may lead to false alarm given by time series that exhibit linear trend but pass the stationarity test: the false alarm reported by Sornette in '10/18/1988’ corresponds to this situation;

The problem with the solving procedure – not solving the constrained optimization problem in a right way – makes results dependent on the searching method, which means no optimal solution is found only because the searching method fails to look at the point but not because it does not exist.

2.3 Enhanced Replication

In the previous section comment 1, we’ve already shown that Tabu algorithm is not a good method in solving this optimization problem. Moreover, due to its stochastic nature, the outcome given by Tabu algorithm suffers from instability.

If the optimization problem defined by Sornette is well-defined and his solving procedure is correct, the results should not depend too much on the searching method. Therefore, when we tried to replicate the procedure reported by Sornette in 2010, we used a different searching method (keeping all the others the same) to see if the results are different when using different searching method.

The new searching method is: directly search all possible combinations of A, B, Beta for a chosen discretization grid. The grid we use is:

A takes 20 equally spaced points in $[\max(\log(p_t)), 2\max(\log(p_t))]$;

B increases by 0.02 from 0.01 to 0.2 and increases by 0.05 from 0.25 to 1. The insight here is that as the denominator, smaller B gives larger value of the second term in equation (2) and offset the linear trend in the first term of equation (2), thus a denser grid for small B’s;

Beta increases by 0.02 from 0.01 to 0.97.
A comparison of results is reported in Table 2.2, while Figure 2.5 provides a plot of log price trajectory of NASDAQ Index with vertical lines indicate huge loss dates.

In general, using the direct searching method, we find more warnings, which means:

- Using a different searching method, we can find more time windows in which the feasible set of the optimization problem defined by Sornette is non-empty. This can be taken as evidence that the Tabu algorithm is not looking at a dense enough grid;
- Using a different searching method, results found by us are different from that found by them, which provides evidence for the second critical comment that Sornette’s optimization problem is ill-posed, making results searching method-dependent.

Results of further analysis on time windows that both Sornette and we flag a warning are reported in figure 2.6 and table 2.3.

Figure 2.6 shows how the demeaned critical time series found by us compare to that found by Sornette. Although parameters found by different searching methods are not comparable, the critical time series generated by different parameters are somehow similar.

Table 2.3 reports the comparison of p-value in the trend test, which shows that and the time series constructed by our parameter estimates better optimize the objective – the p-value in the Trend Test.

### 2.4 Conclusion

According to critical analysis and empirical evidences, the test procedure Sornette uses in parameter estimation suffers from several problems:

- Tabu algorithm is not a suitable search method to solve this problem not only because of its stochastic nature which causes the instability of results, but also that, from sensitivity test result, it does not provide parameter combination that best optimizes the objective;
- The optimization problem is ill-posed by taking restriction as the optimization objective;
- The solving procedure of the optimization problem is of a wrong order and suffers from the drawback that it does not look at as many as possible parameter combinations in the feasible set; and this problem makes the result searching method dependent.

In the next chapter, we will revise the bubble detection method by fixing problems with Sornette’s parameter estimation and additional problems with his bubble detection method.
Figure 2.5: Comparison of Sornette’s warnings with ours. Vertical lines on the upper panel corresponds to the end of time window in which Sornette flags a warning while those on the lower panel are our warnings.

Figure 2.6: Comparison of demeaned critical time series for time windows that both Sornette and we flag a bubble warning. Parameters comparison in Table 2.2 shows that, our parameter estimates are not comparable in these time windows; however, from plots of demeaned critical time series reported here, it can be seen that critical time series generated by different parameters are somehow similar.
Table 2.2: Sornette’s results compared with ours. Shaded ones are those found by both him and us. The first column is the real crash time of the index, the second column is the end of the time window in which a warning is flagged either by Sornette or by us; the third to fifth and the last 3 columns are parameters reported by Sornette and us respectively.

<table>
<thead>
<tr>
<th>Crash</th>
<th>Date</th>
<th>Sornette's</th>
<th>Ours</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>11-May-83'</td>
<td>5.988</td>
<td>0.4185</td>
<td>0.8993</td>
</tr>
<tr>
<td>Oct-87</td>
<td>'9-Jan-87'</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>19-Oct-88'</td>
<td></td>
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<tr>
<td></td>
<td>'15-Nov-90'</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>'1-Jul-98'</td>
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<td>'6-Aug-98'</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>'16-Oct-98'</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>24-Dec-98'</td>
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</tr>
<tr>
<td></td>
<td>10-Mar-99'</td>
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<td>0.8867</td>
</tr>
<tr>
<td></td>
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<td>8.716</td>
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</tr>
<tr>
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<td>7-Sep-99'</td>
<td>10.39</td>
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</tr>
<tr>
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<td>'10-Jun-08'</td>
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<tr>
<td></td>
<td>'16-Jul-08'</td>
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</tr>
</tbody>
</table>

Table 2.3: Comparison of P-value in Trend Test for time series generated by Sornette and us. Results show that time series constructed by our parameter combinations better optimizes the objective.
Chapter 3

Improved Bubble Detection Method

In this chapter, we introduce the revision of the bubble detection method based on the SCT model with a focus on fixing problems with Sornette’s parameter estimation method as reported in Chapter 2 and two additional problems with the bubble detection method – the problem caused by the length of time window and the forward moving predicted crash time.

(1) Fix problems with Sornette’s optimization problem and solving procedure. Based on an analysis of Sornette’s method in Chapter 3, we can see that he tries to solve an ill-defined optimization problem using a problematic procedure, which makes his result search method dependent. To avoid problems with Sornette’s bubble detection method, we redefine the objective and restrictions of the optimization problem and solve it using another searching method following the right procedure.

(2) Deal with the problem caused by the length of estimation time windows. In Sornette’s procedure, the length of time windows is fixed for each index by observing the historical data and choosing an optimal time window length which is neither longer nor shorter than most bubbles. However, when detecting an in-the-making bubble which has not ended yet in real time, it’s difficult to decide the optimal time window length because the length of the current bubble is not known at the current time. The length of the time window may influence the output of warning in that:

- When the time window length is taken longer than the existing period of a bubble, the bubble can not be detected;
- When the time window length is taken much shorter than the existing period of a bubble, warnings appear too long before the deflation of the bubble.

Therefore, taking only one window length based on historical observation makes it highly probable for the method to miss a bubble due to the improper choice of time window length. We need to take certain measures to reduce the probability of missing bubbles caused by the choice of time window length.

(3) Define a warning score that indicates the magnitude of the warning. The SCT model suffers from the drawback that the predicted crash date moves forward as the end of time window moves forward (i.e. time progresses). Therefore, a prediction that changes overtime is not very informative for investors to use in real time. Therefore we would like to define a score which implies the magnitude of warnings.

The rest of the chapter is divided into five parts. Sections 3.1, 3.2 and 3.3 introduce the improvements made concerning (1), (2) and (3) respectively; Section 3.4 concludes and summarizes the new bubble detection method in four steps; Section 3.5 presents in-sample warning results following the revised method.
3.1 Improvements for problems with Sornette’s procedure

In this section, we introduce our revision of the bubble detection method by defining a new optimization problem, using a new searching method, and taking a new solving procedure.

3.1.1 New Optimization Problem

The redefinition of optimization problem consists of two parts: redefinition of restrictions and redefinition of the objective.

By redefining the restrictions, we make sure that the parameter combinations in the feasible set can generate stationary critical time series. When the feasible set is non-empty, it implies that the model detects a bubble in the given time window.

By redefining the optimization objective, we ensure that the parameter combination we select within the feasible set is optimal according to the measure embodied by the objective. Therefore, to redefine the objective, we need to determine what property of (or implied by) the parameter combination we desire the most.

• Change of restrictions and why

Restriction i) \(-25 \leq T_{c,i} - t_{i}^{\text{end}} \leq 250\) changes to \(0 \leq T_{c,i} - t_{i}^{\text{end}} \leq 250\) with lower bound 0 to avoid the situation of predicting a crash that has already happened.

Restriction ii) \(\text{range}(\tilde{T}_{c,i}(t)) < 500\) replaces Sornette’s restriction that \(\tilde{T}_{c,i}(t)\) passes the Variance Homogeneity Test. The reason for this change is that the new restriction can exclude time series with spikes in the middle which can not be excluded by the Variance Homogeneity Test.

• Change of objective and why

When selecting the optimal parameter combination, we would like to pick the one that provide the best fit to the observed log price series. The intuition is simple and clear because the asset price is directly related to a bubble and we would expect the parameter estimate, given as a solution to the optimization problem, to provide a decent fit. To define a proper measure of goodness-of-fit, we require the following two properties:

(i) Independent of time window length: for two fits with the same mean of absolute residual and same average change of log price, the measure should be the same. To provide insight to this property, we consider a simplified example illustrated in figure 3.1, where the two fits are essentially the same: the right one is obtained by taking the first half of the left one. We would like the measure to be the same for these two cases.
(ii) **Removed the scale effect:** we allow larger absolute error when the change in log price is larger. Therefore, the measure for two fits whose absolute error and average change in log price are to the same ratio should be the same. The two fits in figure 3.2 provide an example of such fits: the right one is obtained by multiplying both the fit and the residual in the left by 2.

![Figure 3.1](image1.png)

**Figure 3.1:** Illustration of property (i). The right fit is the first half of the left one. They are actually the same fit and we would like to have the measure for the two fits to be the same.

![Figure 3.2](image2.png)

**Figure 3.2:** Illustration of property (ii) of the measure. Measures of goodness-of-fit are the same for the two fits. The mean of the absolute residual for the left one is half of that for the right, while the average change in log-price is also half of that for the right.

