Baryon asymmetry generation using Affleck-Dine dynamics in the MSSM

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November 19, 2013
Abstract

The Affleck-Dine mechanism is a mechanism that uses flat directions in supersymmetry to generate an asymmetry in particle numbers. Supersymmetric theories contain directions in scalar field space that have a scalar potential that is identically zero at the renormalizable level. During inflation these flat directions get a large vacuum expectation value because of the flatness of the potential and the extremely high energy density.

Because of this supersymmetry breaking operators become important and must be taken into consideration. In both gravity and gauge mediated supersymmetry breaking there are operators that break baryon and lepton number. If these operators have a phase difference or there is a phase difference between the initial and final state the result will be a current that corresponds to the charge density of the field, which in this case is a lepton and/or baryon number. The charge of the field can be extended to include some kind of dark matter particle, resulting in a tight relationship between the amount of dark matter that is produced and the amount of visible matter.

In this thesis I did a numerical analysis of Affleck-Dine dynamics in both gravity mediated supersymmetry breaking and gauge mediated supersymmetry breaking.
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Chapter 1

Introduction

Measurements from the Planck satellite[1] have found that our universe consists of 4.9% ordinary matter, 26.8% dark matter and 68.3% dark energy. This means that we have a good understanding of 4.9% of the universe, some understanding of 26.8% and mostly theoretical work about 68.3% of our universe.

Ordinary matter is understood very well with the use of the extremely successful Standard Model of Particle Physics (Standard Model or SM) for which recently the existence of the Higgs boson has been confirmed.[2] Dark matter on the other hand has been a bit more troublesome, and even though we have a number of models describing what it might be, we still have no real measurements pointing in one certain direction.

One of the better motivated candidates at the moment is that it is the Lightest Supersymmetric Particle (LSP), but until we actually measure a supersymmetric particle we can not be sure. This is also the reason this form of matter is called dark matter, because we can not see it with ordinary light, we have only detected its existence through gravitational interactions. Even though there is excellent agreement of the Theory of General Relativity (GR) with experiments we can not be quite certain that what is measured are indeed some form of particles, all we know is that it is a fluid with (almost) no pressure $p \approx 0$.[21]

And then there is also Dark Energy, which is the remaining 68.3% of the total energy density and is also only measured through gravitational interactions. There is more theoretical understanding about dark matter than dark energy, but there is a lot of theoretical work going on to figure out what it might actually be comprised of and it seems to be responsible for the accelerated expansion of the universe.

So we understand about 5% of our universe and the rest is still being worked out by
Chapter 1. Introduction

physicists and astronomers. One surprising observation — considering the results from the Standard Model — is the fact that there actually is any matter in our universe. When we look at particles and conserved quantities it turns out that electric charge is exactly conserved and baryon number is conserved almost exactly at $T = 0$. Meaning that in every interaction where a particle is created, an anti-particle is also created, leaving the difference in particle numbers, $n_{\text{particle}} - n_{\text{anti-particle}} = 0$, at zero. Then how can there be any particles — particles that we are made out of! — at all? Should they not have been annihilated quite short after the Big Bang?

The answer to this problem is called Baryogenesis, literally meaning the creation of baryons. Although there are a number of theoretical explanations that deal with this problem, all of them have to predict (or postdict) the Baryon asymmetry parameter $\eta$, which tells us how much baryons there are in relation to anti-baryons.

$$\eta = \frac{n_b - n_{\bar{b}}}{n_\gamma},$$  \hspace{1cm} (1.1)

where $n_\gamma$ is the number of photons in the universe. This has been measured and has a value of

$$\eta = 6.1(3) \times 10^{-10}$$  \hspace{1cm} (1.2)

So very roughly for every proton there are about ten billion photons in the universe.

We will focus primarily on a mechanism that can create a non-zero baryon number through the Flat Directions (FDs) of supersymmetric potentials (derived from superpotentials). Therefore we will have to incorporate supersymmetry (SUSY) in our model.

The mechanism used is called the Affleck-Dine mechanism (AD) and has been proposed by the aforementioned authors in 1985 as a mechanism to create a non-zero baryon number, so that there will eventually be a difference between the amount of baryons and anti-baryons in our universe. It uses coherent scalar fields related to the FDs to create a potential in which a rolling field can build up an angular momentum in its internal phase space, which is then related to a baryon number.

The flat direction allows the field to get a large vacuum expectation value (vev), which

---

1Baryogenesis usually includes leptons too, but since the mass of leptons is so much less than that of baryons we often neglect it. For more specific theoretical work on the creation of leptons one could look into Leptogenesis. Even then we still are not sure if there is a lepton asymmetry since we haven’t measured the neutrino background.
will then roll down the potential. During this rolling the potential changes shape since it is time dependent and this in turn causes the phase of the field to change (CP violation), creating an angular velocity, resulting in an angular momentum. This angular momentum is related to a baryon number and we end up with a non-zero number if the field is rotating. At the end the vev is small since the universe has expanded and cooled, and the mechanism ”shuts off” and the CP violation stops, resulting in a conserved baryon number.

The initial conditions are such that the rolling of this field starts during the inflationary epoch of the universe, the stage very early in the universe when it increased at least 60 e-folds in size in a very short time. This baryon number is then conserved at a late stage after inflation and the field decays to regular particles with a baryon number, creating a nett difference between baryons and anti-baryons. In the AD mechanism we can also generalize to a dark particle number $X$ that will lead to a relic density of dark matter. Important to note is that the interaction between visible and dark matter is only significant in the early hot universe and that this interaction is absent in the present universe, otherwise dark matter could decay into visible matter efficiently today.

In this thesis we are going to investigate how the Affleck-Dine mechanism depends on its parameters. We will do this by numerically analysing the created asymmetry as a function of the parameters of the model.
Chapter 1. Introduction
Chapter 2

An introduction to Supersymmetry

2.1 Introduction

Supersymmetry was first proposed in the 60’s and the first versions did not work very well compared to what we have today.\textsuperscript{11} It was a mixture of an internal symmetry and external symmetry, relating states to spacetime like we know it today. In the 70’s it was rediscovered and when people began working on it they realized it was a symmetry between fundamental fields in quantum field theory (QFT). It relates fermionic fields and bosonic fields and was found out that it is also a necessary ingredient of string theory to get fermionic strings in the theory that — back then — just contained bosonic strings.\textsuperscript{12}

Motivation for looking into supersymmetry is threefold.\textsuperscript{9} Firstly, the higgs mass divergencies in QFT cancel very nicely, leaving us with a light mass for the Higgs boson. Secondly the running couplings do not unify in the SM, but in the MSSM they do unify at some large energy scale (GUT scale) so that there is actually only one fundamental force (minus gravity) at high energies. And lastly, we want dark matter candidates, which are automatically incorporated using the existence of superpartner particles. Every particle in your model gets a superpartner particle, which has the same properties except for the spin statistics and the lightest SUSY particle is stable because of R-parity.

The first physical model in particle physics that used supersymmetry properly was the minimal supersymmetric standard model (MSSM), which is the simplest extension of the standard model. It was propsed to solve the hierarchy problem, which is exactly the question why gravity is so much weaker than the other forces, or in the physical sense, why the Higgs mass is so much smaller than the Planck mass, \( m_H \ll m_{Pl} \). Every particle in the SM gets a superpartner particle in this theory, which is a particle with the exact same properties, except that it has the opposite statistics.
For the Affleck-Dine mechanism to work we need scalar fields carrying charges, so we must introduce supersymmetry, because it contains so called flat directions (FDs). We need scalar fields to construct flat directions, but the Higgs potential is the only scalar field in the SM and it is definitely not flat. These flat directions are needed to get a non-zero baryon number.

To construct SUSY we extend the known models that use commuting (bosonic) variables and fields with anti-commuting (fermionic) variables and fields. By defining the transformations in such a way that bosonic fields transform into fermionic fields and vice versa we get a relation between these two different kinds of fields. One can start from the usual Standard Model Lagrangian and work from there to extend it to include supersymmetry. In this way you see how the extension works and how it incorporates all the necessary mathematics. You can also start with the Superfield formalism, in which you use a more geometric approach to incorporate supersymmetry in the theory. We are going to do the latter, but the result should be the same either way.

2.2 Superfield formalism

2.2.1 Grassmann variables

One important ingredient of SUSY are anticommuting numbers also known as Grassmann variables or Grassmann numbers. These variables are characterized by the following behaviour:

\[ \psi \phi = -\phi \psi, \]  
\[ \psi^2 = \psi \psi = -\psi \psi = 0. \]

The square of a Grassmann variable is zero because of the anticommutativity. Along with this we have to introduce the anticommutator \( \{.,.\} \) given by:
2.2. Superfield formalism

\[ \{ \psi, \phi \} = \psi \phi + \phi \psi \quad (2.4) \]

instead of the usual commutator brackets. Grassmann variables are also known as Grassmann odd or fermionic, while regular variables are called Grassmann even or bosonic, because they have the same behaviour as bosonic and fermionic fields respectively. Another way to see this is the fact that even times even should result in an even function, odd time even gives odd and odd times odd is again an even function:

If \( x, y \in \mathbb{R} \) and \( \theta, \psi \in G \)

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where \( G \) is the set of all Grassmann numbers.

This may not be completely clear yet, but below we will give an example in which we show that an odd differentiation operator working on a variable gives a bosonic object and in appendix A I show that contracting two fermionic variables gives us a bosonic object.

### 2.2.2 Derivatives and integrals

Now if one has a function \( f \) of a fermionic variable \( \psi \), and we power expand this function in \( \psi \) it will always terminate because of identity \(2.2\), \( \psi^2 = 0 \).

\[ f(\psi) = f_0 + f_1 \psi + f_2 \psi^2 + ... = f_0 + f_1 \psi. \quad (2.6) \]

So we will always get a function linear in \( \psi \). The derivative is defined as follows:

\[ \frac{df}{d\psi} = f_1. \quad (2.7) \]

Where the differential operator works from the left, and is also anticommuting:

\[ \frac{d(\psi'\psi)}{d\psi} = -\frac{d(\psi'\psi)}{d\psi} = -\frac{d\psi}{d\psi} \psi' = -\psi'. \quad (2.8) \]

The fact that the differential operator is odd can also be seen from
Chapter 2. An introduction to Supersymmetry

\[
\frac{d(-\psi\psi')}{d\psi} = -\frac{d\psi}{d\psi}\psi' = -\psi' = -\psi' \frac{d\psi}{d\psi}. \tag{2.9}
\]

The derivative of the variable is grassmann even, meaning that the differential operator has to be odd to result in an even combination.

Defining integration with respect to \(\psi\) is a little bit less intuitive than differentiation,

\[
\int d\psi = 0, \quad \int d\psi \psi = 1. \tag{2.10}
\]

Meaning that the integral of a function \(f\)

\[
\int d\psi f(\psi) = f_1 \tag{2.11}
\]

is equivalent to the derivative (2.7). Now partial integration can be found by virtue of (2.10):

\[
\int d\psi \frac{df}{d\psi} = \int d\psi f_1 = 0 \tag{2.12}
\]

And the Dirac delta can be used in the usual way:

\[
\int d\psi \delta(\psi - \psi') f(\psi) = f(\psi') \tag{2.13}
\]

But note that we can also use the identity

\[
\delta(\psi - \psi') = \psi - \psi'. \tag{2.14}
\]

It follows that

\[
\int d\psi \delta(\psi - \psi') f(\psi) = \int d\psi (\psi - \psi')(f_0 + f_1\psi) = \\
\int d\psi (\psi f_0 + \psi f_1 - \psi' f_0 - \psi' f_1\psi) = \int d\psi (\psi f_0 - \psi' f_0 + \psi' f_1) = \\
f_0 + \psi' f_1 = f(\psi'). \tag{2.15}
\]
This works because $\psi^2 = 0$ and $\int d\psi = 0$, so we have an explicit function for the Dirac delta.

### 2.2.3 Coordinates and Superfields

Now we are going to add an index to the Grassmann variables so we can contract them and get even functions that have fermionic variables. It is equivalent to Weyl fermions, for further reading see the conventions appendix (A). In the superfield formalism we begin with the usual bosonic spacetime coordinates $x^\mu$: t, x, y, z to which we have to add fermionic coordinates $\theta^\alpha$ and $\theta^\dagger_\dot{\alpha}$ that have a Weyl index:

$$x^\mu, \theta^\alpha, \theta^\dagger_\dot{\alpha}, \quad \alpha = 1, 2 \quad \dot{\alpha} = 1, 2.$$  \hspace{1cm} (2.16)

Where $\theta$ and $\theta^\dagger$ are spinors and we use the two-component Weyl spinor notation as can be seen in appendix A. These anticommuting variables are two-dimensional complex spinors where we differentiate between them with a dot on the index of the conjugate.

Derivatives of anticommuting coordinates are now defined in the same way as the previous section, albeit with an index:

$$\frac{\partial}{\partial \theta^\alpha} \theta^\beta = \delta^\beta_\alpha, \quad \frac{\partial}{\partial \theta^\alpha} \theta^\dagger_\dot{\beta} = 0, \quad \frac{\partial}{\partial \theta^\dagger_\dot{\alpha}} \theta^\dagger_\dot{\beta} = \delta^\dot{\beta}_\dot{\alpha}, \quad \frac{\partial}{\partial \theta^\dagger_\dot{\alpha}} \theta^\beta = 0.$$  \hspace{1cm} (2.17)

And we find for example:

$$\frac{\partial}{\partial \theta^\alpha} \theta^\alpha = \frac{\partial}{\partial \theta^\alpha} \theta^\mu \epsilon^\mu_{\alpha\nu} \theta^\nu = \delta^\alpha_\nu \epsilon^\mu_{\alpha\nu} \theta^\nu - \theta^\mu \epsilon^\mu_{\alpha\nu} \delta^\nu_\alpha =$$

$$\epsilon_{\alpha\nu} \theta^\nu - \theta^\mu \epsilon_{\mu\alpha} = \epsilon_{\alpha\nu} \theta^\nu + \epsilon_{\alpha\nu} \theta^\mu = 2 \theta_\alpha$$  \hspace{1cm} (2.18)

For integration in superspace we have to define a line element, which goes as

$$d^2\theta = -\frac{1}{4} d\theta^\alpha d\theta^\beta \epsilon_{\alpha\beta}, \quad d^2\theta^\dagger = -\frac{1}{4} d\theta^\dagger_\dot{\alpha} d\theta^\dagger_\dot{\beta} \epsilon^{\dot{\alpha}\dot{\beta}}.$$  \hspace{1cm} (2.20)

And results in

$$\int d^2\theta (\theta\theta) = 1, \quad \int d^2\theta^\dagger (\theta^\dagger\theta^\dagger) = 1.$$  \hspace{1cm} (2.21)
Chapter 2. An introduction to Supersymmetry

The algebra associated with supersymmetry — which will be introduced later — has multiple irreducible representations, called supermultiplets. These multiplets contain single-state particles. Different members of the same supermultiplet are called superpartners. In this formalism the supermultiplets are functions of the superspace coordinates which means that translations in this space are equivalent to supersymmetry transformations. This is a generalization of General Relativity (GR). The result yields a theory that is inherently supersymmetric invariant so we do not have to check whether certain transformations are allowed by supersymmetry, we just shift through the superspace!