Traditional measures of goodness-of-fit, such as the R-square in linear regression and the sum of Residual Square, don’t possess the above-mentioned properties.

The pseudo R-square defined in (3.1), which is borrowed from linear regression, doesn’t quality property (i).

\[
R^2 = 1 - \frac{SS_{err}}{SS_{tot}} \tag{3.1}
\]

Where \(SS_{tot} = \sum_i (y_i - \bar{y})^2\) is the total sum of squares which is proportional to sample variance and \(SS_{err} = \sum_i (y_i - f_i)^2\) is the sum of squares of residuals.
Rewriting the right hand side of (3.1), we arrive at

\[
R^2 = 1 - \frac{1}{N} \frac{SS_{err}}{SS_{tot}} = 1 - \frac{\text{mean(residual}^2)}{\text{var(observations)}}
\]  

(3.2)

The mean of Residual Square is the same for the two fits in figure 3.1; however, the variance of the observations is bigger for the left one than that for the right one due to the trend in time in the price series. Therefore, for non-stationary time series, the pseudo R-square is not an ideal measure for the goodness-of-fit.

Obviously, the simple mean of squared residual doesn’t satisfy property (ii) because the mean of the squared residuals is four times of that for the left one.

Since traditional measures of goodness-of-fit do not satisfy the two properties we want, we define our measure of goodness-of-fit \(\text{Re}\) using (3.3) to (3.5). By construction, it satisfies the two properties. By minimizing it, we select the best fit.

Let \(y = \log p\) and \(\text{Re}\) for any time window is defined in (3.3) to (3.5), representing the scale-adjusted mean of absolute difference between model fit and actual observation.

\[
\epsilon_k = \hat{y}_k - y_k \quad (3.3)
\]

\[
\hat{y}_k = A - B(T_c - k)^{1-\beta} \quad (3.4)
\]

\[
\text{Re} = \frac{1}{N} \frac{\sum_{k=0}^{t_{end}} |\epsilon_k|}{\sum_{k=0}^{t_{end}} |\epsilon_k|} = \frac{\sum_{k=0}^{t_{end}} |\epsilon_k|}{y_{\text{end}} \ N} \quad (3.5)
\]

Where \(T_c\) is the mean of the selected critical time series and \(t_{\text{end}}\) denotes the end time of the time window.

**New optimization problem**

In summary, the new optimization problem is defined as follows: for the \(i\)th time window, the estimate of parameter combination is given by solving the following optimization problem:

**Objective:** \(\min(\text{Re}_i)\)

**Restrictions:**

i) \(0 \leq T_{c,i} - t_{i,\text{end}} \leq 250;\)

ii) \(\text{range}(\vec{T}_{c,i}(t)) < \min(500, \frac{N}{2});\)

iii) \(\vec{T}_{c,i}(t)\) passes Trend Test at significance level 95%;

iv) \(\vec{T}_{c,i}(t)\) passes DF Test at significance level 95%;

Where \(T_{c,i}(t)\) is the critical time series, and \(\text{range}(\vec{T}_{c,i}(t)) = \max_{t} (\vec{T}_{c,i}(t)) - \min_{t} (\vec{T}_{c,i}(t))\) is the range of the critical time series; \(\vec{T}_{c,i}(t)\) is the demeaned critical time series; \(T_{c,i}\) is the mean of \(\vec{T}_{c,i}(t)\); and \(t_{i,\text{end}}\) is the end of the \(i\)th time window; \(\text{Re}_i\) is the measure of goodness-of-fit as defined by (3.3) to (3.5).
3.1.2 New Searching Method

Instead of the Tabu searching method, which has a stochastic nature and is not looking at a dense enough grid according to our analysis in Chapter 3, we use the initial point searching method following 2 steps:

i) Fit the observed log price series to the simple power law model (3.6) to obtain estimates for $\hat{A}$, $\hat{B}$ and $\hat{\beta}$, which are denoted by $\hat{A}$, $\hat{B}$ and $\hat{\beta}$.

$$
\log p(t) = A - B(T - t)^{1-\beta} + \tilde{\varepsilon}_t
$$

ii) Construct a parameter grid in the neighbourhood of $\hat{A}$, $\hat{B}$ and $\hat{\beta}$ and search all possible parameter combinations on the grid to identify the feasible set.

The insight of using this searching method is that, the SCT model (3.7) resembles the simple power law model with the only difference that, for simple power law model, the error term $\varepsilon_t$ is in the price; while for SCT, the error term is in time (consider the $\tilde{\varepsilon}_t(t)$ as the error term).

$$
\log p(t) = A - B(t_i + \tilde{\varepsilon}_t(t) - t)^{1-\beta}
$$

Due to this subtle link, the estimates provided by SCT model should not deviate far from that given by the simple power law model, and therefore, by adopting the initial point searching method, we reasonably narrow the searching space of all parameters; and within the searching space, a denser grid is constructed. Details on determination of the searching space and construction of grid is documented in Section 3.1.3, subpoint (2).

3.1.3 New Solving Procedure

To solve the optimization problem, we adopt the following procedure:

(1) **Rescale log price:**
Within each time window, we rescale the log price by subtracting all log price observations within the time window by the log price at the start of the time window using (3.8). After rescaling, the new log price starts from zero and reflects the change of log price over time within one time window; in this way, parameter estimates in different time windows become comparable:

$$
\log p_{new} = \log p(t) - \log p(t_i^{end} - N + 1) , \quad t = t_i^{end} - N + 1...t_i^{end}
$$

After rescaling, $A$ takes value between 0.5 and 2 for most of the time, and the range of $B$ reduces from (0,2] to (0, 0.3]. By rescaling log prices, the nominator $A - \log p(t)$ in equation (3.2) becomes smaller, and to make the fraction remain the same, $B$ is taken to be smaller as well. The narrowing effect of parameter range makes the parameter grid denser with the same computational cost.
(2) Make parameter grids:

Use the initial point searching method to narrow down the searching space for each parameter. Estimates of $A$, $B$, and $\beta$, obtained from fitting log price series to simple power law are denoted by $\hat{A}$, $\hat{B}$ and $\hat{\beta}$. The parameter grid in the second step of the initial point searching method is constructed as follows:

$A$ lies in $[\max\{\log(p_{\text{max}}) ,2\hat{A}\}, \min\{2\log(p_{\text{max}}) ,\hat{A}/2\}]$; $\log(p_{\text{max}})$ denotes the maximum rescaled log price within the time window.

The interval for $A$ means that the neighbourhood is expanded around the estimate $\hat{A}$, while the lower bound is no smaller than the maximum value of the rescaled log price and the upper bound is no larger than 2 times the maximum value of the rescaled log price.

$B$ lies in $[\max\{0.01,2\hat{B}\}, \min\{0.3, \hat{B}/2\}]$, similar with $A$, the interval for $B$ is expanded around the estimate $\hat{B}$, and the lower bound is no smaller than 0.01 while the upper bound is no larger than 0.3. The upper bound of $B$ is small because of the rescaling effect of log-price.

$\beta$ lies in $[\max\{0, \hat{\beta} - 0.2\}, \min\{0.99, \hat{\beta} + 0.2\}]$, similar with $A$ and $B$, the interval of $\beta$ is expanded around $\hat{\beta}$, and the lower bound is no smaller than 0 while the upper bound is no larger than 0.99. The upper bound is taken to be 0.99 because originally, beta lies in $[0,1)$, where the value 1 cannot be taken otherwise the denominator $1 - \beta$ would be 0.

Then 20 equally spaced points are taken within the interval for $A$ and $B$, while for $\beta$, the distance between 2 points on the parameter grid is 0.02.

(3) Construct Time Series:

Construct $Na*Nb*Nbeta$ combinations of parameters, where $Na$, $Nb$, and $Nbeta$ are number of points taken on the parameter grid for $A$, $B$, Beta which depend on the width of parameter interval. For each parameter combination, construct critical time series, and demeaned critical time series using equations (3.3), (3.4) and (3.5).

(4) Check Restrictions and Optimize Objective:

Check restrictions i) to iv) to see if the feasible set for parameter combinations is non-empty. If it is, choose the parameter combination that optimizes the objective within the feasible set.

(5) Flag a warning: If the feasible set in step (4) is non-empty, a warning for bubble is flagged.

(6) Predict a crash: The predicted time of crash is indicated by the mean of the critical time series.
3.2 Improvements on time window length problem

As discussed in the introduction of this Chapter, we design the procedure so that it can reduce the probability of missing a bubble warning due to the choice of time window length, which cannot be easily determined ex-ante in real time.

To this end, for each inspection point, we would like to detect a bubble in time windows of different lengths by keeping the end of all time windows fixed at the current time while letting the start date slide, so that the most recent information in the price series is incorporated. In this way, although we don’t know the optimal window length for the current bubble, by considering several different options, the possibility for us to miss a bubble warning is reduced significantly.

Therefore, when back-testing the bubble detection method, we first determine **Inspection Points** at which we would like to investigate the existence of a bubble. The first inspection point is \( N_{\text{max}} \) trading days – the maximum time window length – away from the start date of our sample period and then slide it forward by 25 trading days.

For each inspection point, we test the existence of a bubble for different time window lengths by sliding the starting date forward so the time window length changes from \( N_{\text{max}} \) to \( N_{\text{min}} \) by a step of \( N_{\text{step}} \), where \( N_{\text{min}} \) and \( N_{\text{max}} \) are the minimum and maximum value of \( N \), and the starting date slides by \( N_{\text{step}} \) each time. Figure 3.3 illustrates the choice of inspection points and the sliding of time window at a fixed inspection point:

**Figure 3.3:** Illustration of choice of inspection points and time window construction for a fixed inspection point.

For each inspection point, there are therefore \( m = \frac{N_{\text{max}} - N_{\text{min}}}{N_{\text{step}}} + 1 \) time windows with different lengths. If there is at least one bubble warning time window, a warning for bubble is flagged at the inspection point. For each inspection point, we pick the optimal time window by minimizing the optimization objective and take the mean of critical time
series in that specific time window as the predicted crash date for the current inspection point.

In sum, concerning the time window construction, we first determine inspection points at which we would like to investigate the existence of a bubble. At each inspection point, we consider multiple time windows by letting the start date of each time window slide by $N_{\text{step}}$ until the $N = N_{\min}$. A warning for bubble is flagged at an inspection point if at least one of its time windows has a warning.

### 3.3 Score of warnings

In this section, we first present the results on NASDAQ Index for the revised bubble detection method discussed in Section 3.1 and Section 3.2. Then we discuss the problem with the revised bubble detection method and propose a solution to the problem.

#### Results and further problems

Following the revised bubble detection method, we obtain bubble warnings for NASDAQ Index from 1 January 1980 to 31 October 2008.

![Figure 3.4. Warning results for NASDAQ Index from 1 January 1980 to 31 October 2008 following the revised bubble detection method. $N_{\text{max}}=900$, $N_{\min}=250$ and $N_{\text{step}}=25$ are taken. On the upper panel, vertical lines indicate the inspection point at which a bubble warning is flagged while vertical lines on the lower panel indicate the predicted time for the bubble to burst.](image-url)
There are mainly two problems with the current results:

- It gives out warnings too long before the bubble bursts; warnings start to appear at the early phase of bubbles since the shortest length of time window is taken as 250, and warnings appear if the bubble last for longer than 250 trading days regardless of how much longer than 250 trading days it actually lasts.

- The predicted time for the bubble to burst, as indicated by the mean of the critical time series (denoted by vertical lines on the lower panel of Figure 3.4), moves forward as the inspection point shifts forward. The forward-moving prediction makes it difficult to base any investment strategy on it in real time.

These two problems make it difficult to generate useful information for investors to avoid losses from a crash. We have observed that, bubble warnings usually appear quite long before the crash/deflation, and if an investor follow bubble warnings at the beginning, he/she would lose the tremendous profit brought by the sharp increase in the asset price; on the other hand, from observation, if it is possible to distinguish stronger warnings from weaker ones, such that the stronger ones correspond to higher probabilities for bubble to resolve in near future, it may be possible for investors to utilize bubble warnings to avoid loss from a crash without losing too much of the profit brought by the bubble price increase.

Then the question becomes: how to distinguish stronger warnings from weaker ones?

Therefore, to answer this question, we would like to develop a warning score, which implies the magnitude of warnings; ideally, within the life-time of a single bubble, the score is large only at the very late phase of the bubble.

**Definition of scores**

We define two scores of warnings which capture different features of the bubble behaviour:

- The persistence of the developing bubble; the concern for this feature is that we have more confidence in the existence of a bubble when it is detectable in different lengths of time windows at the same inspection point;

- The speed of increase of the developing bubble; the concern for this feature is intuitive for the reason that a fast increase can not last for long, and the faster it becomes, the closer it is to its end.

The first feature for the \(i\)th inspection point is captured by \(wnum\), as defined in (3.9) and (3.10), which is the total number of time windows with a warning; big \(wnum\) means that for the same inspection point, asset price behaviour in a number of time windows with different window length exhibit a bubble-like behaviour. For these cases, we expect a larger chance for the bubble to burst in the near future.

\[
A_{ij} = \begin{cases} 
1, & \text{if there is warning for timewindow}_{ij} \\
0, & \text{if there is no warning for timewindow}_{ij} 
\end{cases} 
\]  
(3.9)
\[ wnunm_i = \sum_{j=1}^{m} A_{i,j} \]  \hspace{1cm} (3.10) 

where \( \text{timewindow}_{ij} \) is the \( j \)th time window for the \( i \)th inspection point with window length \( N_{ij} \), and \( m \) is the total number of time windows for the \( i \)th inspection point, for \( i=1,\ldots,n \).

The **second feature** – speed of price increase – is captured by the maximum of average log price increase over all time windows for each inspection point. Let \( \maxslope_i \) denote the maximum of average log price increase over all time windows while \( \maxslope^w_i \) denote the maximum over only time windows with a warning. We define them in (3.11) and (3.12) respectively. Large \( \maxslope^w_i \) corresponds to sharp increase in price and a larger chance for the bubble to burst in the near future.

\[
\maxslope_i = \max_{j=1,\ldots,m} \left\{ \frac{\log p(t_i^{\text{end}}) - \log p(t_i^{\text{end}} - N_{i,j} + 1)}{N_{i,j}} \right\}, \text{ for } i = 1, \ldots, n
\]  \hspace{1cm} (3.11) 

\[
\maxslope^w_i = \max_{j \in W} \left\{ \frac{\log p(t_i^{\text{end}}) - \log p(t_i^{\text{end}} - N_{i,j} + 1)}{N_{i,j}} \right\}
\]  \hspace{1cm} (3.12) 

where \( N_{ij} \) is the length of \( \text{timewindow}_{ij} \), \( W = \{ j \mid A_{i,j} = 1, j = 1,\ldots,m \} \) and \( i \in \{ i \mid wnunm_i > 0, i = 1,\ldots,n \} \).

**Selection of strong warnings:**

For all inspection points with a warning, we calculate \( wnunm_i \) and \( \maxslope^w_i \). Given the two scores, we define two sets of inspection points in (3.13) and (3.14) and define their union in (3.15) to pick stronger warnings according to the two scores from all warnings given by the model. Notably, warnings and inspection points (denoted by \( t_i^{\text{end}} \)) are interchangeable here because warnings are flagged on inspection points, and thus the set of inspection points corresponds to the set of warnings on these dates.

Let

\[ I_{n1} = \{ t_i^{\text{end}} \mid wnunm_i > 0.6m \} \]  \hspace{1cm} (3.13) 

\[ I_{n2} = \{ t_i^{\text{end}} \mid \maxslope^w_i > \maxslope^{0.95} \} \]  \hspace{1cm} (3.14) 

\[ In_{12} = I_{n1} \cup I_{n2} \]  \hspace{1cm} (3.15) 

Where \( \maxslope^{0.95} \) is the 95% quantile of \( \maxslope_i \) given all the information up to the \( i \)th inspection point. To get an initial value of \( \maxslope^{0.95} \), we take 1200 observations of prices prior to the in-sample test period as pre-sample period and calculate \( \maxslope_i \) for inspection points in the period as we do for in-sample period data. Taking the 95%-quantile of \( \maxslope_i \) in the pre-sample period, we obtain the initial value of \( \maxslope^{0.95} \). Within the test period, as inspection point moves forward by 25 trading
days, we include the new max slope for the current inspection point to obtain an updated max slope$^{0.95}$.

Warnings in In_t are either more persistent or increase more sharply and are considered as strong warnings. In figure 3.5, the behaviour of the two scores are plotted in subplot 3.5-a and 3.5-b and warnings correspond to points above the two threshold lines are selected as strong warnings.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.5.png}
\caption{Behaviour of the two scores for all warnings. Red points (from left to right) in subplot 3.5-a corresponds to score 1 (wnum) for the warnings that are indicated by vertical lines in figure 3.4. Similar for 3.5-b. Horizontal lines in the two subplots indicate 60% of the total number of time windows and the 95% quantile of maxslope respectively. Points fall above the two horizontal lines corresponds to scores for stronger warnings.}
\end{figure}

3.4 Summary of the new procedure

Following a 4-step procedure, the revised bubble detection method based on the SCT model is summarized as follows:

1. Time window construction:

In the test-sample period, for a given price time series, set the first inspection point to be $N_{\text{max}}$ trading days away from the start of the test sample period and let it slide forward by 25 trading days as depicted in figure 3.1: $t_i^{\text{end}} = N_{\text{max}}$ and $t_i^{\text{end}} = t_{i-1}^{\text{end}} + 25$, $i > 1$.

For the $i$th inspection point, let the starting date slide by $N_{\text{step}}$ from $t_i^{\text{end}} - N_{\text{max}} + 1$ to $t_i^{\text{end}} - N_{\text{min}} + 1$, as depicted in figure 3.1, therefore: $t_{i,1}^{\text{start}} = t_i^{\text{end}} - N_{\text{max}} + 1$ and $t_{i,j}^{\text{start}} = t_{i,j-1}^{\text{start}} + N_{\text{step}}$, $j > 1$. 

For all inspection points, calculate maxslope, and update maxslope$^{0.95}$, whose initial value is obtained in the pre-sample period.

2. Time series construction:

Within the $j$th time window for the $i$th inspection point, we construct time series following the steps (1) Rescale log price, (2) Make parameter grids and (3) Construct Time Series as described in Section 3.1.3.

3. Parameter estimation:

For each inspection point, within each time window that ends on it:

(1) check restrictions i) to iv) on time series constructed in step 2 to identify the feasible set;

(2) If the feasible set is not empty, pick the parameter combination that optimizes the objective and calculate the score using the chosen parameter; otherwise proceed to the next time window.

4. Flag Strong Warnings:

After (1) and (2) are applied to all time windows for the $i$th inspection point, if there is at least one time window with warning, flag a warning at the $i$th inspection point. For inspection points with a warning, calculate $wnum_i$ and $slope_i^w$, and identify the set of strong warnings using (3.13) to (3.15).