A general superfield $\Phi$ is then a function of these coordinates, and can therefore be power expanded in the given coordinates. Since there are two independent components of $\theta^\alpha$ and similar for $\theta^{\dagger\alpha}$ we can get at most a combination of four fermionic variables: two $\theta^\alpha$’s and two $\theta^{\dagger\alpha}$’s.

The most general superfield $\Phi$ is thus given by:

$$
\Phi(x, \theta, \theta^{\dagger}) = a + \theta\xi + \theta^1\chi^{\dagger} + \theta\theta b + \theta^1\theta^1c + \theta^1\bar{\sigma}^\mu\theta v_\mu + \theta^1\theta^1\theta\eta + \theta\theta\theta^1\zeta^{\dagger} + \theta\theta\theta^1\theta^1d. \quad (2.22)
$$

And we see that it is a function of eight bosonic fields, $a, b, c, d$ and $v^\mu$ and eight fermionic fields, given by two-dimensional spinors $\xi, \chi^{\dagger}, \eta, \zeta^{\dagger}$, so we have 16 degrees of freedom.

And now we can take a look at the coefficients of $\Phi$ by integrating over the coordinate.

$$
\int d^2\theta \Phi(x, \theta, \theta^{\dagger}) = b(x) + \theta^1\zeta^{\dagger}(x) + \theta^1\theta^1d(x), \\
\int d^2\theta \Phi(x, \theta, \theta^{\dagger}) = c(x) + \theta\eta(x) + \theta\theta d(x), \\
\int d^2\theta d^2\theta^{\dagger}\Phi(x, \theta, \theta^{\dagger}) = d(x).
$$

Dirac deltas in two dimensions are extended in an equivalent way, with contractions implied:

$$
\delta^{(2)}(\theta - \theta') = (\theta - \theta')(\theta - \theta'), \quad \delta^{(2)}(\theta^{\dagger} - \theta^{\dagger}) = (\theta^{\dagger} - \theta^{\dagger})(\theta^{\dagger} - \theta^{\dagger}) \quad (2.23)
$$

And result in the expected forms:

$$
\int d^2\theta^{\dagger}\delta^{(2)}(\theta^{\dagger})\Phi(x, \theta, \theta^{\dagger}) = \Phi(x, \theta, 0) = a(x) + \theta\xi(x) + \theta\theta b(x). \quad (2.24)
$$
\[ \int d^2 \theta \delta^{(2)}(\theta) \Phi(x, \theta, \theta^\dagger) = \Phi(x, 0, \theta^\dagger) = a(x) + \theta^\dagger \chi(x) + \theta \theta^\dagger c(x). \] (2.25)

And of course partial integration holds through

\[ \int d^2 \theta \frac{\partial}{\partial \theta^\alpha} f(x, \theta, \theta^\dagger) = 0, \quad \forall f(x, \theta, \theta^\dagger) \] (2.26)

because there can be only two \( \theta \)'s and two \( \theta^\dagger \)'s at most in any function \( f \), and by deriving it will leave you with the product of only one \( \theta \), which then vanishes by integrating over \( d^2 \theta \).

### 2.3 Operators

#### 2.3.1 Supersymmetry generators

Now that we have defined all the basic operations, we can now write down the operator that defines supersymmetry transformations in superspace.

\[ Q_\alpha = i \frac{\partial}{\partial \theta^\alpha} - (\sigma^\mu \theta^\dagger)_\alpha \partial_\mu, \quad Q^\dagger_\dot{\alpha} = -i \frac{\partial}{\partial \theta^\dagger_\dot{\alpha}} + (\theta \sigma^\mu)_\dot{\alpha} \partial_\mu \] (2.27)

The supersymmetry transformation for an infinitesimal \( \epsilon, \epsilon^\dagger \) is now given by

\[
\sqrt{2} \delta \Phi = -i(\epsilon Q + \epsilon^\dagger Q^\dagger) \Phi = \left( \epsilon^\alpha \frac{\partial}{\partial \theta^\alpha} + \epsilon^\dot{\alpha} \frac{\partial}{\partial \theta^\dagger_\dot{\alpha}} + i(\epsilon \sigma^\mu \theta^\dagger + \epsilon^\dagger \bar{\sigma}^\mu \theta) \partial_\mu \right) \Phi \\
= \Phi(x^\mu + i \epsilon \sigma^\mu \theta^\dagger + i \epsilon^\dagger \bar{\sigma}^\mu \theta + \epsilon, \theta^\dagger + \epsilon^\dagger) - \Phi(x^\mu, \theta, \theta^\dagger) \] (2.28)

The reason this holds is because the generator of translations truncates at order one. In general, an operator that generates a transformation is given by the exponent of the generator. A short analogue:

\[ Q = \partial_x, \quad a Q f(x) = a f'(x). \] (2.29)

Where \( a \) is an infinitesimal parameter. Now the complete transformation is the exponential of this generator,
Chapter 2. An introduction to Supersymmetry

\[ T_a = e^{aQ} = (1 + aQ + \frac{1}{2}a^2Q^2 + ...) \]  

(2.30)

Letting this transformation work on a function \( f \) of \( x \),

\[ T_a f(x) = e^{aQ} f(x) = f(x) + af'(x) + \frac{1}{2}a^2f''(x) + ... = f(x + a). \]  

(2.31)

And we see that this results in an infinitesimal translation \( a \). The point is now that power expansions of Grassmann valued functions are always linear, meaning that the operator itself (plus one) is the generator and also gives the transformation. The supersymmetry transformation we defined above is \( e^Q = 1 + Q \), or \( Q = e^Q - 1 \). So the result is that a supersymmetry transformation on \( \Phi \) results in a translation in superspace.

Going back to the supersymmetry transformation \( Q \), we find the defining relation in supersymmetry, the anticommutator of \( Q \):

\[ \{Q_\alpha, Q^\dagger_\beta\} = 2i\sigma^\mu_{\alpha\beta} \partial_\mu = -2\sigma^\mu_{\alpha\beta} p_\mu; \]

\[ \{Q_\alpha, Q_\beta\} = 0, \quad \{Q^\dagger_\alpha, Q^\dagger_\beta\} = 0, \]  

(2.32)

where \( p_\mu \) is the usual momentum operator \(-i\partial_\mu\). This is a closed algebra and moreover notice that this formula relates an internal symmetry (between fermions and bosons) with an external symmetry: spacetime translations.

2.3.2 Covariant SUSY derivative

Now just as in QED where we have to define a covariant derivative that leaves the Lagrangian invariant, we need a covariant derivative that incorporates the supersymmetry transformations. The spacetime derivative \( \partial_\mu \) may commute with \( Q \) and \( Q^\dagger \), but \( \frac{\partial}{\partial \theta^\dagger} \) and \( \frac{\partial}{\partial \bar{\theta}^\dagger} \) certainly do not. So we need a new derivative and it turns out to be:

\[ D_\mu = \frac{\partial}{\partial \theta^\mu} - i(\sigma^\nu \theta^\dagger)_{\alpha} \partial_\mu, \quad D^\mu = -\frac{\partial}{\partial \bar{\theta}^\alpha} + i(\bar{\theta}^\dagger \sigma^\nu)_{\alpha} \partial_\mu \]

\[ D^\dagger_\alpha = \frac{\partial}{\partial \theta^\dagger_\alpha} - i(\sigma^\nu \theta)_{\bar{\alpha}} \partial_\mu, \quad D_\alpha = -\frac{\partial}{\partial \bar{\theta}^\alpha} + i(\theta \sigma^\nu)_{\bar{\alpha}} \partial_\mu \]

which looks an awful lot like the generators \( Q \) and \( Q^\dagger \). But we can check that
2.4. Chiral and vector superfields

\{Q_\alpha, D_\beta\} = \{Q^\dagger_\alpha, D^\dagger_\beta\} = \{Q_\alpha, D^\dagger_\beta\} = \{Q^\dagger_\alpha, D_\beta\} = 0. \tag{2.33}

And in particular

\[
\begin{align*}
\{D_\alpha, D^\dagger_\beta\} &= 2i\sigma^\mu_{\alpha\beta} \partial_\mu = -2\sigma^\mu_{\alpha\beta} p_\mu, \\
\{D_\alpha, D_\beta\} &= 0, \quad \{D^\dagger_\alpha, D^\dagger_\beta\} = 0.
\end{align*}
\tag{2.34}
\]

So it follows from (2.33) that when they work on superfields the resulting object is also a superfield because the $D$-operators commute with the $Q$-operators but among them they generate the same transformation. Now we can use the covariant derivatives to make a Lagrangians in superspace, so we can make a theory that includes supersymmetry.

2.4 Chiral and vector superfields

2.4.1 Chiral superfields

Now we want to find the irreducible representations (irreps) for the algebra (2.32) because the representation we found before (2.22) is not an irrep. It turns out that the irreps are what we call chiral and vector superfields.\[9\]

Chiral supermultiplets are described by the constraints

\[
D^\dagger_\alpha \Psi = 0, \quad D_\alpha \Psi^\dagger = 0. \tag{2.35}
\]

Where $\Psi$ is called chiral (left chiral) and $\Psi^\dagger$ is called antichiral (right chiral). But before continuing we will define the coordinates $y^\mu = x^\mu + i\theta^\dagger \bar{\sigma}^\mu \theta$ and $\bar{y}^\mu = x^\mu - i\theta \sigma^\mu \theta^\dagger$ to make our lives easier. So from now on we have two sets of coordinates that depending on the situation, we change to and from. In the $(y, \theta, \theta^\dagger)$ representation, the covariant derivative becomes:

\[
D_\alpha = \frac{\partial}{\partial \theta^\alpha} - 2i(\sigma^n \theta^\dagger)^\alpha \frac{\partial}{\partial y^n}, \quad D^\alpha = -\frac{\partial}{\partial \theta^\alpha} + 2i(\theta^\dagger \bar{\sigma}^\mu)^\alpha \frac{\partial}{\partial y^\mu},
\]

\[
D^{\dagger\alpha} = \frac{\partial}{\partial \theta^{\dagger\alpha}}, \quad D^\dagger_\alpha = -\frac{\partial}{\partial \theta^{\dagger\alpha}}.
\]
And in the \((y^*, \theta, \theta^\dagger)\) representation we find the covariant derivatives

\[
D_\alpha = \frac{\partial}{\partial \theta^\dagger}, \quad D^\alpha = -\frac{\partial}{\partial \theta}, \quad D^{\dagger\alpha} = \frac{\partial}{\partial \theta^\dagger} - 2i(\sigma^\mu \theta)^\dagger \frac{\partial}{\partial y^\mu}, \quad D_{\dagger\alpha} = -\frac{\partial}{\partial \theta} + 2i(\theta \sigma^\mu)_{\dagger} \frac{\partial}{\partial y^\mu}.
\]

Which means the constraint in equation (2.35) becomes in the new coordinates:

\[
-\frac{\partial}{\partial \theta^\dagger} \Psi(y, \theta, \theta^\dagger) = 0,
\]

which has a solution that is only dependent on \(y\) and \(\theta\). The general solution for \(\Psi\) is given by

\[
\Psi(y, \theta) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta \theta F(y),
\]

where \(F\) is an auxiliary term, \(\phi\) is a boson and \(\psi\) a fermion. This auxiliary term follows from the equations of motion but does not change the content of the theory. This term is the same as in the general derivation of supersymmetry as seen in literature[10]. This gives us immediately

\[
\Psi^*(y^*, \theta^\dagger) = \phi^*(y^*) + \sqrt{2} \theta^\dagger \psi^\dagger(y^*) + \theta^\dagger \theta^\dagger F^*(y^*).
\]

Now that we have found a general solution in \(y\) coordinates, we transform back to \(x\), to find

\[
\Psi = \phi(x) + i \theta^\dagger \sigma^\mu \theta \partial_\mu \phi(x) + \frac{1}{4} \theta \theta \theta^\dagger \theta^\dagger \partial_\mu \partial^\mu \phi(x) + \sqrt{2} \theta \psi(x)
\]

\[
-\frac{i}{\sqrt{2}} \theta \theta \theta^\dagger \sigma^\mu \partial_\mu \psi(x) + \theta \theta F(x),
\]

\[
\Psi^* = \phi^*(x) - i \theta^\dagger \sigma^\mu \theta \partial_\mu \phi^*(x) + \frac{1}{4} \theta \theta \theta^\dagger \theta^\dagger \partial_\mu \partial^\mu \phi^*(x) + \sqrt{2} \theta^\dagger \psi^\dagger(x)
\]

\[
-\frac{i}{\sqrt{2}} \theta^\dagger \theta^\dagger \partial_\mu \theta \partial^\mu \psi^\dagger(x) + \theta^\dagger \theta^\dagger F^*(x).
\]

An important remark we would like to make here is that the product of chiral superfields is also a chiral superfield. One can confirm that the following holds
\[ \Psi_i \Psi_j = \phi_i \phi_j + \sqrt{2} \theta (\phi_i \psi_j + \psi_i \phi_j) + \theta \theta (\phi_i F_j \phi_j - \psi_i \psi_j) \] (2.39)

where we have used the \( y \) representation to keep it compact. We see that this is of the same form as a general chiral superfield, so that the equation \( D^\dagger (\Psi_i \Psi_j) = 0 \). It is evident that we can also do this for all the conjugated fields, and get the equation \( D_\alpha (\Psi_i^\dagger \Psi_j^\dagger) = 0 \). So we can extend the multiplication of superfields up to generic order, and we will get a field that has the same expansion as the general field \( \Psi \) because of equation (2.33).

Another note is that we can construct chiral and antichiral fields using the covariant Laplacian on a general superfield,

\[ \Psi = D^\dagger D^\dagger \Phi = D^{\dagger\alpha} D^{\dagger\alpha} \Phi, \quad \Phi^\dagger = D D \Phi = D^\alpha D_\alpha \Phi. \] (2.40)

Since the chiral covariant derivative is anticommuting, it will disappear at the cubic term (which can also be seen from the anticommutation equations) and a quadratic term is the highest we can get. So we can get a chiral field from a superfield, but the opposite is also true, for any chiral field \( \Psi \) you can write down, there is a corresponding superfield \( \Phi \) that you can find. Moreover, you can even make chiral superfields as a holomorphic function \( W(\Psi_i) \) of other chiral superfields \( \Psi_i \). But since it’s holomorphic, you can not use any conjugated fields.