3.5 In-Sample Warning Results

In this section, we report the in-sample test data and bubble warning results with short analysis on the results.

In-Sample Data:

- NASDAQ: 01-Feb-1980 to 31-July-2008, with pre-sample period from 5-Aug-1974 to 31-Jan-1980;
- SP500: 01-Feb-1980 to 31-Oct-2008, with pre-sample period from 1-July-1975 to 31-Jan-1980;

Warning results: In table 3.1, inspection points of warnings for the three indices are reported. In figure 3.6, we present warning results for the three indices where vertical lines indicate the inspection points on which warnings are flagged.
Warning Results Analysis

- The revised bubble detection method gives out warnings for crashes which follow bubbles with either long life-time or sharp increase.


- Warnings are not flagged for some huge crashes, for example the 1997 crash for Hang Seng. Warnings prior to the crash were not selected as strong warnings because judging from the build up of the bubble; it doesn’t have a long history or a sharp increase. According to strong warnings given by the bubble detection method, the existence of an endogenous bubble is not the only cause of the severe crash, and in this case, the external cause is the 1997 Asia Financial crisis that started with the financial collapse of the Thai baht in July 1997.

In chapter 4, we will develop a test to quantify the performance of the bubble detection method and apply it to both in-sample data and out-of-sample data.
Figure 3.6: Strong warnings for the three indices. Vertical lines indicate the inspection points on which warnings are flagged.

<table>
<thead>
<tr>
<th>NASDAQ</th>
<th>SP500</th>
<th>Hang Seng</th>
</tr>
</thead>
<tbody>
<tr>
<td>'28-Apr-83'</td>
<td>'23-Mar-87'</td>
<td>'25-May-92'</td>
</tr>
<tr>
<td>'3-Jun-83'</td>
<td>'13-Aug-87'</td>
<td>'I-Jul-92'</td>
</tr>
<tr>
<td>'30-Sep-99'</td>
<td>'9-Jul-97'</td>
<td>'6-Aug-92'</td>
</tr>
<tr>
<td>'10-Dec-99'</td>
<td>'13-Aug-97'</td>
<td>'30-Dec-93'</td>
</tr>
<tr>
<td>'18-Jan-00'</td>
<td>'18-Sep-97'</td>
<td>'3-Feb-94'</td>
</tr>
<tr>
<td>'23-Feb-00'</td>
<td>'23-Oct-97'</td>
<td>'14-Mar-94'</td>
</tr>
<tr>
<td>'29-Mar-00'</td>
<td>'24-Apr-98'</td>
<td>'17-Jun-99'</td>
</tr>
<tr>
<td>'4-May-00'</td>
<td>'11-May-07'</td>
<td>'26-Feb-07'</td>
</tr>
<tr>
<td></td>
<td>'18-Jun-07'</td>
<td>'25-Jul-07'</td>
</tr>
<tr>
<td></td>
<td>'24-Jul-07'</td>
<td>'5-Oct-07'</td>
</tr>
<tr>
<td></td>
<td></td>
<td>'12-Nov-07'</td>
</tr>
<tr>
<td></td>
<td></td>
<td>'17-Dec-07'</td>
</tr>
<tr>
<td></td>
<td></td>
<td>'24-Jan-08'</td>
</tr>
</tbody>
</table>

Table 3.1: Inspection points with warnings for the three indices.
Chapter 4
Model Test Description

In this chapter, the test of bubble warnings generated by the revised bubble detection method based on the SCT model (called the Model afterwards for short) is introduced. The difficulty of designing a statistical significance test of bubble warnings lies in that:

- It depends on the way of translating bubble warning clusters into crash signals;
- It depends on the definition of crash/deflation;
- It depends on the recognition of a successful crash signal given the realization of prices after the crash signal;

These difficulties, however, are not independent. For instance, the recognition of a successful crash signal not only depends on the determination of a crash signal which is related to the first difficulty but also depends on the determination of crash/deflation which is related to the second difficulty.

Given these difficulties, the design of a rigorous statistical significance test of bubble warnings cannot be perfectly objective. The test we designed in this chapter tries to take into account different situations; also, it is designed based on the principle that it is implementable in practice in real-time.

To provide an indicator of the statistical significance of bubble warnings generated by the Model, the test provides a ratio of successful crash signals translated from bubble warnings according to our translation rule, definition of crash/deflation, and recognition of successful crash signals. It is designed by simulating the way bubble warnings would be used to generate crash signals in practice and the test procedure provides plausible but not perfect ways to deal with the above-mentioned difficulties.

In Section 4.1, three translation rules on how to convert bubble warnings into crash signals are introduced. It’s necessary to develop a translation rule for the reason that there is usually more than one bubble warnings flagged prior to a crash. Since there is no fixed relationship between the appearance timing of bubble warnings and starting time of crash/deflation, it’s difficult to define a perfect translation rule; thus, we design three of them with their own advantages and disadvantages.

In Section 4.2, the test of crash signals is introduced by simulating the way that bubble warnings would be utilized in practice in real-time. Success Ratios of crash signals are reported with comparison to Success Ratio of crash signals generated by a naïve model which assumes that there is a bubble warning at all inspection points. By testing crash signals generated by bubble warnings, the success ratios of bubble warnings are tested indirectly.

In Section 4.3, the in-sample test results and simple analysis are included.

Figure 4.1 provides an illustration of the structure of the test.
4.1 Translation Rules

The first step of the statistical significance test on the crash signals would be to determine how to convert bubble warnings into crash signals given more than one bubble warning prior to a same crash is flagged.

The reason for the translation of bubble warnings into crash signals is that, we need to decide the time point at which we react to bubble warnings (hedge long positions in the assets by buying put options or simply reduce positions). However, from analysis of in-sample bubble warning results, bubble warnings only indicate the existence of a possible bubble; and for the timing of a forthcoming crash, the information contained in bubble warnings is limited.

Two observations from in-sample warning results:

i) When bubble warnings are generated, sometimes they stand alone, and sometimes they appear as warning clusters (Definition 4.1).

ii) Bubble warnings stop before or short after the crash happens.

**Definition 4.1: (Warning Clusters):** Two successive warnings are considered as the same warning cluster if the distance between them is less than 25 trading days. We call the 25 trading days the Cluster Distance.

Since the appearance time and pattern of bubble warnings are not unanimous from observation, it’s difficult to develop a simple translation rule which can perfectly translate.
bubble warnings into crash signals. Therefore, we designed three of them, each of which has its own advantages and disadvantages.

In Table 4.1, a pro-con list of the three crash signal translation rules is presented, and more detailed descriptions of each translation rule are introduced below.

<table>
<thead>
<tr>
<th>Crash Signal Translation rule</th>
<th>Pros</th>
<th>Cons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Crash signal at the first warning</td>
<td>Avoided reacting too late</td>
<td>Reacted too early</td>
</tr>
<tr>
<td>2. Crash signal at 25 trading days after the end of a warning cluster</td>
<td>Avoided reacting too early</td>
<td>Reacted too late</td>
</tr>
<tr>
<td>3. Crash signal only if a small loss happens after a bubble warning</td>
<td>Trade-off between reacting too early and missing crash; Included price information into decision making;</td>
<td>Suffered from the loss at the beginning of the crash which can already be huge</td>
</tr>
</tbody>
</table>

Table 4.1 Pro-con list of crash signal translation rules

Translation Rule 1:
The first simplest way to translate bubble warning into crash signals would be: **flag a crash signal at the beginning of a warning cluster**.
This method is easy to implement but suffers from the drawback that it may lose the sharp increase in the price before the crash for reacting too early.

Translation Rule 2:
To avoid reacting too early, another possible way to translate bubble warnings into crash signals would be: **flag a crash signal at the end of a warning cluster**.
A potential problem with this method is that, to conclude that the current warning is the last one of the cluster, one needs to wait for 25 trading days to make sure that there is no bubble warning anymore. However, it’s highly probable that the crash would happen within the 25 trading days for that the warning is actually the last of the cluster, but one can not know that for sure at that time point when the future is unknown.

Translation Rule 3:
Obviously, losing the potential increase and missing the crash due to the waiting time are both undesirable, and therefore, a third way of translating bubble warning into crash signals is developed as a trade-off between the two: **monitor the price behaviour after each bubble warning, and if a small-loss (Definition 4.2) occurred, flag a crash signal; otherwise proceed to the next bubble warning.**
**Definition 4.2: (Small-Loss):** A small-loss happens at time $t$ if the 5-day-return falls below $-1.5$ times the volatility at the time point. The small loss Indicator defined in (4.1) takes value of 1 if a small-loss happens.

\[
\text{Indicator} = I_{(nr_{t,5} < -1.5)} \quad (4.1)
\]

\[
nr_{t,5} = \frac{r_{5,t}}{5\sigma_i} \quad (4.2)
\]

\[
\sigma_i = \sqrt{\lambda (\sigma_{i-1})^2 + (1-\lambda)\sigma_{t-1}^2} \quad (4.3)
\]

Where $nr_{t,5}$ and $r_{5,t}$ are the 5-day-normalized-return and 5-day-return at time $t$, respectively; $\sigma_i$ is the Exponentially Weighted Moving Average (EWMA)\(^3\) estimate of daily volatility obtained using (4.3), where $\lambda = 0.95$ is taken for daily data and $\sigma_i$ is obtained using the ordinary volatility estimate (standard deviation of return) in the pre-sample period; estimate for 5-day-return volatility is given by $\sqrt{5}\sigma_i$.