### 2.4.2 Vector superfields

Moving on to Vector Superfields, these fields are defined by the reality constraint

\[ V = V^\dagger. \] (2.41)

This results in the following relations for the superfield components:

\[ a = a^\dagger, \quad \xi^\dagger = \chi^\dagger, \quad b^\dagger = c, \quad v_\mu = v_\mu^\dagger, \quad \zeta^\dagger = \eta^\dagger, \quad d^\dagger = d, \] (2.42)

and a potential of the form

\[
V(x, \theta, \theta^\dagger) = a(x) + \sqrt{2} \theta \xi(x) + \sqrt{2} \theta^\dagger \xi^\dagger(x) + \theta \theta b(x) + \theta^\dagger \theta^\dagger b^\dagger(x) + \theta \sigma^\mu \theta^\dagger v_\mu(x) + \theta \theta \theta^\dagger \eta^\dagger(x) + \theta^\dagger \theta^\dagger \theta \eta(x) + \frac{1}{2} \theta \theta \theta^\dagger \theta^\dagger d(x)
\]
So from these constraint we see that \( a, v_\mu \) and \( d \) must be real fields and that \( b \) is a complex scalar field and that \( \xi \) and \( \eta \) are complex spinor fields.

Next we define the fields

\[
\eta_\alpha = \lambda_\alpha - \frac{i}{2}(\sigma^\mu \partial_\mu \xi_\alpha), \quad v_\mu = A_\mu, \quad d = \frac{1}{2} D + \frac{1}{4}(\partial_\mu \partial^\mu a), \tag{2.43}
\]

so that we can rewrite the potential as

\[
V(x, \theta, \theta^\dagger) = a + \theta \xi + \theta^\dagger \xi^\dagger + \theta \theta b + \theta^\dagger \theta^\dagger b + \theta^\dagger \bar{\sigma}^\mu \theta A_\mu + \theta^\dagger \theta^\dagger (\lambda - \frac{i}{2} \sigma^\mu \partial_\mu \xi_\alpha) + \theta \theta^\dagger (\lambda^\dagger - \frac{i}{2} \bar{\sigma}^\mu \partial_\mu \xi^\dagger) + \theta \theta^\dagger \theta^\dagger (\frac{1}{2} D + \frac{1}{4} \partial_\mu \partial^\mu a).
\]

Seeing as a vector superfield has to be real, we can also construct them from chiral superfields using \((\Psi + \Psi^\dagger), (\Psi - \Psi^\dagger)\) and \((\Psi \Psi^\dagger)\). Now we can actually construct a gauge transformation and in particular, we define a vector superfield to be invariant under certain transformations in vector superfield space. The simplest supergauge transformation is a \(U(1)\) gauge like in the SM and it is given by:

\[
V \rightarrow V + i(\Lambda - \Lambda^\dagger), \tag{2.44}
\]

where \( \Lambda \) is a chiral superfield and \( i(\Lambda - \Lambda^\dagger) \) is a vector superfield. In practice this comes down to the transformations:

\[
\begin{align*}
a &\rightarrow a + i(\phi - \phi^*), \\
\xi_\alpha &\rightarrow \xi_\alpha - i\sqrt{2} \psi_\alpha, \\
b &\rightarrow b - i F, \\
A_\mu &\rightarrow A_\mu + \partial_\mu (\phi + \phi^*), \\
\lambda_\alpha &\rightarrow \lambda_\alpha, \\
D &\rightarrow D.
\end{align*}
\]

So we see that the vector field \( A_\mu \) can have the "usual" \(U(1)\) gauge transformation from the standard model, but we could make it more complicated, for example an \(SU(2)\) transformation would also suffice this. Moreover, we can now use the supergauge transformation to simplify the vector superfield. The Wess-Zumino gauge for vector superfields is found by gauging \( \phi - \phi^* = ia, \psi_\alpha = -\frac{i}{\sqrt{2}} \xi_\alpha \) and \( F = -ib \). The resulting vector \( V = (0, 0, 0, A_\mu, \lambda, D) \) then gives us the W-Z gauge:
2.4. Chiral and vector superfields

\[ V_{WZ} = \theta^\dagger \sigma^\mu \theta A_\mu + \theta^\dagger \theta^\dagger \theta \lambda + \theta \theta \theta^\dagger \lambda^\dagger + \frac{1}{2} \theta \theta \theta^\dagger \theta^\dagger D. \quad (2.45) \]

In this gauge \( A_\mu \) can be identified as a gauge field with the restriction \( \partial_\mu A^\mu = 0 \) and \( \lambda \) corresponds to a gaugino, the supersymmetric partner of a gauge field. \( D \) is an auxiliary field that can be eliminated from the Lagrangian through the equations of motion. So in this gauge we are left with only physical quantities.

2.4.3 Action and superpotentials

To create an action, we need to find real-valued fields from the chiral- and vector superfields. Of course we have the usual reality condition on action \( S \). To get these real-valued fields from all these supersymmetric objects, we have to integrate out the fermionic coordinates. We do not see fermionic space and therefore it is not physical. Just as complex numbers are a tool to find real measurable quantities, so do we use the grassmann variables as a means to end up with a result that can be interpreted physically.

So we integrate out the fermionic fields in the vector superfield to find

\[ [V]_D = \int d^2 \theta d^2 \theta^\dagger V(x, \theta, \theta^\dagger) = V(x, \theta, \theta^\dagger) \bigg|_{\theta \theta^\dagger \theta^\dagger} = \frac{1}{2} D + \frac{1}{4} \partial_\mu \partial^\mu a \quad (2.46) \]

where the \( \theta \theta^\dagger \theta^\dagger \) means that this operation is equivalent to projecting out this part of \( V(x, \theta, \theta^\dagger) \).

This is called the D-term, and this quantity can also be found through the classic way of building up supersymmetry. Note that the full derivative in the D-term will disappear upon integration.

In the same way we can also find an F-term in the action boundary term. The F-term was included in the chiral superfield, and we can find it explicitly by integrating it in the following way:

\[ [\Psi]_F = \int d^2 \theta \Psi \bigg|_{\theta^\dagger = 0} = \int d^2 \theta d^2 \theta^\dagger \delta^{(2)}(\theta^\dagger) \Psi = F. \quad (2.47) \]

We have to set \( \theta^\dagger \) to zero, otherwise we end up with a few extra terms that are functions of said \( \theta^\dagger \), which is what we want to avoid if we want a physical answer. And in general, we also have a chiral field \( \Psi^\dagger \) where \( F \) is complex. So we can use that \( F + F^* \) is a real field, or in other words
Another way to find \( F \) is to use the following property of \( \Psi \):

\[
[\Psi]_F = \frac{1}{4} DD\Psi \bigg|_{\theta = \theta^\dagger = 0}.
\] (2.49)

This works because F-component transforms into itself and gets a spacetime derivative. So when we integrate over \( d^4x \) the spacetime part will be identically zero and cancel, so the action is invariant and the transformation of \( F \) as well.

The \( F \) and \( D \) terms are invariant under SUSY transformations up to a total derivative that vanishes in the action, so now we know that we have a supersymmetric action.

Because the F- and D-term are independent of \( \theta \) and \( \theta^\dagger \) we can now switch freely between the \( x^\mu \) and \( y^\mu \) coordinates. In the general chiral and vector superfields this is not possible, since they are completely dependent on these coordinates, but now there is no difference. So in difficult calculations we can switch to the coordinates that are better suited for the problem, and then when we go to the action, it will not be relevant anymore, since the fermionic coordinates are integrated out.

A more general rule for chiral superfields is the fact that the product of chiral superfields is also a superfield. Or more mathematically: any holomorphic function where the chiral superfields are treated as complex variables is a chiral superfield. This means that you can define an object called superpotential \( W \), from which you can then distill the F-terms to find a proper Lagrangian. Usually we take a superpotential with dimension \([\text{mass}]^3\) so that potential stays renormalizable and is given by:

\[
W(\Psi_i) = h_i \Psi_i + \frac{1}{2} m_{ij} \Psi_j \Psi_j + \frac{1}{6} f_{ijk} \Phi_i \Psi_j \Psi_k, \quad (2.50)
\]

where \( m_{ij} \) and \( f_{ijk} \) are symmetric in their indices. The Lagrangian now becomes

\[
\mathcal{L} = [V]_D + [W(\Psi) + W^\dagger(\Psi^\dagger)]_F = \left( \frac{1}{2} D + \frac{1}{4} \partial_\mu \partial^\mu a \right) + (F + F^\dagger). \quad (2.51)
\]

And we also define

\[
W_i(\phi) = \frac{\partial W}{\partial \Psi_i}, \quad \bar{W}_i(\bar{\phi}) = \frac{\partial W^\dagger}{\partial \Phi^\dagger_i}. \quad (2.52)
\]
2.4. Chiral and vector superfields

where the \( | \) means that we evaluate \( \theta = \theta^\dagger = 0 \). This way we get the scalar part of the superpotential without too much of a hassle.

So now we can write down the simplest action that does not contain a gauge field,

\[
A = \int d^4x d^2\theta d^2\theta^\dagger \mathcal{L} = \int d^4x d^2\theta d^2\theta^\dagger (|V|_D + [W(\Psi) + W^\dagger(\Psi^\dagger)]_F). \tag{2.53}
\]

This action is SUSY invariant by construction, meaning \( \delta_\xi A = 0 \). Since the action has to be real we know it contains terms such as \( V, \Psi + \Psi^*, \Psi\Psi^* \).

The Lagrangian is explicitly something of the form

\[
\mathcal{L} = -\partial_\mu \phi \partial^\mu \phi^* + i\bar{\psi}^\dagger \tilde{\sigma}^\mu \partial_\mu \psi + FF^* \tag{2.54}
\]

where \( F \) is an auxiliary term as mentioned earlier in this chapter. With the equations of motion it becomes clear that

\[
F_i = -W_i^* \tag{2.55}
\]

resulting in a scalar potential that looks like

\[
V_F = |W_i|^2. \tag{2.56}
\]

In equation (2.54) we now have a kinetic term, a fermionic term and a potential. More generally we need gauge invariant terms that result in force carrying fields. For this we define

\[
\mathcal{W}_\alpha = \frac{1}{4} D^\dagger D^\dagger D_\alpha V. \tag{2.57}
\]

Notice that since this definition uses the superfield formalism this object is automatically chiral invariant, it’s also gauge invariant since \( V \) obeys (2.44) and it carries a spinor index.

The resulting action will look something like

\[
\int d^4x \frac{1}{4} ([\mathcal{W}_\alpha \mathcal{W}^\alpha]_F + \text{c.c.}) + [\phi^* e^{2g\alpha} V \phi]_D + [W(\phi) + \text{c.c.}]_F. \tag{2.58}
\]

The first term gives us a kinetic term for the gauge field, so the force carriers can actually
travel. The second term is gauge invariant and is the kinetic term for the matter fields that carry the charge. The final term once again yields a scalar part for the potential.

An explicit form for $W_\alpha$ is given in the W-Z gauge:

$$W_\alpha = \lambda_\alpha + \theta_\alpha D + \frac{i}{2} (\sigma^\mu \bar{\sigma}^\nu \theta)_{\alpha} F_{\mu\nu} + i \theta \theta (\sigma^\mu \partial_\mu \lambda^\dagger)_{\alpha}.$$  

(2.59)

Where $F_{\mu\nu}$ is the usual electromagnetic field tensor. In our action this gives

$$W_\alpha W^\alpha = \frac{1}{2} D^2 + i \lambda_\alpha \bar{\sigma}^\mu \partial_\mu \lambda - \frac{1}{4} F_{\mu\nu} F^{\mu\nu},$$  

(2.60)

where we see a $D$ term for our scalar potential, a gaugino part — the SUSY partner of the gauge field — and the normal $U(1)$ gauge field. An explicit form found using the equations of motion for the $D$ term is

$$D^\alpha = - g (\phi^\dagger T^\alpha \phi)$$  

(2.61)

where $T^\alpha$ is the generator of the gauge group and $g$ the coupling constant, and works for $SU(N)$ as well.

In the end, the resulting scalar potential found for a supersymmetric theory is found using the equations of motion (2.55) and (2.61) and thus

$$V = F_i F^i + \frac{1}{2} \sum_\alpha D^\alpha D_\alpha = W_i^* W^i + \frac{1}{2} \sum_\alpha g^2_\alpha (\phi^\dagger T^\alpha \phi)^2.$$  

(2.62)
Chapter 3

Flat Directions in Supersymmetry

3.1 Introduction

In this chapter we will take a look at flat directions (FDs), what they are and how we will use them to get the Affleck-Dine mechanism.

For the mathematicians we can make a very compact statement:

Flat directions are non-compact lines and surfaces in the space of scalar fields along which the scalar potential vanishes.\[13\]

Collectively they are also known as moduli space.

For the people that don’t know too much about compactness in mathematics, such as the author, we can say that flat directions are the directions in the space of scalar fields for which the scalar potential is identically zero.

\[ V(\phi) = 0. \] (3.1)

Meaning we have infinitely degenerate vacua. However these degeneracies can be lifted by non-perturbative effects, but these effects are suppressed exponentially and are only important at very high energies when fields have large vacuum expectation values. Large vevs will turn out to be one of the important ingredients for the Affleck-Dine mechanism, since it works during and just after inflation, when the universe is still very energetic. Because of the flat directions a field $\phi$ can get a large vev. This is possible since $\phi$ does not
have to go up a steep potential, since the potential is flat. So the field $\phi$ can get a large vev after which all kinds of interesting phenomena can occur.

### 3.2 Scalar fields

#### 3.2.1 Superpotential and scalar potential

We have some idea what a flat direction is, but what is it precisely? In this section we will give a bit more background and see what this means for the theory.

In chapter 2 we saw that a chiral superfield can be written by

$$\Phi = \phi + \sqrt{2} \theta \tilde{\psi} + \theta \bar{\theta} F,$$  \hspace{1cm} (3.2)

where we have a scalar field $\phi$, a fermion $\psi$ and an auxiliary field $F$.

A general superpotential as in chapter 2 was given by

$$W(\Psi_i) = h_i \Psi_i + \frac{1}{2} m_{ij} \Psi_i \Psi_j + \frac{1}{6} f_{ijk} \Phi_i \Psi_j \Psi_k.$$  \hspace{1cm} (3.3)

Translated to the Minimal Supersymmetric Standard Model (MSSM) this becomes

$$W_{\text{MSSM}} = \lambda_u Q_H \bar{u} + \lambda_d Q_H \bar{d} + \lambda_e L_H \bar{e} + \mu H_u H_d$$  \hspace{1cm} (3.4)

where $Q, L, \bar{u}, \bar{d}, \bar{e}, H_u$ and $H_d$ are chiral superfields, and $\lambda_i$ are dimensionless constants.

The fields denoted by small letters are $SU(2)$ singlets and the fields with a capital are $SU(2)$ doublets (barred and unbarred). We need a $H_u H_d$ term because $H^\dagger$ is forbidden in the superpotential. Combinations of $H_i H_i^*$ are not allowed because the superpotential is analytical in the chiral fields, and both $H_u$ and $H_d$ are needed for the quark masses and also the cancellation of gauge anomalies.\[13\]

The supersymmetric scalar potential is composed of the D- and F-terms and is

$$V = \sum_i |F_i|^2 + \frac{1}{2} \sum_j g_j^2 D^a D^a.$$  \hspace{1cm} (3.5)

$$F_i = \frac{\partial W}{\partial \Phi_i}, \quad D^a = \phi^\dagger T^a \phi$$  \hspace{1cm} (3.6)
where $T^a$ are the generators of the Lie algebra of the gauge group $G$ under which the fields $\phi_i$ transform.

Technically we can still add some terms to the superpotential (3.3) that are gauge invariant and analytic in the chiral superfields, but these terms actually violate lepton number $L$ or baryon number $B$. If we include these terms we get the most general gauge invariant superpotential that is also renormalizable at the same time. In the MSSM they are also forbidden by $R$-parity which gives a $-1$ for fermions and a $+1$ for bosons. The terms that we omitted in equation (3.3) are

\begin{align}
W_{\Delta B=1} &= \frac{1}{2} \lambda^{ijk} \bar{u}_i \bar{d}_j \bar{d}_k \\
W_{\Delta L=1} &= \frac{1}{2} \lambda'^{ijk} L_i L_j \bar{e}_k + \lambda''^{ijk} L_i Q_j \bar{d}_k + \mu' L_i H_\mu
\end{align}

where the indices are the generation indices. The numbers carried by the chiral fields are $L = +1$ for $L_i$ and $L = -1$ for $\bar{e}_i$ for leptons. For baryons we have $B = +1/3$ for $Q_i$ and $B = -1/3$ for $\bar{u}_i, \bar{d}_i$. Other terms have either $B = 0$ or $L = 0$.

Looking at equation (3.7) we see the total baryon number that will be violated is $+1$. In equation (3.8) we see that every term gives a violation of $+1$ in lepton number. If $\lambda$ and $\lambda''$ are not small we would have a large decay of protons, which means these constants are very much suppressed because we do not observe protons decaying.

### 3.2.2 Flat directions in the MSSM

Stepping back to flat directions, the question becomes: "How do we find the flat directions of a SUSY theory?" We want the potential to be flat, so $V = 0$. In other words, we have to solve

\begin{align}
D^a &\equiv \Phi^\dagger T^a \Phi = 0, & F_{\Phi_i} &\equiv \frac{\partial W}{\partial \Phi_i} = 0,
\end{align}

simultaneously for the chiral fields $\Phi_i$ of the SUSY theory. This is also called D-flat and F-flat. The D-flat part can be found by just looking at the gauge invariant monomials of the chiral superfields, while for the F-flat parts we must explicitly solve $F_{\Phi_i} = 0$. We know that a product of chiral superfields is also a chiral superfield, so a flat direction can also be a product of $k$ chiral superfields: $\Phi_m = \Phi_1 \Phi_2 \ldots \Phi_m$. 
Chapter 3. Flat Directions in Supersymmetry

From the definition of a flat direction we see that we only have to look at the scalar part, not the full chiral superfield. In the AD mechanism only the scalar fields are relevant because of this. This means that a single flat direction must carry a $U(1)$ symmetry which corresponds to the invariance of the lagrangian under the transformation $\phi \rightarrow e^{i\alpha} \phi$, and this field can therefore carry a corresponding particle number.

In the MSSM the global $U(1)$ symmetry is $B - L$. An example of a flat direction in the MSSM which carries this quantum number is $QL\bar{d}$.

$$Q_1^a = \frac{1}{\sqrt{3}} \begin{pmatrix} \phi \\ 0 \end{pmatrix}, \quad L_1 = \frac{1}{\sqrt{3}} \begin{pmatrix} 0 \\ \phi \end{pmatrix}, \quad \bar{d} = \frac{1}{\sqrt{3}} \phi. \quad (3.10)$$

In this example the upper index is the color and lower is the generation of the field, $\phi$ parametrizes the flat direction. Now this flat direction is of the form $\Phi = Q_1 L_1 \bar{d}_2$ so we get a dimension $n = 3$. The scalar component of this superfield is then parametrized by $\Phi = c\phi^n$ which is in this case $\Phi = c\phi^3$.

Another example of a flat direction is given by the direction $\Phi_2 = LH_u$.

$$H_u = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi \end{pmatrix}, \quad L = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi \\ 0 \end{pmatrix} \quad (3.11)$$

where $\phi$ is once again a complex field that parametrizes the flat direction. We also set all other fields to zero.

When we look at the $\sigma_3$ component of the D-term we first note that the generator is in this case $SU(2)$ and we find

$$D_{SU(2)}^3 = H_u^\dagger \sigma_3 H_u + L^\dagger \sigma_3 L \quad (3.12)$$

$$= \frac{-1}{2} \begin{pmatrix} 0 & \phi^* \\ \phi & 0 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} \phi^* & 0 \\ 0 & \phi \end{pmatrix} \quad (3.13)$$

$$= \frac{1}{2} |\phi|^2 - \frac{1}{2} |\phi|^2 = 0. \quad (3.14)$$

So the D-term is zero. The same holds for the $\sigma_1$ and $\sigma_2$ components, and also the $U(1)$ gauge group. Using equation (3.9) the F-terms become

$$F_{H_u} = \frac{\partial W}{\partial H_u} = \lambda_u Q\bar{u} + \mu H_d = 0, \quad (3.15)$$

$$F_L = \frac{\partial W}{\partial L} = \lambda_d H_d\bar{e} = 0. \quad (3.16)$$

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3.3 Breaking supersymmetry

But we also have the term

\[ F_{H_d} = \frac{\partial W}{\partial H_d} = \mu H_u \neq 0 \] (3.17)

because we have \( H_u \neq 0 \). This term is generally also flat because the parameter \( \mu \) is about the same size as the weak scale. The contribution to the potential is of the order of the soft supersymmetry breaking masses. F-terms that only contribute in the supersymmetric limit are usually included in flat directions.\[5\] So there is some flexibility in the flatness of a direction.

Almost all flat directions of the MSSM are either baryonic or leptonic. Only the ones involving the Higgs field are thus \( H_u H_d \) and \( LH_u \). As seen below we have some purely leptonic directions like \( L\tilde{\ell} \) and purely baryonic such as \( \tilde{u}d\tilde{d} \). But there are also mixed directions for example \( QL\tilde{d} \). In all these FDs we have suppressed the generation indices, meaning that we can make permutations like \( L_2 L_3 \tilde{\ell}_1 \) and the resulting directions are all flat.

We can now wonder if fields can have multiple flat directions simultaneously and the answer is yes, but this is quite rare. This is because the F-flatness is generally not maintained in multiple directions. The relation between the scalar field \( \phi^m \) and so called composite operators is in general highly non-linear.\[5\] This means that we usually only have one direction to work with and do not need multiple direction, which we can always do in principle. And it turns out that usually the lowest dimensional operator that generates the potential for the flat direction determines the largest share of the baryon to entropy ratio. So we will only consider one flat direction, but as we stated before, we can expand to multiple directions (if they exist).

3.3 Breaking supersymmetry

3.3.1 Non-renormalizable terms

The vacuum degeneracy can be broken by supersymmetry breaking terms and by non-renormalizable operators. The non-renormalizable operators give rise to superpotential terms that are of the form

\[ W = \frac{h}{dM^{u-3}} X^k = \frac{h}{dM^{u-3}} \phi^n, \quad n \geq 4 \] (3.18)

where \( X \) is an invariant operator of the form \( \phi^m \), \( M \) is some large mass scale, such as the
GUT or Planck scale. We have \( n = mk \) and \( h \) is a constant coupling constant of order one: \( |h| \sim \mathcal{O}(1) \). There is also another type that looks quite similar

\[
W = \frac{h'}{dM^{n-3}} \psi \phi^{n-1},
\]

(3.19)

and lifts because the F-term is nonzero, \( F_\psi \neq 0 \). The scalar potential gets a contribution that is

\[
V(\phi) = \frac{|\lambda|^2}{M^{2n-6}} |\phi|^{2n-2}
\]

(3.20)

with \( |h|^2 + |h'|^2 = |\lambda|^2 \). Although this term lifts the flat direction, it does not break \( U(1) \) symmetry so \( B - L \) will not be violated by this term.

### 3.3.2 Soft SUSY breaking terms

The other way to lift the vacuum degeneracy is through SUSY soft breaking terms such as

\[
V(\phi) = m^2 |\phi|^2 + \left( \frac{\lambda A m_{3/2}}{n M^{n-3}} \phi^n + \text{h.c.} \right).
\]

(3.21)

where the mass of the field is of the order of the gravitino mass \( m_{3/2} \) when we have gravity mediated supersymmetry breaking but this is often called Planck mediated SUSY breaking (PMSB). The A-term is the term that is going to violate the \( B - L \) symmetry during the Affleck-Dine baryogenesis and is of order \( \mathcal{O}(1) \). The parameter \( \lambda \) is in general complex with an arugment \( \theta_{\lambda} \), and as such it will yield a angular dependence of the potential. When we write \( \phi = |\phi| e^{i\theta} \) we get a term of the form \( \cos(n\theta + \theta_{\lambda}) \) and so the \( U(1) \) symmetry is broken with \( d \) different angular minima.

### 3.3.3 Hubble induced terms

When the vacuum has a finite energy, supersymmetry is spontaneously broken. This can be seen from the vacuum expectation value (vev) using the SUSY operators and the definition of unbroken supersymmetry.

\[
H |0\rangle = E |0\rangle = 0
\]

(3.22)
3.3. Breaking supersymmetry

defines whether supersymmetry is broken. If it is zero, it is unbroken. The full Hamiltonian in terms of the supersymmetry generating operators is

\[ H = p^0 = \frac{1}{4} \left( Q_1 Q_1^\dagger + Q_2 Q_2^\dagger + Q_1^\dagger Q_1 + Q_2^\dagger Q_2 \right). \]  

(3.23)

The vev now becomes

\[ \langle 0 | H | 0 \rangle = \frac{1}{4} \left( |Q_1| |0\rangle|^2 + |Q_1^\dagger| |0\rangle|^2 + |Q_2| |0\rangle|^2 + |Q_2^\dagger| |0\rangle|^2 \right) \geq 0, \]  

(3.24)

which is non-zero when we have a non-zero potential, such as the inflation potential. So the SUSY operators working on the vacuum will give a non-zero value since we have a non-zero vev, meaning that supersymmetry is broken.

If we have a non-zero inflationary potential the inflaton field also gives rise to SUSY breaking terms of the scale of the Hubble parameter. Since the Hubble parameter goes as \( H \propto \frac{1}{t} \) we get a very large value at early times. So the Hubble term will dominate the soft SUSY breaking terms at first. These terms are of the form

\[ V(\phi) = -cH^2 |\phi|^2 + \left( \frac{\lambda a H}{n M^{n-3}} \phi^n + \text{h.c.} \right) \]  

(3.25)

where \( a \) is a complex parameter that depends on the coupling to inflation but we take it to be of order \( \mathcal{O}(1) \). \( c \) is a real number whose size and sign also depends on the coupling to inflation and we take it also to be of order \( \mathcal{O}(1) \). The \( c \)-term is again \( U(1) \) invariant, but the Hubble-term, the \( a \)-term, breaks the \( U(1) \) invariance of the field.

Notice that since \( H \) is time dependent this term will first dominate but later we can neglect it.

### 3.3.4 The full Affleck-Dine potential

Now we collect all terms and find the full AD potential in the case of gravity mediated supersymmetry breaking.

\[ V(\phi) = (m_\phi^2 - cH^2) |\phi|^2 + \left( \frac{\lambda a H}{n M^{n-3}} \phi^n + \text{h.c.} \right) \]  

\[ + \left( \frac{\lambda A m^{3/2}}{n M^{n-3}} \phi^n + \text{h.c.} \right) + \frac{|\lambda|^2}{M^{2n-6}} |\phi|^{2n-2}. \]  

(3.26)
Some general properties that we can see immediately are for example that during inflation, the $H$ term is large, and we can neglect the $m_{\phi}$-term. In the radial direction this will give a minimum, because every part of the potential is positive except the $c$-part. After $H$ becomes smaller the minimum will become smaller and eventually disappear.

In the angular direction we notice the $U(1)$ breaking terms, which will give $\cos (n\theta + \theta_{\lambda} + \theta_{a})$ terms and $\cos (n\theta + \theta_{\lambda} + \theta_{A})$ terms — spot the difference! — which will give $n$ angular minima.

For AD in the MSSM we have the bounds

\[ 4 \leq n \leq 9. \]  \hspace{1cm} (3.27)

because for $n < 4$ there are non-flat directions and in the MSSM there are no flat directions with $n > 9$ but in general it could be that $n > 9$.\[13\]
Chapter 4

Cosmology

In this chapter we will give a brief introduction to cosmology so we have the necessary background to be able to understand why the Affleck-Dine mechanism comes about and how it works into the whole story. We will take a look at the Einstein equations, the metric that we use for our calculations and finally how the field equations come about that make it all possible.

4.1 The cosmos

4.1.1 The metric

When we peer into the universe we see that it is isotropic and homogeneous. So everywhere we look we see the same composition and in every direction it is identical on large scales. Moreover we also observe that the universe is expanding, which can be described by a Friedmann-LeMaître-Robertson-Walker (FLRW) metric [20]

\[ ds^2 = -dt^2 + a(t)g_{ij}dx^idx^j = -dt^2 + a(t)d\mathbf{x}^2. \]  

(4.1)

Here \( a(t) \) is some function of time that describes the size of the universe and usually called the expansion factor or scale factor, since it could also describe a shrinking universe.

4.1.2 The field equations

Using the Euler-Lagrange equations
we want to know the equations of motion of a scalar field \( \phi \). We have the Lagrangian

\[
\mathcal{L} = \sqrt{-g} \left( \frac{1}{2} \dot{\phi}^2 + V(\phi) \right)
\]

(4.3)

with the metric given by (4.1) \( g_{\mu\nu} = \text{diag}(-1,a,a,a) \) resulting in \( \sqrt{-g} = a^{3/2} \).

A scalar field in this universe has the equations of motion

\[
\ddot{\phi} + 3H \dot{\phi} + \frac{\partial V}{\partial \phi} = 0.
\]

(4.4)

where \( H \) is the Hubble parameter and is given by

\[
H = \frac{\dot{a}}{a}
\]

(4.5)

and today it has a value of \( 67.80 \pm 0.77 \text{ (km/s)/Mpc} \). This value tells you how fast an object moves one megaparsec away, so if an object is 1 Mpc away from the observer, it moves away from the observer with a speed of 67.80 km/s due to the expansion of the universe. We can also calculate (4.4) using brute-force methods with Christoffel symbols.