One possible problem with this method is that, it suffers from the loss at the beginning of a crash, especially when the crash happens in a short time period by losing 30% to 40% of its value. Therefore, adopting this method only reduces the possibility to miss the whole crash but still suffers from the loss at the beginning of a crash, which can also be huge. Also, it’s possible that the price drops a little bit and starts to increase again.

To conclude, there are three ways of translating bubble warnings to crash signals, and none of them is perfect. We would like to test all of them to see if any of them would generate crash signals with a high success ratio.

### 4.2 Test of Crash Signals

To test the statistical significance of crash signal, we need to investigate:

- The **Success Ratios** of crash signals which is the ratio of signals followed by a crash/deflation over the total number of signals;

As the other half of the test, it would also be necessary to investigate the detection ratio of crash signals. The reason why we do not look at it is:

- The bubble detection method is designed to detect endogenous crashes and thus the detection ratio should be calculated on the basis of only endogenous crashes. However, since there is no established rule for the classification of endogenous and exogenous crashes, it would be quite subjective to distinguish them from each other.

---

\(^3\) A short introduction to the Exponentially Weighted Moving Average (EWMA) estimate of daily volatility is included in Appendix 2.
Also, to provide an indicator of the economic significance of crash signals, we would investigate:

- The distributions of j-day-return after crash signals to see if there is an obvious shift-to-the-left effect in the distributions.

**Test of Success Ratios**

Before introducing the test procedure, we first give the definition for success ratio and k-day-huge loss, which is recognized as a crash/deflation.

**Definition 4.3 (Success Ratio):** The success ratio of crash signals is:

\[
\text{success ratio} = \frac{\# \text{ successful signals}}{\# \text{ total signals}}
\]  

This measures the ratio of signals followed by a crash over the total number of signals. A crash signal is recognized as successful if compared to the price at the time when the crash signal is given, a \textit{k-day-huge-loss} (Definition 4.4) occurs within 250 trading days.

**Definition 4.4 (k-day-huge loss):** A \textit{k-day-huge-loss} Indicator at time \( t \) is defined as:

\[
\text{Indicator} = I_{\{nr_{t,k} < -2\}} \quad k=1, 250
\]  

Where \( nr_{t,k} \) is the \textit{k-day-normalized return} at time \( t \) defined in the same way as the 5-day-normalized return as in equation (4.2). Here, the \textit{k-day-huge-loss} is recognized as a crash/deflation.

- **Determination of Successful Crash Signals**

In Section 4.1, we introduced three ways of translating bubble warnings to crash signals. In practice, once a crash signal is generated, precautious measures would be taken, and thus, bubble warnings that are flagged after the crash signal become irrelevant. Also, if no crashes/deflations happen after a crash signal for a certain time period, decisions of further strategies should be made.

To simulate the strategies we would take in practice, we introduce a procedure based on a \textit{crash mode} to determine the success ratio of crash signals.

**Definition 4.5 (Crash Mode):** A crash mode is a special period that we enter after each crash signal and during it we expect a crash to happen and won’t react to any forthcoming bubble warnings until the crash mode is exited.

The procedure is illustrated in figure 4.2. Description of the procedure, which is introduced in combination with the third translation rule, follows the figure.

Signal translation rules 1 and 2 can be considered as two special cases of the third one, for which the crash mode is entered immediately after the received\_warning without entering a monitor mode. The difference between these two methods lies in the determination of the received\_warning: for the former, it is the first warning of a cluster; while for the latter, it is 25 trading days after the last warning of a cluster. The rest of the test procedure remains the same.
Figure 4.2 Procedure of the test procedure based on a crash mode. For signal translation rule i) and ii), the part in the dashed frame is not considered.

Description of the Procedure:

Given all the bubble warnings for a certain asset, we start from the first bubble warning and received_warning1 is set to be the first bubble warning.

1. Enter monitor mode 1 after received_warning1 and monitor the 5-day-normalized-return within 25 trading days;

2. If a small loss happens at \( i^* \) during the monitor mode, s.t. \( nr_{received\_warning, vi^*} < -1.5 \), flag crash signal1 at received_warning1 + \( i^* \) (crash signal1 = received_warning1 + \( i^* \)) and enter crash mode 1 one day after the flag of the crash signal; otherwise reset the received_warning1 to be the next bubble warning until the small loss happens and the crash mode is entered;

3. During Crash Mode1, investigate j-day-normalized-return \( nr_{signal, j^*} \). If a huge-loss happens at \( j^* \) during the crash mode such that \( nr_{signal, j^*} < -2 \), signal1 is counted as a successful signal, and the crash mode is exited after \( j^* + 1 \) trading days; otherwise, the crash mode is exited after 250 trading days.

4. The next received_warning is the first warning after the crash mode is exited. Repeat the procedure until there is no warnings remained.

- Critical Value of Success Ratio

Following the above-mentioned procedure, the success ratio of crash signals translated from bubble signals generated by the Model using three translation rules can be calculated. To shed light on the statistical significance of these crash signals, we compare success
ratios for crash signals translated from bubble warnings generated by the Model (called model bubble warnings or model warnings for short afterwards) with that from model warnings are compared with signals based on naïve bubble warnings which are flagged at all inspection time points.

Names of crash signals translated from model/ naïve bubble warnings using the three different translation rules are concluded in Table 4.2.

<table>
<thead>
<tr>
<th>Type warnings</th>
<th>Type Translation Rules</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model warnings</td>
<td>Model Signals 1</td>
</tr>
<tr>
<td></td>
<td>Model Signals 2</td>
</tr>
<tr>
<td></td>
<td>Model Signals 3</td>
</tr>
<tr>
<td>Naïve warnings</td>
<td>Naïve Signals 1</td>
</tr>
<tr>
<td></td>
<td>Naïve Signals 2</td>
</tr>
<tr>
<td></td>
<td>Naïve Signals 3</td>
</tr>
</tbody>
</table>

*Table 4.2: Name of signals translated from different types of warnings using different types of translation rules.*

For Translation Rule 1 and 3, success ratios for Naïve Signals 1 and 3 can be taken as critical values to test the statistical significance of Crash Signals 1 and 3. If success ratios for Model Signals 1 and 3 are lower than that for Naïve Signals 1 and 3, we conclude that there is no statistical significance for bubble warnings generated from the bubble detection method in predicting a forthcoming crash/deflation.

For Translation Rule 2, the success ratio for Naïve Signal 2 is not meaningful because all the naïve bubble warnings are recognized as the same cluster according to Definition 4.1 since the distance between all naïve bubble warnings is 25-trading days. Therefore, we compare the success ratio for Crash Signals 2 with that for Crash Signals 1 and 3 to get an indirect implication of its statistical significance.

**Distribution of j-day-return after crash signals**

Suppose there are in total M crash signals for all price series that are tested. For signal\(_m\), we get a j-day-return series:

\[
r_{signal\_m+j,j} = \frac{p_{signal\_m+j} - p_{signal\_m}}{p_{signal\_m}}, \quad j=1, 250
\]

Therefore, for each j, we have m different j-day-returns, which form the distribution of the j-day-return after crash signals. It is desirable if there is an obvious shift-to-the-left effect in the distribution, which means returns after crash signals become more negative than ordinary time periods.

For in-sample test data, since observation is limited, it’s difficult to develop such a distribution. However, for out-of-sample test data, we can report the distribution by generating histograms of j-day-returns after crash signals for different j’s.
4.3 In-Sample Test Result

Crash signals results

In Figure 4.3, Model Signals 1 are indicated by vertical lines in the plot of the three in-sample test price series. For Model Signals 2 and 3, please refer to Appendix 3.

*Figure 4.3 Model Signals 1 for three in-sample test series. Vertical lines indicate Model Signals 1.*

Take the first three crash signals for Hang Seng as an example, the first crash signal is flagged in May-1992, and since the severe crash happens in Oct-1992, it was too early and counted as a false alarm. The second one was counted as a successful signal because a severe short-term crash happens immediately after the crash signal, where the 10-day-normalized-return fall below -2. Therefore the crash mode is exited immediately and the third warning was received in Feb-94.
**Success Ratio Results**

In Table 4.3, the *success ratios* of different types of crash signals are reported for in-sample data.

<table>
<thead>
<tr>
<th>Model Signals 1</th>
<th>Naive Signals 1</th>
<th>Model Signals 2</th>
<th>Naive Signals 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.46</td>
<td>0.25</td>
<td>0.11</td>
<td>0.31</td>
</tr>
<tr>
<td>13</td>
<td>79</td>
<td>9</td>
<td>71</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
<td>1</td>
<td>22</td>
</tr>
</tbody>
</table>

**Table 4.3: Success-ratio of different types of crash signals for In-sample data**

From the in-sample test results, we can see that, the success ratios for model signals 1 and model signals 3 are higher than that for naïve signals 1 and 3, respectively. Also, the first translation rule outperforms the other two; however, due to the small test sample size, it’s difficult to draw the final conclusions from it.

Probably, this out-performance is caused by the small sample size because it’s not always the case that the crash happens immediately after the first bubble warning (e.g. HangSeng-2007 and NASDAQ-2000).

In the next Chapter, we introduce the data for out-of-sample test, and carry out the test on a larger scale to draw the final conclusions.
Chapter 5
Test of Bubble Warnings

In this chapter, we report the data and result of the out-of-sample test using the test introduced in Chapter 4. In Section 5.1, we introduce the out-of-sample test data; in Section 5.2, we report the results with analysis of the results. The last section concludes.