For a field \( \phi \) the equations of motion are given by

\[
D_\mu D^\mu \phi + \frac{\partial V}{\partial \phi} = 0.
\]

(4.6)

where \( D_\mu \) is the covariant derivative \( D_\mu = \partial_\mu + \Gamma^\nu_{\mu\rho} \).

On a scalar object the d’Alembertian yields

\[
D_\mu \partial^\mu \phi = (\partial_\mu + \Gamma^\nu_{\mu\rho}) \partial^\rho \phi.
\]

(4.7)

For the symbols we find

\[
\Gamma^i_{i,0} = \frac{\dot{a}}{a} = H, \quad i \neq 0
\]

(4.8)
4.1. The cosmos

and every other symbol is zero because φ is only a function of t, not of the spatial coordinates. So those symbols disappear. Once again we find the field equations for a scalar field φ in an expanding universe.

\[ \ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0. \]  

(4.9)

4.1.3 The expansion factor

Now we know how a scalar field behaves in the FLRW universe, but we still do not know much about the universe itself. So we are going to look at the equation of state and the behaviour of the expansion factor a.

We begin with the energy-momentum tensor which is diagonal because of homogeneity and isotropy of the universe.

\[ T^\mu_\nu = \text{diag}(-\rho, p, p, p) \]  

(4.10)

with ρ the energy density and p the pressure of the stuff in our universe. \( \text{Tr} T^\mu_\nu = T^\mu_\mu = -\rho + 3p \). The continuity equation gives us the condition

\[ D_\mu T^\mu_\nu = 0. \]  

(4.11)

The zero-th component yields a condition,

\[ D_\mu T^\mu_0 = -\partial_0 \rho - 3\frac{\dot{a}}{a}(\rho + p) = 0. \]  

(4.12)

With the equation of state

\[ p = w\rho \]  

(4.13)

we find for the continuity equation

\[ \frac{\dot{\rho}}{\rho} = -3(1 + w)\frac{\dot{a}}{a}. \]  

(4.14)

When we integrate this with the assumption that w is constant we get
Now we have an equation that relates the energy density to the expansion factor of the universe.

The values of $w$ are given in the table below, and show that different kinds of matter have different behaviour for their energy density.\[20\]

<table>
<thead>
<tr>
<th></th>
<th>$w$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>matter</td>
<td>0</td>
<td>$a^{-3}$</td>
</tr>
<tr>
<td>radiation</td>
<td>$\frac{1}{3}$</td>
<td>$a^{-4}$</td>
</tr>
<tr>
<td>vacuum</td>
<td>-1</td>
<td>$a^0$</td>
</tr>
</tbody>
</table>

Now we take a look at the Einstein field equations

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{4.16}$$

and remember that we can write these equations\[20\] as

$$R_{\mu\nu} = 8\pi G \left( G_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right). \tag{4.17}$$

From the different components we can find the results

$$\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho, \tag{4.18}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p), \tag{4.19}$$

which are known as the Friedmann equations, and they describe the FRLW universe.

The expansion factor can behave in different ways, depending on what part of the Einstein equations dominates the expansion of the universe. There are matter dominated, radiation dominated and vacuum energy ($\Lambda$) dominated eras.

<table>
<thead>
<tr>
<th></th>
<th>$a$</th>
<th>$H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>matter</td>
<td>$t^{2/3}$</td>
<td>$1/2t$</td>
</tr>
<tr>
<td>radiation</td>
<td>$t^{1/2}$</td>
<td>$3/2t$</td>
</tr>
<tr>
<td>vacuum</td>
<td>$e^{Ht}$</td>
<td>cst</td>
</tr>
</tbody>
</table>
4.1. The cosmos

After inflations and reheating the universe started out in a radiation dominated era, then after about 70,000 years went to a matter dominated era and finally after $5 \times 10^9$ years ended up in a $\Lambda$ dominated era where it still is today\[22\].

**4.1.4 The density parameter**

While we’re at it we are also going to define the density parameter $\Omega$, which describes the topology of the universe.

$$\Omega = \frac{8\pi G}{3H^2}\frac{\rho}{\rho_{\text{crit}}}$$  \hspace{1cm} (4.20)

where the critical density is defined by

$$\rho_{\text{crit}} = \frac{3H^2}{8\pi G}.$$  \hspace{1cm} (4.21)

The density parameter tells us about the geometry of universe we are and there are three possibilities.

- $\rho < \rho_{\text{crit}} \iff \Omega < 1 \Rightarrow$ open universe
- $\rho = \rho_{\text{crit}} \iff \Omega = 1 \Rightarrow$ flat universe
- $\rho > \rho_{\text{crit}} \iff \Omega > 1 \Rightarrow$ closed universe.

An open universe has a hyperbolic geometry is non-compact and without boundary, a closed universe is compact and without boundary. A flat universe has Euclidian geometry.

This means in practise that a closed universe will collapse upon itself, an open one will expand infinitely which will result in at least the heat death\[23\], when the universe is at thermodynamic equilibrium everywhere and no more work can be done. A flat universe will expand but the expansion will slow down and asymptotically approach zero. This universe will have the same fate as an open universe.

**4.1.5 The flatness: inflation**

We noted above that the universe is almost completely flat, isotropic and homogeneous. The most plausible explanation for this observation is that the universe went through a phase of rapid inflation where the size of the universe increased by a factor of at least $10^{78}$
Chapter 4. Cosmology

during the time of $t = 10^{-36}$ s to $t = 10^{-33}$ s depending on the inflation model. This is called cosmic inflation and it is driven by a negative-pressure vacuum energy density. It also set the density parameter to one, $\Omega \approx 1$, making the universe almost certainly flat ($\Omega - 1 < 10^{-4}$).\[1\] Inflation happens when the inflaton field slowly rolls through a potential that is almost flat. During this time the universe expands rapidly. Afterwards the field rolls into a harmonic potential where it starts to oscillate. During this oscillation the universe is dominated by the inflaton matter so the Hubble parameter will be $H = 2/3t$. After this oscillation the field decays and creates particles that will fill the universe and puts energy into the heat bath.

Remember that after this inflation the universe continued expanding, but not at the same rate.

Now that we have some background in cosmology we can go forward with the production of matter in our universe after inflation.

4.2 The Sakharov conditions for Baryogenesis

In this section we will look into the Sakharov conditions and how they are necessary for the production of excess matter so that we may end up with a universe that is not empty.

4.2.1 The conditions

Under the CPT-theorem we would expect that during the Big Bang there would be an equal amount of matter and anti-matter produced, and that this would eventually annihilate each other resulting in a universe with no matter, only radiation. But we observe that there is no large amount of anti-matter in the universe only matter and radiation.\[18\] So what happened? There could be a slight preference for matter at the start of our universe or there could be a symmetric universe with a small preference towards matter instead of anti-matter in physical phenomena resulting in more creation of matter. These asymmetric initial conditions have been excluded by inflation because the universe is empty at the end of inflation and then the inflaton field decays into particles, so we need a dynamic mechanism to provide us with a way to create this asymmetry.

To find the answer to this remarkable observation we need Baryogenesis theories, which describe such phenomena so we can find a compatible theory. We are focusing on theories in which phenomena have a slight balance towards matter and see how this can possibly result in this asymmetric universe.
4.2. The Sakharov conditions for Baryogenesis

We first need a general principle that can result in an abundance of one particle over its anti-particle, and the necessary conditions were first postulated by Sakharov.\textsuperscript{[7]} The Sakharov conditions are:

1. Baryon number $B$ violation.
2. C- & CP-symmetry violation.
3. Out of thermal equilibrium interactions.

Baryon number violation is obvious, we need to produce more baryons than anti-baryons in our process, otherwise we could never get an abundance of one particle over another. The CP-violation is needed so that interaction that produce more baryons than anti-baryons are not compensated by interactions that produce more anti-baryons than baryons. Otherwise we would still end up with no nett baryon number. Finally the ”out of equilibrium” condition is there to make sure this process can’t go reverse, so that once we are in equilibrium we have a definite asymmetry that can not be removed.

Now we already have some CP-violation in the SM, but this is believed to not be enough to get the abundance of baryons we see in the universe today.

### 4.2.2 The baryon asymmetry

To measure how large the asymmetry between baryons and anti-baryons is we need to define the baryon asymmetry parameter $\eta$.

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} \quad (4.22)$$

where we take the difference of baryons and anti’s and divide by the number of cosmic radiation background photons, which is constant in time.

As noted in the introduction, this value has been measured\textsuperscript{[3]} and is

$$\eta = 6.1(3) \times 10^{-10}. \quad (4.23)$$

So ideally we want a theory that predicts this number. But there are many theories. There’s Planck scale baryogenesis, GUT scale baryogenesis, electroweak scale baryogenesis, leptogenesis and coherent scalar field baryogenesis (Affleck-Dine). And there are of course all kinds of subcategories, but we are going to focus on the Affleck-Dine mechanism.
4.2.3 The mechanism and the Sakharov conditions

Now we want to relate the Sakharov conditions to the Affleck-Dine mechanism, so that we can be sure that we really do have a theory that can produce a nett baryon number.

The CP-violation can be seen from the angular minima that break the $U(1)$ symmetry, the angle of this phase changes during evolution of the field, which is exactly what CP-violation is. This yields a preference of one angular momentum over the other, because the angular difference in the parameters of the theory dictate in which direction the field will rotate. The baryon number violation comes from the fact that we end up with a non-zero "angular momentum", which corresponds to a baryon number. This is caused by the B violating non-renormalizable terms in the potential, $\phi^n$ since they break the $U(1)_{B-L}$ symmetry. And the fact that there is a time ordering of the events (deformation of the potential) yields an arrow of time which is equivalent to being out of thermal equilibrium. This time ordering is the fact that the AD field $\phi$ rolls down the potential and can not go back up to the initial values, since the potential evolves through time and the universe expands.
Chapter 5

Evolution of the scalar Affleck-Dine field

In this section we will look into the evolution of the scalar field that will lead to the build-up of a non-zero baryonic charge. This baryonic charge will then result in an asymmetry in baryons and anti-baryons after the field decays and it can also be used for the creation of dark matter using the same principle. We use the potential that is generated by gravity mediated supersymmetry breaking.

5.1 The Noether current

This charge is build up in the period when the supersymmetry breaking parts of the potential are important. The general current is given by

\[ j_\mu = i\phi \overset{\leftrightarrow}{\partial}_\mu \phi^* = i\phi(\partial_\mu \phi^*) - i(\partial_\mu \phi)\phi^* \]  

(5.1)

and the charge is

\[ Q = \int d^4x j_0 = \int d^4x (i\phi(\partial_0 \phi^*) - i(\partial_0 \phi)\phi^*). \]

(5.2)

The symmetry that will be broken is the \( U(1) \) symmetry of the current. We can see that it is invariant under \( U(1) \) transformation:
Chapter 5. Evolution of the scalar Affleck-Dine field

\[ \phi \rightarrow \phi e^{i\alpha} \]  
\[ j_{\mu} \rightarrow j'_{\mu} = i\phi e^{i\alpha} (\partial_{\mu}\phi^* e^{-i\alpha}) - i(\partial_{\mu}\phi e^{i\alpha})\phi^* e^{-i\alpha} = j_{\mu} \]  

for some angle \( \alpha \). According to the Noether theorem every symmetry implies a conserved quantity. We can find this quantity - in this case! - by rewriting our field as below.

When we write \( \phi = \rho e^{i\theta}/\sqrt{2} \) we get the following:

\[ j_0 = \frac{1}{2} i\rho e^{i\theta} (\dot{\rho} - \rho \dot{\theta}) e^{-i\theta} - \frac{1}{2} i(\dot{\rho} + \rho \dot{\theta}) e^{i\theta} e^{-i\theta} \]  
\[ = \frac{1}{2} i\rho (\dot{\rho} - \rho \dot{\theta}) - \frac{1}{2} i(\dot{\rho} + \rho \dot{\theta}) \rho = \rho^2 \dot{\theta}. \]  

(5.4)

(5.5)

Using the usual coordinates \( \rho^2 = x^2 + y^2, \ \theta = \arctan(y/x) \)

\[ \dot{\theta} = \frac{1}{1 + \frac{y^2}{x^2}} \frac{\dot{y}x - \dot{x}y}{x^2 + y^2} = \frac{\dot{y}x - \dot{x}y}{x^2 + y^2}, \]  

(5.6)

so \( \rho^2 \dot{\theta} = \dot{y}x - \dot{x}y = L_z \), which is exactly the angular momentum! We have found that the conserved current for this field is the internal angular momentum of the field \( \phi \).

5.2 The particle number

By rearranging the expression for particle number \( n_X \) we can get a differential equation that describes the evolution of particle number.

The particle number is given by the Noether current:

\[ n_X = j_0 = i q_X (\dot{\phi}^* \phi - \phi^* \dot{\phi}), \]  

(5.7)

where \( q_X \) is the charge of the field \( \phi \). Differentiating gives

\[ n'_X = i q_X (\ddot{\phi}^* \phi + \dot{\phi}^* \dot{\phi} - \dot{\phi}^* \dot{\phi} - \phi^* \ddot{\phi}) = i q_X (\ddot{\phi}^* \phi - \phi^* \dot{\phi}) \]  

(5.8)

\[ ^1\text{In this thesis we will interchange "angular momentum" and "baryon number" quite frequently. We are almost always talking about the angular momentum of the field } \phi. \]
5.3. The scalar potential

After rewriting we find

\[ \dot{n}_X + 3Hn_X = iq_X (\ddot{\phi}^i \phi - \phi^* \ddot{\phi} + 3H \dot{\phi} \dot{\phi} - 3H \phi^* \phi) \] (5.9)

Using the equations of motion (4.4) equation (5.9) becomes

\[ \dot{n}_X + 3Hn_X = iq_X (-\frac{\partial V}{\partial \phi} \phi + \frac{\partial V}{\partial \phi^*} \phi^*). \] (5.10)

The potential is

\[ V(\phi) = (m_\phi^2 - cH^2)|\phi|^2 + \left( aH\lambda \frac{\phi^n}{nM^{n-3}} + \text{h.c.} \right) + \left( Am_{3/2}\lambda \frac{\phi^n}{nM^{n-3}} + \text{h.c.} \right) + |\lambda|^2 \frac{|\phi|^{2n-2}}{M^{2n-6}} \] (5.11)

Filling out this potential we find

\[ \dot{n}_X + 3Hn_X = -2q_X \left( |A||\phi|^n \frac{m_{3/2} \sin (n\theta + \theta_A)}{M^{n-3}} + \frac{|a||\phi|^n H \sin (n\theta + \theta_a)}{M^{n-3}} \right) \] (5.12)

and we see that if we have no angular difference this immediately becomes zero.

5.3 The scalar potential

We start off with the general equation of motion for a scalar field \( \phi \) in an expanding universe described by the FLRW metric.

\[ ds^2 = -dt^2 + a(t)\delta_{ij}dx^idx^j \] (5.13)

\[ \ddot{\phi} + 3H \dot{\phi} + \frac{\partial V}{\partial \phi} = 0 \] (5.14)

Notice that this equation is very similar as the equation for a harmonic oscillator. In particular, we can use the properties of the harmonic oscillator to classify the behaviour of
the field by taking $H$ small or large. The potential is constructed from the MSSM superpotential, and includes non-renormalizable pieces that lift the flat directions. The entire potential is given in equation (7.1).

Not every term of the potential is relevant at every epoch in the evolution of the field. In general we can neglect some terms at every point in time. For example, the Hubble parameter is very large but nearly constant during inflation, $H \approx H_I$, so this yields certain constraints on the equation, but after inflation the parameter becomes a dynamic quantity $H(t)$, resulting in stages that do and do not have a dependence on $H$ and $t$.

5.4 Inflation: Large damping

5.4.1 Rolling down the potential

During inflation the Hubble parameter is roughly constant but also extremely large compared to the mass term of the Affleck-Dine potential: $H^2 \gg V''(\phi)$. Looking at equation (5.14) we see that the damping term is large so the Affleck-Dine field will display very damped oscillations and it will move towards a minimum. The only relevant parts of potential (7.1) are now the first and the last one, the rest is negligible of size. This is because $H^2$ is quadratically bigger and $\phi^{2n-2}$ has a very large vacuum expectation value from the flat direction. Moreover $H \gg m_\phi$, $H \gg m^{3/2}$ and the constant $c$ is of order $O(1)$. The potential is of the form

$$V(\phi) = -cH^2|\phi|^2 + |\lambda|^2 \frac{|\phi|^{2n-2}}{M^{2n-6}}$$ (5.15)

when we neglect the angular minimum and look just at the radial part.

Now the field will roll towards the minimum and this decay goes exponentially fast, so it will stay in the minimum without oscillating too much the rest of this epoch. The calculation of this decay is at the end of this section.

5.4.2 The minimum

The entire potential in this epoch is given by

$\text{With FDs we talk of the lifting of flat directions, in the same sense we will call it lifting when we speak of the minimum of a potential that is deformed due to some mechanism.}$
5.4. Inflation: Large damping

Figure 5.1: The field starts out with a large vacuum expectation value and slowly rolls down due to inflation. The large value of $H$ makes sure that after rolling down to the minimum the oscillation will be damped.

\[ V(\phi) = -cH^2|\phi|^2 + \left( aH\lambda \phi^n + \text{h.c.} \right) + |\lambda|^2 \frac{|\phi|^{2n-2}}{M^{2n-6}}, \]  \hspace{1cm} (5.16)

The minimum in the $\rho$-direction can be found by parameterizing the field as $\phi = \rho e^{i\theta}$ and solving for $\frac{\partial}{\partial \rho} V = 0$.

\[ V(\rho) = -cH^2\rho^2 + \left( \frac{aH\lambda \rho^n e^{in\theta}}{nM^{n-3}} + \text{h.c.} \right) + |\lambda|^2 \frac{\rho^{2n-2}}{M^{2n-6}}, \]  \hspace{1cm} (5.17)

\[ = -cH^2\rho^2 + \left( \frac{aH\lambda \rho^n \cos (n\theta + \theta_\lambda + \theta_a)}{nM^{n-3}} \right) + |\lambda|^2 \frac{\rho^{2n-2}}{M^{2n-6}}, \]  \hspace{1cm} (5.18)

where we introduce $\theta_\lambda$ and $\theta_a$ as the arguments of the complex parameters $\lambda$ and $a$ and $\theta$ is the argument of the complex field $\phi$.

\[ \frac{\partial V}{\partial \rho} = -2cH^2 \rho + \left| a \right| |\lambda| H\rho^{n-1} \cos (n\theta + \theta_\lambda + \theta_a) \frac{\rho^{n-2}}{M^{n-3}} + \left| \lambda \right|^2 \frac{(2n-2)}{M^{2n-6}} \rho^{2n-3} = 0. \]  \hspace{1cm} (5.19)

For $\rho \neq 0$ we get

\[ \frac{n-1}{M^{2n-6}} \rho^{2n-4} + \left| a \right| |\lambda| H \cos (n\theta + \theta_\lambda + \theta_a) \rho^{n-2} - cH^2 = 0. \]  \hspace{1cm} (5.20)

This is a polynomial in $\rho^{n-2}$ of order 2, so we can solve it analytically, the solutions are

\[ \rho^{n-2} = \frac{|a||\lambda|HM^{3-n}C^2 \pm \sqrt{|a|^2|\lambda|^2H^2M^{6-2n}C^2 + 4|\lambda|^2(n-1)cH^2M^{6-2n}}}{2|\lambda|^2(n-1)/M^{2n-6}} \]  \hspace{1cm} (5.21)
where I have substituted \( \cos(n\theta + \theta_\lambda + \theta_a) \) with \( \mathcal{C} \) and inverted all the \( M^{2n-6} \) terms to \( 1/M^{-2n+6} \). After pulling out all relevant terms in the square-root and cancelling them with the denominator we get

\[
\rho^{n-2} = \frac{|a|HM^{n-3}\mathcal{C} \pm HM^{n-3}\sqrt{|a|^2\mathcal{C}^2 + 4(n-1)c}}{2|\lambda|(n-1)}
\]

\[
= \frac{|a|\mathcal{C} \pm \sqrt{|a|^2\mathcal{C}^2 + 4(n-1)c}}{2|\lambda|(n-1)} HM^{n-3}.
\]

And finally, the minimum of the potential fully written out is

\[
\phi_0 = \left(\frac{|a| \cos(n\theta + \theta_\lambda + \theta_a) \pm \sqrt{|a|^2 \cos(n\theta + \theta_\lambda + \theta_a)^2 + 4(n-1)c}}{2|\lambda|(n-1)} H M^{n-3}\right)^{\frac{1}{n-2}}.
\]

If we minimize the cosine to \(-1\), which would be the smallest minimum, and choose the minus-sign as a solution we would find a complex solution to \( \phi_0 \) because we would have to take the square root of a negative number - note the \( 1/n-2 \) exponent - so to get a proper solution, one with a positive radius \( \rho \), we take the solution with a plus sign.

When we neglect the angular part we would have found

\[
\phi_0 = \left(\sqrt{\frac{c}{n-1}|\lambda|} H M^{n-3}\right)^{\frac{1}{n-2}},
\]

so we see that we get a correction of order \( \mathcal{O}(a) \) for the minimum in equation (5.24).

The minimum \( \phi_0 \) of the potential is thus given by

\[
\phi_0 = \left(\frac{-|a| \pm \sqrt{|a|^2 \cos(n\theta + \theta_\lambda + \theta_a)^2 + 4(n-1)c}}{2|\lambda|(n-1)} H M^{n-3}\right)^{\frac{1}{n-2}}
\]

\[
= \text{const} \times \left(H M^{n-3}\right)^{\frac{1}{n-2}}
\]
and so we see that the constant in front of this quantity only depends on the order of the monomial of the flat direction, the previous established constant $c$, which was of order $O(1)$ and the parameters $a$ and $\lambda$ but they are also of order $O(1)$. So the actual minimum depends on the size of the Hubble parameter and of the large mass parameter, which can be on the Planck or the GUT scale. In the next chapter we will see how different values of these parameters impact the baryon number $n_B$.

5.4.3 Initial value

Now at the start of the evolution — during inflation — $\phi$ can be arbitrary, but using the fact that the energy density of the AD field has to be smaller than the inflation energy density, we find the condition:

$$\rho_{\text{inf}} = 3H^2M_p^2 > \rho_\phi$$

(5.28)

Where $\rho_{\text{inf}}$ and $\rho_\phi$ now represent the energy densities of the respective fields, not the minima. Because there is new physics at the Planck scale we take $M = M_p$. But if we want to make a general statement on the upper bounds of the initial value of $\phi$ we will want to take the mass scale a bit more general.

For $\phi_0$ from equation (5.27)

$$|\phi_0| \propto (HM_n^{-3})^{\frac{1}{n-2}}.$$  

(5.29)

We can make it even simpler and just look at the potential, to find bounds for the field $\phi$ and from potential $V$ we get the upperbound

$$\frac{\phi^{2n-2}}{M^{2n-6}} \leq H^2M_p^2$$

(5.30)

meaning

$$|\phi|^{2n-2} \leq H^2M^{2n-4} \Rightarrow |\phi| \leq (HM_n^{-2})^{\frac{1}{n-2}}.$$  

(5.31)

The upperbound of the initial value of the field is then

$$\frac{|\phi|}{|\phi_0|} = (HM_n^{-2})^{\frac{1}{n-2}}$$

(5.32)
Chapter 5. Evolution of the scalar Affleck-Dine field

where we will take the Planck mass $M_p$ as the upper bound on the mass scale $M$, but we could also take for example the GUT mass. But first

$$\frac{H^{\frac{1}{n-1}}}{H^{\frac{1}{n-2}}} \text{ with } \frac{1}{n-1} - \frac{1}{n-2} = \frac{-1}{(n-1)(n-2)} \text{ gives } \frac{1}{H^{\frac{1}{(n-1)(n-2)}}},$$

(5.33)

and

$$\frac{M^{\frac{2}{n-3}}}{M^{\frac{2}{n-2}}} \text{ with } \frac{n-2}{n-1} - \frac{n-3}{n-2} = \frac{1}{(n-1)(n-2)} \text{ gives } M^{\frac{1}{(n-1)(n-2)}}.$$

(5.34)

This yields

$$\frac{\phi}{\phi_0} = \left(\frac{M}{H}\right)^{\frac{1}{(n-1)(n-2)}}. \tag{5.35}$$

If we fill in for example $H = 10^{13} \text{ GeV}, M = 10^{18} \text{ GeV}$ and $n = 4$, we find that the boundary condition on the starting value of the field is $\phi < 10\phi_0$, which is reasonably close because it does not dominate the energy and it will minimize before the epoch ends because it goes exponentially to the minimum while inflation takes at least 60 e-folds.

5.4.4 Exponential decay of $\phi$

Next we want to know how fast the field settles in the minimum, so we look at the energy dissipation. Since the system starts out with a large VEV we use the large value approximation of the potential: $V = k\phi^{2n-2}$ for some $k$.

$$\frac{dE}{dt} = \frac{d}{dt}(T + V) = \left(\frac{dT}{d\phi} + \frac{dV}{d\phi}\right)\frac{d\phi}{dt} \tag{5.36}$$

$$= \dot{\phi}\left(\frac{d^2\phi^2}{d\phi} + \frac{dV}{dt}\right) = \dot{\phi}\left(\frac{d\phi}{dt}\right) + \frac{dV}{dt} = \dot{\phi}\left(\ddot{\phi} + \frac{dV}{dt}\right). \tag{5.37}$$

Using the equations of motion (4.4) we find that

$$\frac{dE}{dt} = \dot{\phi}\frac{d}{d\phi}(T + V) = -3H\dot{\phi}^2 = -6HT. \tag{5.38}$$
5.4. Inflation: Large damping

The Virial Theorem states that for a potential that is of the form \( V(r) \propto c r^n \), the kinetic energy is related by

\[
2 \langle T \rangle = n \langle V \rangle. \tag{5.39}
\]

Our potential is of the form \( \phi^{2n-2} \), so after averaging over time we find that

\[
\langle T \rangle = (n - 1) \langle V \rangle \tag{5.40}
\]

This result combined with eq. (5.38) and averaged over a period so we can use the Virial Theorem yields

\[
\dot{\phi} \frac{d}{d\phi} (nV) = -6H(n - 1)V, \tag{5.41}
\]

and using \( V = k\phi^{2n-2} \) it easily follows that

\[
\dot{\phi} = -\frac{3H}{n} \phi \tag{5.42}
\]

\[
\phi = e^{-\frac{3m}{n} \phi_i} \tag{5.43}
\]

with the initial field value \( \phi_i \). So we see that the field decays exponentially, and it will be near a minimum within a few e-foldings since the Hubble parameter is so large. In conclusion, the field value will be at the minimum \((5.27)\) of the potential when inflation ends. The field also has a phase, dependent on the initial condition, which we have neglected so far in our calculations, since it does not affect the field value and amplitude of the minimum. Now we have a range of possibilities for the phase, since \( H \) is large, only the \( aH \) term will play a role. When \( a = 0 \) the angular parts will be approximately flat, since the \( Am_{3/2} \) term is very small compared to all other parts of the potential. This means that the initial phase of the field is random and does not necessarily have to be that of \( Am_{3/2} \). The other option is that \( a \neq 0 \), which means there is an angular minimum in which the field will settle exponentially fast, comparable to the radial Higgs field. Afterwards the phases between \( a \) and \( A \) might be different resulting in CP violation and thus creating a baryon number, but more on this in the next sections.
Chapter 5. Evolution of the scalar Affleck-Dine field

5.5 After Inflation: following the minimum

When inflation ends, the Hubble parameter becomes time dependent and since the inflaton oscillations in the (approximately) quadratic potential will dominate the universe, we get a matter domination Hubble parameter, it is of the form

\[ H = \frac{2}{3t}. \]  

(5.44)

The field equation now becomes

\[ \ddot{\phi} + \frac{2}{t} \dot{\phi} + \frac{\partial V}{\partial \phi} = 0. \]  

(5.45)

And for the potential we use \([7.1]\) where we neglect the a-term. This term is of order \(O(1)\) and only provides us a phase of the initial condition which can be added back later. The main difference with the potential we now have is that \(H\) is time dependent, and thus the potential is time dependent. Moreover, because of this time dependence the minimum of the potential, \(\phi_0\) will start to move.