5.1 Out-of-Sample Data Description

Generally, we test the bubble detection method on indices. We divide the out-of-sample test data into four categories: sector-to-market, currencies, commodities and Emerging Market Indices.

5.1.1 Sector to Market

Data source:

**Fama-French 48 Industry** daily return data, downloaded from online data library at: [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).

**Market Benchmark**: S&P 500 index, downloaded from Bloomberg.

Sample Period: Pre-Sample Period: 1-Sep-1978 to 10-Jun-1983

Data Description:


The 48 industry portfolios can be seen as data for different industry sectors, and we are interested in the relative performance of each sector compared to the whole market.

Sectors to Market Data Construction:

Let $r_{m,t}$ denote the daily return of the S&P500 and $r_{j,t}$ denote the daily return of jth industry portfolio, for j=1…48. The relative daily return of the jth industry portfolio and the recovered price series from relative daily return are calculated using (5.1) and (5.2):

\[
1 + r'_{j,t} = \frac{1 + r_{j,t}}{1 + r_{m,t}} \quad (5.1)
\]

\[
p'_{j,t} = \prod_{s \leq t} p'_{j,s}(1 + r'_{j,s}) \quad (5.2)
\]

Where $r'_{j,t}$ and $p'_{j,t}$ are the relative daily return and recovered price series, respectively and $p'_{j,t} = 1$ for all j’s.
5.1.2 Currencies
For currencies, we test for 20 combinations of G10 Currencies and 16 combinations of Emerging Market currencies.

Data source: Bloomberg.

Sample period:
G10 Currencies: Pre-Sample-Period: 1-Jan-80 to 3-Aug-84;
Test-Sample-Period: 6-Aug-84 to 5-Feb-09;
Emerging Market currencies: Pre-Sample-Period: 4-Sep-95 to 6-Apr-00;
Test-Sample-Period: 7-Apr-00 to 25-Jul-08.

Data Description:
G10 Currencies are considered as the 10 most liquid currencies in the world, which includes US Dollar (USD), Canadian Dollar (CAD), Japanese Yen (JPY), Australian Dollar (AUD), New Zealand Dollar (NZD), British Pound (GBP), Euro (EUR), Swiss Franc (CHF), Swedish Krona (SEK), and Norwegian Krone (NOK).

The 20 combinations that we considered are: EURUSD, USDJPY, GBPUSD, AUDUSD, NZDUSD, USDCAD, EURGBP, EURCHF, EURNOK, EURSEK and their mirror series.

As for Emerging Market currencies, we consider the following 8 currencies: Brazilian Real (BRL), Hungarian Forint (HUF), South Korean Won (KRW), Mexican Peso (MXN), Turkish Lira (TRY), South Africa Rand (ZAR), Czech Koruna (CZK), and Polish złoty (PLN).

We compare each of them with the US Dollar and also the mirror series. Therefore, we have 16 time series for Emerging Market Currencies, which are USDBRL, USDHUF, USDKRW, USDMXN, USDTRY, USDZAR, USDCZK, USDPLN, BRLUSD, HUFUSD, KRWUSD, MXNUSD, TRYUSD, ZARUSD, CZKUSD, and PLNUSD.

5.1.3 Commodity

Data source: Bloomberg.

Sample period:
Copper: from 30-Sep-83 to 15-Jul-09 with pre-sample-period 2-Jan-79 to 29-Sep-83;
Crude Oil: from 2-Jul-86 to 3-May-93 with pre-sample-period 4-May-93 to 4-Feb-09.
Metal: from 30-Sep-83 to 15-Jul-09 with pre-sample-period 2-Jan-79 to 29-Sep-83;
Goldman Sachs Commodity Index (GSCI): from 8-Aug-83 to 5-Feb-09 with pre-sample-period from 2-Jan-79 to 5-Aug-83.
5.1.4 Emerging Equity Market Indices

Data source: Bloomberg.

Data Description and Sample Period:
- **IBOV**: Brazil Bovespa Index, from 27-Nov-98 to 3-Jul-09 with pre-sample-period from 20-Jan-94 to 26-Nov-98;
- **KOSPI100**: Korean KOSPI 100 Index, from 21-Nov-02 to 15-Jul-09 with pre-sample-period from 6-Jan-98 to 20-Nov-02;
- **MERVAL**: *MERcado de VALores*, the most important index of the Buenos Aires Stock Exchange, from 27-Nov-98 to 3-Jul-09 with pre-sample period from 20-Jan-94 to 26-Nov-98;
- **MEXBOL**: Mexico IPC Index, from 27-Nov-98 to 3-Jul-09 with pre-sample-period from 20-Jan-94 to 26-Nov-98.

Also, we tested the Morgan Stanley Capital International Index, which is a stock market index of 1500 'world' stocks from 30-Sep-83 to 15-Jul-09 with pre-sample-period 2-Jan-79 to 29-Sep-83.

5.2 Out-of-Sample Test Result

In this section, we present the out-of-sample test result for the Model.

5.2.1 Warning results

In total, we carry out the out-of-sample test on 93 price series for the five asset classes mentioned above. Warning results for two selected assets in each asset class are presented in figure 5.1 where bubble warnings are indicated by dashed vertical lines. Bubble warnings results of all the 93 price series are presented in Appendix 4.

From plots in figure 5.1, it can be seen that bubble warnings generated by the Model usually appear when there is a super-exponential growth in asset prices; however, the behaviour of asset prices after the flag of a bubble warning varies from case to case.

Generally, there are three possible behaviours of price after bubble warnings:
- Crash immediately in a short term;
- Deflate slowly;
- Sustain without crash/deflation.

Take bubble warnings for *Metal* as an example:

Price behavior after the bubble warnings cluster in 1987 to 1988 belongs to the first category, which decreased sharply during the warning cluster;
Price behavior after the bubble warnings cluster in late 1994 and early 1995 belongs to the first category, which decreased gradually and lasted for approximately 2 years;
Price behavior after the bubble warnings cluster in 2004, the asset price went down a little bit and continued to soar; also price behavior after the bubble warnings cluster in 2006, the asset price fluctuated fiercely for two year before it crashed in Apr-2008;
The simple eye-ball analysis indicates that when a super-exponential increase in the asset price is identified, it’s not necessary that the asset price will crash immediately. In the next section, we provide the results for the test of crash signals which are translated from bubble warnings.

Figure 5.1: Out-of-Sample warning results
### 5.2.2 Test Results

1. **Success Ratio results:**

Table 5.1 presents the success ratio results of the out-of-sample test.

<table>
<thead>
<tr>
<th>Sectors-Market</th>
<th>Success Ratio</th>
<th># signals</th>
<th># suc-signals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model Signals 1</td>
<td>0.21</td>
<td>193</td>
<td>40</td>
</tr>
<tr>
<td>Naive Signals 1</td>
<td>0.27</td>
<td>1258</td>
<td>339</td>
</tr>
<tr>
<td>Model Signals 2</td>
<td>0.25</td>
<td>163</td>
<td>40</td>
</tr>
<tr>
<td>Model Signals 3</td>
<td>0.26</td>
<td>147</td>
<td>38</td>
</tr>
<tr>
<td>Naive Signals 3</td>
<td>0.34</td>
<td>1095</td>
<td>372</td>
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</table>

<table>
<thead>
<tr>
<th>G10 Currencies</th>
<th>Success Ratio</th>
<th># signals</th>
<th># suc-signals</th>
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<tbody>
<tr>
<td>Model Signals 1</td>
<td>0.22</td>
<td>55</td>
<td>12</td>
</tr>
<tr>
<td>Naive Signals 1</td>
<td>0.28</td>
<td>531</td>
<td>150</td>
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<tr>
<td>Model Signals 2</td>
<td>0.32</td>
<td>47</td>
<td>15</td>
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<td>Model Signals 3</td>
<td>0.18</td>
<td>34</td>
<td>6</td>
</tr>
<tr>
<td>Naive Signals 3</td>
<td>0.31</td>
<td>435</td>
<td>137</td>
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<table>
<thead>
<tr>
<th>EM Currencies</th>
<th>Success Ratio</th>
<th># signals</th>
<th># suc-signals</th>
</tr>
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<tbody>
<tr>
<td>Model Signals 1</td>
<td>0.44</td>
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<tr>
<td>Naive Signals 1</td>
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<td>5</td>
</tr>
<tr>
<td>Model Signals 2</td>
<td>0.38</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>Model Signals 3</td>
<td>0.43</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>Naive Signals 3</td>
<td>0.37</td>
<td>27</td>
<td>10</td>
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</table>

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Success Ratio</th>
<th># signals</th>
<th># suc-signals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model Signals 1</td>
<td>0.12</td>
<td>26</td>
<td>3</td>
</tr>
<tr>
<td>Naive Signals 1</td>
<td>0.24</td>
<td>113</td>
<td>27</td>
</tr>
<tr>
<td>Model Signals 2</td>
<td>0.19</td>
<td>26</td>
<td>5</td>
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<tr>
<td>Model Signals 3</td>
<td>0.25</td>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td>Naive Signals 3</td>
<td>0.39</td>
<td>108</td>
<td>42</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>EM equity</th>
<th>Success Ratio</th>
<th># signals</th>
<th># suc-signals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model Signals 1</td>
<td>0.30</td>
<td>43</td>
<td>13</td>
</tr>
<tr>
<td>Naive Signals 1</td>
<td>0.30</td>
<td>179</td>
<td>54</td>
</tr>
<tr>
<td>Model Signals 2</td>
<td>0.24</td>
<td>33</td>
<td>8</td>
</tr>
<tr>
<td>Model Signals 3</td>
<td>0.27</td>
<td>30</td>
<td>8</td>
</tr>
<tr>
<td>Naive Signals 3</td>
<td>0.37</td>
<td>155</td>
<td>58</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Aggregate</th>
<th>Success Ratio</th>
<th># signals</th>
<th># suc-signals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model Signals 1</td>
<td>0.22</td>
<td>326</td>
<td>72</td>
</tr>
<tr>
<td>Naive Signals 1</td>
<td>0.27</td>
<td>2109</td>
<td>575</td>
</tr>
<tr>
<td>Model Signals 2</td>
<td>0.26</td>
<td>277</td>
<td>71</td>
</tr>
<tr>
<td>Model Signals 3</td>
<td>0.25</td>
<td>238</td>
<td>60</td>
</tr>
<tr>
<td>Naive Signals 3</td>
<td>0.34</td>
<td>1820</td>
<td>619</td>
</tr>
</tbody>
</table>

*Table 5.1: Success Ratio results for the out-of-sample test. There are six sub-tables containing results for five different asset classes and also the aggregate success ratios results. In each sub-table, the first column indicates the type of crash signals and the second to last column contain success ratio, number of signals and number of successful signals for different types of signals.*
From the success ratio results, we can see that:

- The success ratios for Model Signals 1 to 3 are low (around 0.25) for out-of-sample test data.
- In general (except for the EM currencies), signals translated from the naïve bubble warnings have a higher success ratio than that from the model bubble warnings, which means that the possibility to translate a successful crash signal from the bubble warnings given by the method developed in Chapter 4 is lower than that using the naïve model which gives out a bubble warning at all inspection points.

Analyses of the two observations from out-of-sample test results are reported below.

**Cause of low Success Ratio of Crash Signals:**

The low success ratio of crash signals are caused by the following two uncertainties – *three possible behaviors of the asset price and the varying relationships between the timing of crash and timing of bubble warnings*, which make it difficult to use bubble warnings for crash prediction.

**Varying relationships between the timing of crash and the appearance of bubble warning clusters**

From observation of bubble warning results, there are basically three different relationships between the timing of crash and appearance of bubble warnings given that a crash/deflation happens during or not long after a cluster of bubble warnings are generated:

- Crashes/deflations happen right after the first bubble warning;
- Crashes/deflations happen after the last bubble warning;
- Crashes/deflations happen but warnings don’t stop.

The three different relationships that make it difficult to develop a universal translation rule from bubble warnings to crash signals. Therefore, even a crash happens when bubble warnings are generated, it’s still difficult to use bubble warnings to generate crash predictions.

**Three possible behaviors of the asset price after a cluster of bubble warnings**

The three possible behaviors of the asset price are recorded in subsection 5.2.1.

The first situation – crashes after bubble warnings – is the most desirable one. The test we designed in Chapter 5 can easily recognize the bubble signals prior to the first situation as a successful one.

However, it’s more complicated for the second and third situations. For the second situation, only when the magnitude of the deflation is large enough within the crash mode period (250 trading days after the signal) would the crash signal prior to it be recognized as a successful one; while for the third one, if the crash happens too long after the crash signal (longer than the length of crash period – 250 trading days), it would be recognized as a failure even though the crash finally happens.
Cause for out-performance of naïve bubble warnings:

The cause for the out-performance of naïve bubble warnings is that: the bubble detection method based on the SCT model only gives out warning when the asset price exhibits an upward trend, while the naïve model gives out warning both in upward trend and downward trend. Therefore, during the same long-term downward trend, model warnings stop because there is no super-exponential growth in the price anymore, however, naïve warnings continue to be flagged, and more than one naïve signals are counted as successful signals, causing the outperformance of the naïve crash signals.

In figure 5.2, a comparison of model signals 3 (upper panel) and naïve signals 3 (lower panel) is presented, where signals are indicated by vertical lines. Only one signal out of five signals on the upper panel was considered as a successful signal; while eight out of twenty-four are recognized as successful signals on the lower panel, where we can see several cases of multiply successful crash signals during the same downward trend.

**Figure 5.2** Comparison of signals translated from model bubble warnings (upper) and naïve bubble warnings (lower). Price trajectory of Metal is plotted with vertical lines indicating the timing of signals.
2. Distribution of j-day-return

Simple analysis on the distribution of j-day-return (for j=25, 50, 225) after crash signals also shows that there is no obvious shift-to-the-left effect in the distribution.

In figure 5.3, the histogram for j-day-return (for j=25, 50, 225) after signals generated from model bubble warnings using translation rule 3 are plotted. Histogram of j-day-return after signals generated using translation rule 1 and method 2 are reported in Appendix 3.

![Figure 5.3: Histogram for j-day-return (for j=25, 50, ..., 225) after signals generated from model bubble warnings using translation rule 3.](image-url)
In figure 5.4, the median of j-day-return distributions after crash signals are plotted. Solid line and dashed line correspond to model signals and naïve signals respectively.

![Figure 5.4](image)

**Figure 5.4** Plot of median of j-day-return distributions over j for model crash signals compared with naïve crash signals. The horizontal solid line indicates the level of 0.

From the plot of median, minimum and maximum of j-day-return distributions, it can be seen that:

- The median of j-day-return distributions after model crash signals 1 and 3 are more to the right than that after naïve crash signals;
- The median of j-day-return distributions after model crash signals 2 fall below zero slightly for j larger than 125.

In general, there is no obvious shift-to-the-left effect for return distributions after any of the three crash signals.
5.3 Conclusions

Analyses of out-of-sample test results show that:

1. Success Ratios of crash signals translated from model bubble warnings are low due to the varying relationships between the timing of crash and timing of bubble warnings and the unforeseeable behaviors of asset price after bubble warnings.
2. The fact that the Success Ratios of model crash signals are lower than that for naïve crash signals shows that there is no statistical significance of the crash prediction indicated by those crash signals.
3. There is no obvious shift-to-the-left effect on the j-day-return distribution after crash signals.

Therefore, detection of a bubble-like behavior in the asset price (whether it corresponds to the existence of a bubble is not testable) has little implication on the future behavior of the asset price.
Chapter 6

Summary and Conclusions

In this Chapter, we first make a brief summary of researches that have been carried out in this thesis, and then we draw the final conclusion based on research results by answering the research question we formulated in Chapter 1.

6.1 Summary

In this thesis, we have analyzed the problems with the bubble detection method based on the Stochastic Critical Time (SCT) model put forward by Lin and Sornette in 2010. By mending problems that made Lin & Sornette’s results not reproducible, we proposed a revision of the bubble detection method. What’s more, we designed a test of bubble warnings and carried out the test on 93 asset price series of four different asset classes.

1. Problem analysis of the bubble detection method reported by Lin and Sornette

The SCT model assumes that the bubble price increases in a super-exponential way and the critical time, which is the most probable time for a crash to happen, can be expressed as a function of calendar time and asset price with three unknown parameters. When the crash is not far away, as time progresses, the critical time is assumed to form a stationary time series that fluctuates around its mean without straying away. Therefore, a bubble detection method is constructed by searching for a parameter combination which can generate stationary critical time series. If such parameter combination exists, according to the model assumptions, there is a developing bubble in the asset.

The bubble detection method designed by Lin and Sornette suffers from several problems. The procedure they used to estimate the three unknown parameters suffers from the following problems:

- Maximized the objective without a lower bound;
- Adopted a searching algorithm that is not suitable for solving the optimization problem;
- Failed to cover the whole solution space which makes the result searching method dependent.

What’s more, the fixed time window length taken for a given asset is derived by observing the optimal length of historical bubble. However, when applied in real-time, this information is not available.
2. **Revision of the bubble detection method**

To revise the bubble detection method designed by Lin and Sornette, we did the following:

- Reformulated the optimization problem by setting the measure of goodness-of-fit as the optimization objective and tests that ensure the mean-reverting property of critical time series as restrictions;
- Used the initial point search method to narrow down the range of each parameter so that a dense parameter grid can be constructed given a certain computational power.
- Solved the reformulated optimization problem by first identifying the feasible set of parameter combinations and then selecting the optimal parameter combination that optimizes the objective if the feasible set is non-empty;
- For each inspection time point, constructed time window of different lengths, and if bubble warnings are flagged within at least one time window, flag a bubble warning at the inspection time point.
- Selected stronger bubble warnings at inspection time points which are either more persistent or have faster price increase than most of the others.

The revised bubble detection model is capable of identifying super-exponential growth and generating bubble warnings not long from the crash time for the in-sample data set.

3. **Test of bubble warnings**

Lin and Sornette did not develop a test on the bubble warnings, leaving their results untested. However, the evaluation of the model results is important to provide information on the significance and contribution of the method to both the academic world and the financial industry.

Therefore, we designed such a test by doing the following:

- Designed three Translation Rules to convert bubble warnings into crash signals. Due to the varying relationship between the appearance of bubble warnings and the timing of a crash, there is no perfect Translation Rule which can find the optimal time to flag a crash signal.
- Designed a test procedure by simulating the way of receiving and reacting to bubble warnings in real-time. Once a crash signal is flagged, enter a crash mode; if a huge loss happens during the crash mode, exit the crash mode and count the crash signal as a successful one; otherwise, exit the crash mode after 250 trading days which is the maximum length of the crash mode. During the crash mode, no bubble warnings are received.
- Compared the success ratio of signals translated from model bubble warnings using Translation Rule 1 and 3 with that from naïve bubble warnings, which are flagged at every inspection time point.
- Reported the distribution of j-day-return after crash signals to see if there is an obvious shift-to-the-left effect.
6.2 Conclusions

The research question that we formulated in Chapter 1 was:

- Is it possible to make crash prediction using bubble warnings generated by the SCT bubble detection method in real-time?