5.5.1 Transforming \(\phi\)

To see this we transform our coordinates to the following:

\[ z = \log t, \quad \phi = \xi \phi_0(t) = \xi \left( \frac{\beta}{\lambda} M^{n-3} e^{-z} \right)^{\frac{1}{n-2}}. \]  

(5.46)

This means that \(\frac{1}{t} = e^{-z}\) and we have also defined \(\beta = \sqrt{\frac{c'}{n-1}}\) with \(c' = \frac{4}{5}c\) to simplify the form and \(\xi\) is a function of \(z\). The line element transforms as:

\[ dz = \frac{dt}{t} \Rightarrow \frac{d}{dt} = \frac{1}{t} \frac{d}{dz}. \]  

(5.47)

To simplify the calculation we let the time-derivative work on \(\phi_0(z)\) and find

\[ \frac{d}{dz} \left( \frac{\beta}{|\lambda|} M^{n-3} e^{-z} \right)^{\kappa} = -\kappa \left( \frac{\beta}{|\lambda|} M^{n-3} e^{-z} \right)^{\kappa-1} \left( \frac{\beta}{|\lambda|} M^{n-3} e^{-z} \right), \]  

(5.48)

\[ = -\kappa \left( \frac{\beta}{|\lambda|} M^{n-3} e^{-z} \right)^{\kappa}. \]  

(5.49)
5.5. After Inflation: following the minimum

The $\frac{\partial}{\partial z}$ operator is working on an eigenstate with eigenvalue $-\kappa$. Now we define $\phi_0 = \alpha^{\frac{1}{n-2}}$ and find for the time derivatives

\begin{align}
\dot{\phi} &= \xi \alpha^{\frac{1}{n-2}} \\
\ddot{\phi} &= \frac{1}{t} \dot{\xi} \alpha^{\frac{1}{n-2}} + \frac{-1 - \frac{1}{n-2} - 2}{n-2} \xi \alpha^{\frac{1}{n-2}} \\
\dddot{\phi} &= \frac{1}{t^2} \ddot{\xi} \alpha^{\frac{1}{n-2}} + \frac{-2}{n-2} \frac{1}{t^2} \dot{\xi} \alpha^{\frac{1}{n-2}} + \frac{1}{(n-2)^2} \xi \alpha^{\frac{1}{n-2}} + \frac{2}{t^2} \frac{1}{(n-2)^2} \xi \alpha^{\frac{1}{n-2}} \tag{5.50}
\end{align}

There is also the potential part of the equation of motion, so we have to rewrite

\[ V'(\phi) = -2cH^2 \phi + \frac{|\lambda|^2}{M^{2n-6}} (2n-2) \phi^{2n-3}. \tag{5.53} \]

Where $-2cH^2 = -2c \frac{4}{M^2} = -2c'$.

Using (5.50) we get

\begin{align}
V' &= -\frac{2c'}{t^2} \xi \alpha^{\frac{1}{n-2}} + \frac{|\lambda|^2}{M^{2n-6}} (2n-2) \xi^{2n-3} \alpha^{\frac{2n-3}{n-2}} \\
&= -\frac{2c'}{t^2} \xi \alpha^{\frac{1}{n-2}} + \frac{|\lambda|^2}{M^{2n-6}} (2n-2) \xi^{2n-3} \alpha^{\frac{1}{n-2}} \\
&= -\frac{2c'}{t^2} \xi \alpha^{\frac{1}{n-2}} + \frac{|\lambda|^2}{M^{2n-6}} (2n-2) \left( \frac{c'}{n-1} \frac{1}{|\lambda|^2} M^{2n-6} e^{-2z} \right) \xi^{2n-3} \alpha^{\frac{1}{n-2}} \\
&= -\frac{2c'}{t^2} \xi \alpha^{\frac{1}{n-2}} + \frac{2c'}{t^2} \xi^{2n-3} \alpha^{\frac{1}{n-2}}. \tag{5.54}
\end{align}

Now we add all the pieces together and find the equation of motion becomes

\begin{align}
\dddot{\phi} + \frac{2}{t} \ddot{\phi} + V' &= \frac{1}{t^2} \dddot{\xi} \alpha^{\frac{1}{n-2}} + \frac{1}{t^2} \dot{\xi} \alpha^{\frac{1}{n-2}} \frac{-2}{n-2} + \frac{1}{t^2} \ddot{\xi} \alpha^{\frac{1}{n-2}} \frac{1}{(n-2)^2} + \frac{2}{t^2} \frac{1}{(n-2)^2} \xi \alpha^{\frac{1}{n-2}} + \\
&+ \frac{1}{t^2} \xi \alpha^{\frac{1}{n-2}} - \frac{1}{n-2} \frac{-2c'}{t^2} \xi \alpha^{\frac{1}{n-2}} + \frac{2c'}{t^2} \xi^{2n-3} \alpha^{\frac{1}{n-2}} \\
&= \frac{1}{t^2} \dddot{\xi} \alpha^{\frac{1}{n-2}} + \frac{2}{t^2} \ddot{\xi} \alpha^{\frac{1}{n-2}} - \xi \frac{2n-3}{n-2} \alpha^{\frac{1}{n-2}} + \\
&+ \frac{-2c'}{t^2} \xi \alpha^{\frac{1}{n-2}} + \frac{2c'}{t^2} \xi^{2n-3} \alpha^{\frac{1}{n-2}} = 0. \tag{5.55}
\end{align}
Which, after multiplying with $t^2$ and $\alpha^{-1}$, becomes:

$$\ddot{\xi} + 2\dot{\xi} \frac{n-3}{n-2} - \xi \frac{2n-3}{(n-2)^2} - 2c'\xi + 2c'\xi^{2n-3} = 0 \quad (5.56)$$

The resulting equation of motion is

$$\ddot{\xi} + \dot{\xi} \left( \frac{2n-6}{n-2} \right) - \xi \left( \frac{2n-3}{(n-2)^2} + 2c' \right) + 2c'\xi^{2n-3} = 0 \quad (5.57)$$

Notice that this equation is not explicitly dependent on either $t$ or $\phi$, these have both dropped from the equation using our transformation. This means that we have found a different field that tells us something about the evolution of the Affleck-Dine field. Namely, we notice from equation (5.57) that $\xi$ has a stable point at

$$\bar{\xi} = \left( 1 + \frac{2n-3}{2c'(n-2)^2} \right)^{\frac{1}{n-4}} \quad (5.58)$$

### 5.5.2 A stable point

This means that once $\phi$ has settled in the minimum during the inflationary epoch after this epoch ends and the minimum will start to walk, $\phi$ will follow this minimum by sitting in the stable point. Evaluation of $\bar{\xi}$ shows that it is of order $O(1)$. In the table below we have taken $c = 1$.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$\xi(c = 1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2.1</td>
</tr>
<tr>
<td>4</td>
<td>1.3</td>
</tr>
<tr>
<td>5</td>
<td>1.1</td>
</tr>
<tr>
<td>6</td>
<td>1.1</td>
</tr>
<tr>
<td>7</td>
<td>1.0</td>
</tr>
</tbody>
</table>

When $\phi$ starts in a stable point — such as a minimum — it will stay there and track the minimum since $\phi(t) = \xi \phi_0(t) = \bar{\xi} \phi_0(t)$ and so $\phi(t)$ will follow $\phi_0(t)$ and lag a little bit behind because $\bar{\xi}$ is a bit larger than 1.

Up to this point the angular part of the minimum was not of much concern, since in the Inflationary epoch it rolls to the minimum and stays there due to the expansion of the universe and the large Hubble constant. And during the period just after inflation the field will follow the minimum around until the A term becomes important enough that it can
5.6. Late stage: build-up of baryonic charge

In this epoch the most interesting behaviour starts to show, the build up of baryonic charge, resulting in a baryon number that is non-zero. And after the Affleck-Dine field decays, this non-zero number will be conserved and it results in an asymmetry in the number of baryons and anti-baryons.

5.6.1 General properties

Inspecting the full potential

\[ V(\phi) = (m^2 - cH^2)|\phi|^2 + \left( \frac{(Am^2/2 + aH)\lambda}{nM^{n-3}} \phi^n + \text{h.c.} \right) + |\lambda|^2 \frac{|\phi|^{2n-2}}{M^{2n-6}} \]

where the Hubble parameter is time dependent since the universe is dominated by inflaton matter: \( H = 2/(3t) \) and the constants \( c, A \) and \( a \) are of order \( \mathcal{O}(1) \) and \( \lambda \) can be absorbed into \( \phi \) using a field redefinition so its angle is not of great importance. \( A \) and \( a \) are not necessarily Real numbers. In particular, \( A \) and \( a \) should be non-real, or at least one of them should be, so that you get two minima with different angles \( \theta_a \) and \( \theta_A \). These angles are just the arguments of complex numbers \( a = |a|e^{i\theta_a} \) and \( A = |A|e^{i\theta_A} \), which appear in the potential when you explicitly write out the conjugate part: \( a + a^* = 2|a| \cos(\theta_a) \), \( A + A^* = 2|A| \cos(\theta_A) \).

\[ \text{Before we took } A \text{ to be Real, but it does not have to be real at all. In the end the important thing is that there is a phase difference.} \]
5.6.2 The mechanism

The idea is as follows: in the early universe when the Hubble parameter is very large we can throw away the $A$-term. So $\phi$ will roll to a minimum at angle $\theta_a$ and follow it around during the Post-Inflation epoch. As time progresses, the Hubble parameter $H$ will get smaller by the $t^{-1}$ relation. The Late Stage will start when $H$ is of order $m_{3/2}$ and both contribute to the potential.

When $H \sim m_{3/2}$ the minimum of the potential shifts from angle $\theta_a$ to $\theta_A$ and $\phi$ will roll towards the new minimum. In the previous epochs $\phi$ started out with a non-zero VEV and was rolling in the radial direction towards (0, 0), but now the minimum has shifted in the $\theta$-direction and the field will pick up angular momentum trying to roll towards the new minimum at angle $\theta_A$. This will yield a non-zero $\dot{\theta}$.

At the same time the Hubble parameter has shrunk enough so that the damping term is relatively small. Simultaneously the $cH^2$ term is also getting smaller and the $m^2|\phi|^2$ term will come to dominate in this epoch, meaning that the local minimum at (0, 0) will slowly turn into the global minimum, where $\phi$ will try to roll towards.

\[
(m_\phi^2 - cH^2)|\phi|^2 \approx -cH^2|\phi|^2, \quad H \gg m_\phi \geq m_{3/2}. \quad (5.60)
\]
\[
(m_\phi^2 - cH^2)|\phi|^2 \approx m_\phi^2|\phi|^2, \quad H \ll m_{3/2} \leq m_\phi. \quad (5.61)
\]

This means that $\phi$ will pick up an angular momentum and during this will roll towards the middle, after which it is stuck with said momentum and circles around the minimum.

5.6.3 Analytical solution

In section \([5.5]\) we calculated the minimum from which we can find the initial condition for the velocity.
5.6. Late stage: build-up of baryonic charge

\[ \phi_0(t) = \left( \sqrt{\frac{c}{n-1} 3t} \frac{2 M^{n-3}}{|\lambda|} \right)^{\frac{1}{n-2}} \]  
\[ \dot{\phi}_0(t) = \frac{1}{n-2} \left( \sqrt{\frac{c}{n-1} 3t} \frac{2 M^{n-3}}{|\lambda|} \right)^{\frac{1}{n-2}-1} \left( \sqrt{\frac{c}{n-1} 3t^2} \frac{-2 M^{n-3}}{|\lambda|} \right) \]

When we look at the full equation of motion (4.4) with a complex field this will give us in practice the two equations

\[ \ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0 \]  
\[ \ddot{\bar{\phi}} + 3H\dot{\bar{\phi}} + \frac{\partial V}{\partial \bar{\phi}} = 0. \]

But a quick glance at the potential (7.1) will show us that these equation can not be solved analytically.

\[ \frac{\partial V}{\partial \phi} = 2(m^2 - cH^2)\bar{\phi} + \left( \frac{Am_{3/2} + aH}{M^{n-3}} \right) \lambda \phi^{n-1} + \frac{(2n-2)|\lambda|^2}{M^{2n-6}} |\phi|^{2n-4} \bar{\phi} \]

\[ \ddot{\phi} + 3H\dot{\phi} + (m_{\phi}^2 - cH^2)\bar{\phi} + \left( \frac{Am_{3/2} + aH}{M^{n-3}} \right) \lambda \phi^{n-1} + \text{h.c.} \]
\[ + (2n-2)|\lambda|^2 \frac{|\phi|^{2n-4} \bar{\phi}}{M^{2n-6}} = 0. \]

This part would give a 5th order piece for \( n = 4 \) including a complex conjugated part, which by itself is already very hard to solve analytically.\(^4\)

5.6.4 Numerical solution

We will have to use numerical tools to solve the equations to study the behaviour of the model. To do so we prefer to work with the field equations that are of the form

\(^4\)Remember from chapter 3 that \( n = 4 \) is the lowest physical value.
\[ \ddot{\phi}_x + 3H \dot{\phi}_x + \frac{\partial V}{\partial \phi_x} = 0 \] (5.68)  
\[ \ddot{\phi}_y + 3H \dot{\phi}_y + \frac{\partial V}{\partial \phi_y} = 0. \] (5.69)

So instead of just the parametrization \( \phi = \rho e^{i\theta} \) we are also going to use the canonical fields \( \phi = \frac{\phi_x + i\phi_y}{\sqrt{2}} \), and to complete the computation we will have to switch here and there to make it a bit faster.

The derivatives of the potential are given by
\[
\frac{\partial V}{\partial \phi} = 2(m^2 - cH^2)\tilde{\phi} + \left( \frac{A_{m3/2} + aH}{M^{n-3}} \right) \lambda \phi^{n-1} + \frac{(2n-2)|\lambda|^2}{M^{2n-6}} |\phi|^{2n-4} \tilde{\phi}, \] (5.70)
\[
\frac{\partial V}{\partial \bar{\phi}} = 2(m^2 - cH^2)\phi + \left( \frac{\bar{A}_{m3/2} + \bar{a}H}{M^{n-3}} \right) \bar{\lambda} \phi^{n-1} + \frac{(2n-2)|\lambda|^2}{M^{2n-6}} |\phi|^{2n-4} \phi. \] (5.71)

Rewriting \( \phi \), where we use \( \rho = |\phi| \):
\[
\frac{\partial V}{\partial \phi} = 2(m^2 - cH^2)(\phi_x - i\phi_y) + \left( \frac{A_{m3/2} + aH}{M^{n-3}} \right) \lambda (\phi_x + i\phi_y)^{n-1} \] 
\[ + \frac{(2n-2)|\lambda|^2}{M^{2n-6}} \rho^{2n-4}(\phi_x - i\phi_y), \] (5.72)
\[
\frac{\partial V}{\partial \bar{\phi}} = 2(m^2 - cH^2)(\phi_x + i\phi_y) + \left( \frac{\bar{A}_{m3/2} + \bar{a}H}{M^{n-3}} \right) \bar{\lambda} (\phi_x - i\phi_y)^{n-1} \] 
\[ + \frac{(2n-2)|\lambda|^2}{M^{2n-6}} \rho^{2n-4}(\phi_x + i\phi_y). \] (5.73)

For simplicity we will use \( a = 0 \) in the next couple of steps to keep the computation compact, but the principle is the same. Parameter \( a \) adds an extra phase to the evolution of the field, by setting it to zero we just remove this phase temporarily.

Now with this parametrization we get \( \phi + \bar{\phi} = 2\phi_x \) and \( \phi - \bar{\phi} = 2i\phi_y \).