Judging from the low success ratios of the three types of model crash signals, which are lower than those for naïve crash signals, the bubble warnings generated by the Model have no significance in providing crash signals which indicate the timing of forthcoming crashes.

Also, judging from the analysis of return distributions after model crash signals, there is no significantly negative return after such signals, which provides further evidence that the crash prediction based on bubble detection is unsuccessful.

By analyzing the behavior of price series after the identification of a bubble-like behavior by the Model, we investigated the causes that lead to the low success ratios of model crash signals.

**Why is it impossible to predict a crash based on the bubble detection method?**

- Three possible behaviors of the asset price after a cluster of bubble warnings

  In Chapter 1, we mentioned that, if there is a bubble, it would end either via a crash or a deflation. However, from observation of price behavior after the Model identifies a bubble-like behavior, it’s found that there are three (instead of two) possible asset price behaviors: crash immediately, deflate over a longer time period, sustain without crash/deflation.

  The observation is not in consistency with our assumption for the bubble behavior when it disappears. One reason for the discrepancy is that: the bubble detection method detects bubble-like price behavior, and whether a bubble-like price behavior corresponds to a bubble is difficult to test.

  We didn’t test whether a bubble-like price behavior corresponds to a bubble not only because the difficulty in obtaining the fundamental value of an asset makes it not easy to test the existence of a financial bubble, but also that we are more concerned about the consequence of a bubble-like behavior than the existence of a bubble itself.

  The observation of the three possible price behaviors after bubble-like price pattern explains the low success ratios of crash signals to a certain extent and also indicates that the existence of bubble-like price pattern has little implication on future price pattern.

- Difficult to determine the timing of crash/deflation

  Another fact that leads to the low success ratios of model crash signals is that, even when a crash/deflation happens after a bubble warning generated by the Model, it’s still difficult to provide a timing of the crash.
In Chapter 3, we’ve already observed that the estimated critical time moves forward as the inspection point moves forward, which makes the estimated critical time generated by the model unreliable. Instead of looking at estimation of critical time, we looked at stronger warnings which are flagged not long before the crash/deflation from the in-sample warning results. However, the varying relationships between the timing of crash and the appearance of bubble warning clusters – crash/deflation can happen in the middle of, immediately after or long after a warning cluster – makes it difficult to develop a universal Translation Rule from bubble warnings to crash signals.

Therefore, detecting a bubble-like behavior has little indication on the timing of a crash/deflation.

In sum, the bubble detection method is capable of identifying bubble-like (super-exponential growth) price pattern and the model fits the historical price trajectory quite well in hindsight. However, fitting the model into historical data and using it to generate prediction in real-time are two different things and the latter is much more difficult. From test results, we conclude that bubble warnings generated by the Model have no significance in predicting a crash/deflation.
Reference


Appendix 1
Derivation of the LPPL model

In [Johansen, Ledoit, Sornette, 2000], the LPPL model was introduced using concepts and tools from natural science and the Rational-Expectation (RE) model from social science. Concepts and tools were borrowed from statistical physics and studies of complex system to describe how individual behavior can affect the aggregate behavior of a system; and the RE model was used to incorporate the positive feedback loop between the investors’ expectation and market price, where the expectation of investors about the market is an important ingredient that distinguishes the social science from the natural science. Below is a brief introduction on the relationship between bubble price and the crash risk represented by the hazard rate and the derivation of LPPL model by studying the behavior of hazard rate in complex systems.

A.1.1 Bubble price and hazard rate

The dynamics of a pure speculative bubble are modeled by:

\[ dp(t) = \mu(t)p(t)dt - \kappa p(t)dj(t) \tag{2.1} \]

where \( dj(t) \) is a jump term, which is zero before the crash, and jumps to one when crash happens. \( \kappa \) is the amplitude of the crash.

Let \( Q \) be the risk neutral measure and suppose that the risk free rate is zero. Under the risk neutral measure, for there being no arbitrage, we have:

\[ E_Q[dp(t) \mid F_t] = E_Q[p(t + dt) - p(t) \mid F_t] = 0 \tag{2.2} \]

Substituting the expression for \( dp(t) \) in (2.1) into (2.2), we have:

\[ \mu(t)p(t)dt = \kappa p(t)E_Q[j(t + dt) - j(t) \mid F_t] \tag{2.3} \]

Define the hazard rate \( h(t) \) to be the probability for a crash to happen in a unit time \( (dt) \) given that it has not occurred up to time \( t \). By definition of \( h(t) \) and \( dj(t) \), we have that:

\[ E[ j(t + dt) - j(t) \mid F_t] = h(t)dt \tag{2.4} \]

Combining (2.3) and (2.4), we have that:

\[ \mu(t) = \kappa h(t) \tag{2.5} \]

Therefore, conditioned on that no crash has happened yet, (2.1) can be rewritten as:

\[ dp(t) = \kappa h(t)dt \tag{2.6} \]

Thus, we can arrive at the relationship between hazard rate and the market price of an asset:

\[ \log[\frac{p(t)}{p(t_0)}] = \kappa \int_{t_0}^{t} h(t')dt' \tag{2.7} \]
This result is in consistency with observation: the risk embodied by the hazard rate drives the price of the bubble to soar. Therefore, to study the price process, it’s necessary to study the behaviour of the hazard rate.

A.1.2 Behavior of hazard rate

Consider the hazard rate as a characteristic feature of a complex system (e.g. financial market) which captures the stability of the system before its critical times (e.g. crash time). The bigger it is, the less stable the system is, and thus more susceptible to small external disturbances.

The behaviour of hazard rate depends on the structure of the network in a complex system, which refers to how the individual in such a system is connected with others and how information passes through the system would affect the aggregate behaviour of hazard rate.

In [Johansen, Ledoit, Sornette, 2000], the ‘Hierarchical Diamond Lattice’ model (see figure 2.1) was selected as the model for network structure in the financial market. It assumes that the market has different levels of investors and the least-connected investors have \( 2^{p-1} \) times fewer neighbours than the most connected ones, where \( p \) is the number of hierarchy in the network. Solution to such a model was given by Derrida et al. (1983), which leads to the expression for \( h(t) \)

\[
h(t) \approx B_0(t_c - t)^{-\alpha} + B_1(t_c - t)^{-\alpha} \cos[\omega \log(t_c - t) + \psi].
\]  

(2.7)

And thus from expression (4) and (5), one can derive that

\[
\log[p(t)] = \log(p_c) - \frac{\kappa}{\beta} \{B_0(t_c - t)^\beta + B_1(t_c - t)^\beta \cos[\omega \log(t_c - t) + \phi] \}.
\]  

(2.8)

Renaming parameters, the model appeared more often as

\[
\log[p(t)] = A + B(t_c - t)^\beta \{1 + C \cos[\omega \log(t_c - t) + \phi] \}.
\]  

(2.9)

In sum, the LPPL model was built by first deducing the relationship between hazard rate for the crash in an asset to happen with the expected return of the asset using the rational expectation theory. Then, it relates the hazard rate in the financial market with the characteristic feature of a complex system, which has already been studied in statistical physics, and borrows the mathematical expression here to represent the hazard rate. Lastly, by integrating the differential equation conditioned on that the crash has not happened yet, the LPPL model for asset price before the crash is given.
Figure A.1 The first three steps of constructing the ‘Hierarchical Diamond Lattice’. Quoted from [Johansen, Ledoit, Sornette, 2000], Figure 1. Each time the line between two points are expanded to a diamond, and the hierarchy in the network increases by one. Therefore, the number of neighbor points for points on the $p$th hierarchy would be 2 times of on the $(p-1)$th hierarchy. As in the figure, points in the third step that have already existed in the second one have four neighbors; while those new points introduced in the third step have only two.
Appendix 2

Exponentially Weighted Moving Average estimate of daily volatility

The EMWA estimate of volatility is given by:

\[ \sigma_t = \sqrt{\lambda (\sigma_{t-1})^2 + (1-\lambda)r_{t-1}^2} \]  

(A.1)

where \( \lambda = 0.95 \) is taken for daily data, \( \sigma_t \) is usually obtained by the standard deviation of return in a earlier sample period, and \( r_{t-1} \) is the return on the previous trading day.

There are two reasons to use the EWMA estimate of the daily volatility:

i) Compared to the volatility estimate obtained by calculating the standard deviation of daily-return in the whole sample period, it is a real-time volatility estimate that avoids the problem of using the future information that is not available at time point \( t \).

ii) Compared to the standard deviation of the return in a rolling time window of \( N \) days, it is better in that it uses all the information in the past and the further away the information is in the history, the less weight it is assigned. In this way, the more recent information (more relevant) is given a higher weight, and it avoids the abrupt change in the volatility estimate when an extreme event moves out of the estimation window.

Figure A.1 gives a plot of weight assigned to the square of return 1 to 20 trading days before the current time point.

![Figure A.2 Illustration of weight assigned to square of daily return.](image-url)
Appendix 3
Model Signals 2 and 3 for the three in-sample test series

Figure A.3 Model Signals 2 for the three in-sample test series.

Figure A.4 Model Signals 3 for the three in-sample test series.
Appendix 4: Out-of-Sample Warning Results (Figure A.5)
Appendix 5

Histograms of j-day-normalized return after Crash Signals 1 and 2

Figure A.6: Histogram for j-day-return (for j=25, 50, …, 225) after Crash Signals 1.
Figure A.7: Histogram for j-day-return (for j=25, 50, … , 225) after Crash Signals 2.