Realizing that the time derivative works on field piece-wise we see \( \dot{\phi} = \dot{\phi}_x + i\dot{\phi}_y \) and \( \dot{\phi} = \bar{\phi}_x + i\bar{\phi}_y \) we find
5.7. Dimensionless field equations

\[ \ddot{\phi} + 3H \dot{\phi} + \frac{\partial V}{\partial \phi} = (\ddot{\phi}_x + i\ddot{\phi}_y) + 3H(\dot{\phi}_x + i\dot{\phi}_y) + \frac{\partial V}{\partial \phi} = 0, \]  
(5.74)

\[ \ddot{\bar{\phi}} + 3H \dot{\bar{\phi}} + \frac{\partial V}{\partial \bar{\phi}} = (\ddot{\bar{\phi}}_x - i\ddot{\bar{\phi}}_y) + 3H(\dot{\bar{\phi}}_x - i\dot{\bar{\phi}}_y) + \frac{\partial V}{\partial \bar{\phi}} = 0. \]  
(5.75)

Equations (7.42) + (7.43) / 2 yields

\[ \ddot{\phi}_x + 3H \dot{\phi}_x + (m^2 - cH^2)\phi_x + \left( \frac{Am^{3/2}}{M^{n-2} \lambda} \right) \rho^{n-1} \cos ((n-1)\theta + \theta_{\lambda} + \theta_{A}) 
+ \frac{(2n-2)|\lambda|^2}{M^{2n-6}} \rho^{2n-4} \phi_x = 0, \]  
(5.76)

and (7.42) - (7.43) / 2i results in

\[ \ddot{\phi}_y + 3H \dot{\phi}_y + (m^2 - cH^2)\phi_y + \left( \frac{Am^{3/2}}{M^{n-2} \lambda} \right) \rho^{n-1} \sin ((n-1)\theta + \theta_{\lambda} + \theta_{A}) 
+ \frac{(2n-2)|\lambda|^2}{M^{2n-6}} \rho^{2n-4} \phi_y = 0. \]  
(5.77)

These equations can be integrated numerically if we use the coordinates

\[ \rho^2 = \phi_x^2 + \phi_y^2, \]  
(5.78)

\[ \theta = \arctan \frac{\phi_y}{\phi_x}, \]  
(5.79)

we get two coupled differential equations that describe the behaviour of the Affleck-Dine field.

5.7 Dimensionless field equations

Now we will try to make the equations of motion of this model dimensionless, to see what kind of dependencies it has and how the parameters are related. We start with

\[ \phi = \alpha X, \quad t = \beta \tau \]  
(5.80)
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giving us the relations

\[
\frac{\partial}{\partial t} = \frac{1}{\beta} \frac{\partial}{\partial \tau}, \quad H = \frac{2}{3t} = \frac{1}{\beta^2} \frac{\partial}{\partial \tau} \frac{1}{\beta} h(\tau) \tag{5.81}
\]

\[
\frac{\partial \phi}{\partial t} = \frac{\alpha}{\beta} X', \quad \frac{\partial^2 \phi}{\partial t^2} = \frac{\alpha}{\beta^2} X''. \tag{5.82}
\]

Now we take a look at the potential,

\[
V(\phi) = (m^2 - cH^2)|\phi|^2 + (aH + \tilde{A}m_{3/2}) \frac{\lambda \phi^n}{n M^{n-3}} + \text{h.c.} + |\lambda|^2 \frac{\phi^{2n-2}}{M^{2n-6}} \tag{5.83}
\]

\[
\frac{\partial V}{\partial \phi} = (m^2 - cH^2)\phi + (aH + \tilde{A}m_{3/2}) \frac{\lambda \phi^{n-1}}{n M^{n-3}} + |\lambda|^2 (n-1) \frac{\phi^{2n-4}}{M^{2n-6}}. \tag{5.85}
\]

Our field equations are

\[
\ddot{\phi} + 3H \dot{\phi} + \frac{\partial V}{\partial \phi} = 0 \tag{5.87}
\]

and become of the form

\[
\frac{\alpha}{\beta^2} X'' + \frac{\alpha}{\beta^2} 3h(\tau) X' + (m^2 - c \frac{1}{\beta} h(\tau)^2) \alpha X
\]

\[
+(\alpha \frac{1}{\beta} h(\tau) + \tilde{A}m_{3/2}) \frac{\lambda \phi^{n-1} X^{n-1}}{M^{n-3}} + |\lambda|^2 (n-1) |\alpha|^{2n-4} |X|^{2n-4} \frac{X}{M^{2n-6}} = 0. \tag{5.88}
\]

Now we rewrite it so that we can get a good perspective on the relation between the parameters \(\alpha\) and \(\beta\).
5.7. Dimensionless field equations

\[ X'' + 3h(\tau)X' + (\beta^2m_\phi^2 - ch(\tau)^2)X = 0 \] (5.89)

\[ + (\bar{a}\beta h(\tau) + \bar{A}\beta m_{3/2}/m_{3/2} \frac{\lambda n^{-1} X^{n-1}}{M^{n-3}} + |\lambda|^2(n - 1)|\alpha|^{2n-4}\beta^2 \frac{|X|^{2n-4} X}{M^{2n-6}} = 0. \]

We choose parameter \( \beta \) such that \( \beta^2m_\phi^2 = 1 \), or \( \beta = m_\phi^{-1} \). And we see in the last term of equation (5.89) the combination

\[ \frac{|\alpha|^{2n-4}}{m_\phi^{2n-6}} \] (5.90)

which has to be dimensionless. Dimensional analysis gives \([\alpha] = \text{GeV}\), so we know it has to be a mass. The obvious choice from \( \bar{A}m_{3/2}/m_{3/2} \) is \( \alpha = m_{3/2} \), giving us

\[ X'' + 3h(\tau)X' + (1 - ch(\tau)^2)X = 0 \] (5.91)

\[ + (\bar{a}h(\tau) + \bar{A}m_{3/2}/m_{3/2} \frac{\lambda n^{-1} X^{n-1}}{m_\phi M^{n-3}} + |\lambda|^2(n - 1)\frac{m_{3/2}^2 |X|^{2n-4} X}{m_\phi^{2n-6}} = 0. \]

Now we define parameter \( b \)

\[ b^{n-2} = \frac{m_\phi M^{n-3}}{\lambda m_{3/2}} \] (5.92)

where we have taken \( \lambda \) to be real as we could already absorb it into \( \phi \).

And we find our final result for the equations of motion

\[ X'' + 3h(\tau)X' + (1 - ch(\tau)^2)X = 0 \] (5.93)

\[ + (\bar{a}h(\tau) + \bar{A}m_{3/2}/m_{3/2} \frac{X^{n-1}}{b^{n-2}} + (n - 1)\frac{|X|^{2n-4} X}{b^{2n-4}} = 0. \]

We see that the parameters still in the equation are \( c, a, A \), the combination \( \frac{m_{3/2}}{m_\phi} \) and our new parameter \( b \). So in general we expect that \( n_X \) will depend on all these parameters. However, in our case we take a few parameters of order \( \mathcal{O}(1) \), these parameters are \( c, a, A, \lambda \).
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With the complex angles of $a$ and $A$ this results in a general dependence on $\theta_a, \theta_A, m_{3/2}, b$ and of course $n$:

$$n_X = n_X(b, c, \theta_a, \theta_A, \frac{m_{3/2}}{m_\phi}, n). \quad (5.94)$$

### 5.8 Asymmetry parameter $\eta$

Before we go on with the actual computation of the data and trying to make sense of it all we want to know the physical relevance of all these equations. We might get a lot of information from the behaviour of the system, but in the end, we will have to relate the particle number created during AD baryogenesis to something that can be measured. As mentioned in the introduction, a measurable quantity is the asymmetry parameter $\eta$ and it is defined as follows for baryons, leptons and dark matter by

$$\eta_X = \frac{n_X - n_{\bar{X}}}{s}. \quad (5.95)$$

In the numerator we have the difference in particle and anti-particle density and in the denominator the entropy density of the universe. For baryons we have measured

$$\eta_{B,measured} = \frac{n_B - n_{\bar{B}}}{n_\gamma} = 6.1(3) \times 10^{-10} \quad (5.96)$$

which is related by $n_\gamma \approx 7s$ or $\eta_{B,measured} = \frac{1}{7} \eta_{B,calculated}$.

The reason we use this is because it is invariant in time during the radiation domination era of the universe. We can see this from the following

$$\frac{n_X - n_{\bar{X}}}{s} = \frac{(N_B - N_{\bar{B}})/\text{Vol}}{S/\text{Vol}}. \quad (5.97)$$

$N_B$ and $N_{\bar{B}}$ are the baryon- and anti-baryon number, which nowadays are conserved in SM interactions since the $\bar{B}$-operators decouple after baryogenesis, and the entropy of the universe is constant and maximal since we are in thermal equilibrium since reheating.

So we can say $N_B = n_X(t_{osc})a^3(t_{osc}) = n_X(t_R)a^3(t_R)$, where the field starts oscillating at $t_{osc}$ and the baryon number is generated. This leads to

$$n_X(t_R) = n_X(t_{osc})\frac{a_{osc}^3}{a_R^3}. \quad (5.98)$$
5.8. Asymmetry parameter $\eta$

After inflation we are in an inflaton matter dominated universe, so the Hubble parameter is

$$\frac{\dot{a}}{a} = H = \frac{2}{3t}. \quad (5.99)$$

This means of course that $a$ as function of $t$ is given by

$$a \propto t^{2/3} \propto H^{-2/3}. \quad (5.100)$$

in other words

$$V \propto a^3 \propto \frac{1}{H^2}. \quad (5.101)$$

Putting this all together we find

$$n_X(t_R) = n_X(t_{osc}) \frac{H^2_R}{H^2_{osc}}. \quad (5.102)$$

After reheating the entropy density becomes $s(t_R) = 4H^2_R M^2_{pl}/T_R$ which yields

$$\eta_X = \frac{n_X(t_R)}{s(t_R)} = \frac{n_X(t_{osc})T_R}{4H^2_{osc} M^2_{pl}}. \quad (5.103)$$

Plugging in $S_R = \frac{2\pi^2}{45} g_R T^3_{R} a^3_{R}$ for the reheating entropy and $H^2_R = \frac{g_R T^4_{R}}{M^2_{pl}}$ for the Hubble parameter\[21\] at reheating we find

$$\eta_X = \frac{n_X(t_{osc})g_R T^3_{R} H^2_{osc} M^2_{pl}}{2\pi^2} = \frac{n_X(t_{osc})T_R}{2\pi^2 H^2_{osc} M^2_{pl}}. \quad (5.104)$$

Which is the same besides the numerical factor $\frac{2\pi^2}{45} \approx 0.4$.

We know that $n_X = g_X \phi^2 \dot{\theta}$ which we will try to find in terms of the parameters of the model. Therefore if we can express the baryon density $n_X$ in terms of its parameters we have found a formula that can be completely evaluated. In the next chapter we will investigate the behaviour of the field numerically.
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Chapter 6

Numerical solution of the Affleck-Dine field

Now that we have equations that can be solved numerically, we can start inspecting the field evolution and adjust the boundary conditions to see their influence on the behaviour of the field $\phi$. 

6.1 Toy model

The general way a field evolves is shown in figure (6.1).

In this very basic example we took trivial parameters like we are dealing with a toy model, just to solve it easily without numerical problems. The parameters are

$$n = 4, \ c = 1, \ m_\phi = 1 \text{ GeV}, m_{3/2} = 1 \text{ GeV}, \ M = 1 \text{ GeV},$$
$$a = 0.8 + 0.2i, \ A = 0.8, \ \lambda = 0.8 + 0.2i, \ H = 1 \text{ GeV.} \quad (6.1)$$

The first thing we notice is that when we take $m_\phi \lesssim m_{3/2}$ as parameters we get a situation that results in the field rolling towards a local minimum. Not the minimum at the origin, but the angular minimum that is associated with the supersymmetry breaking and non-renormalizable operators.

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\[1\text{In the first few paragraphs we are only interested in the phenomenology of the system so our plots do not use proper units until section 6.2.6.}\]
Chapter 6. Numerical solution of the Affleck-Dine field

Figure 6.1: (a) A characteristic path of the field in $\phi$-space, rolling towards the middle. (b) Using comoving volumes we see that energy is conserved. Red is total energy of the system while green is potential and blue kinetic energy of the fields. We notice that there is a difference between $T + V$ and $E$, this is the rotational energy that the field carries.

Figure 6.2: (a) The angular momentum density builds up quickly after a kick but decreases due to the expansion of the universe. (b) Using comoving volumes we clearly see that the angular momentum gets a kick and is conserved shortly after.

Since our universe is expanding the baryon number density becomes smaller and smaller because the baryon number $N_B$ is conserved after generation. The volume gets larger and larger, resulting in $n_B = N_B/V$ decreasing. To get meaningful quantities we sometimes have to get rid of this expansion factor and we do this by using comoving volumes. This basically means we get rid of the expansion factor hidden in our fields.

In other words, every quantity has a density in space, but this space is expanding. It is expanding with the expansion factor $a(t)$, which are known for the different eras of the universe. In particular, $V \propto a(t)^3$. During the radiation dominated era, $a(t) \propto t^{\frac{1}{2}}$ and during the matter dominated era, $a(t) \propto t^{\frac{2}{3}}$. We want to remove this spatial expansion. For some density $n_X$ in an expanding space,

$$n_X = \frac{N_X}{V(t)} \rightarrow n_X(t') = \frac{N_X}{V(t')}, \text{ with } t' > t \Rightarrow V(t') > V(t). \tag{6.2}$$
6.2. Behaviour of the field

To remove this time dependent volume we multiply by $a(t)^3$.

$$n_X = (t^3)^3 n_X(t) = t^2 n_X(t).$$

(6.3)

So now we have a general idea of how the field behaves and moves through the potential, and what the behaviour is of the energy and the angular momentum of the field, and how to use comoving volumes to find conserved quantities.

6.2 Behaviour of the field

Now we are going to look at the behaviour of the baryon density of the field as function of its parameters. We use as definitions

$$n_B(t) = \phi(t)^2 \dot{\theta}(t),$$

(6.4)

$$a^3(t)n_B(t) = t^2 n_B(t).$$

(6.5)

6.2.1 Angular dependence

The dependence of $n_B$ on the parameters $a$, $A$, and $\lambda$ is shown in figure (6.3).

Figure 6.3: (a) The angular dependence of $n_B$ as a function of $\theta_a$. (b) The angular dependence of $n_B$ as a function of $\theta_A$. In both cases have we taken the minimum to be in the first quadrant of the potential. Except for the deviation around $\theta_{a,A} = \pi$ they are almost a perfect sines.

As we can see, the angle of parameter $a$ and $A$ influence the final baryon number that is generated in the system. Moreover it looks quite like a sine with a small deviation at $\theta = \pi$. So we can tweak the angle to adjust the baryon number to get the value closest to the physical value.
Chapter 6. Numerical solution of the Affleck-Dine field

In the figure (6.4) (a) the difference (or rather the addition) is shown of (6.3) (a) and (b): 

\[ n_B(\theta_a) + n_B(\theta_A). \]

We see that it is very close to zero - even more so compared to the values of \( n_B \) - so this is probably due to the numerical errors that are exhibited in such calculations. Therefore it only depends on the difference between the angles, \( (\theta_a - \theta_A) \). In figure (b) we see the influence of \( \theta_\lambda \), which is even less than in figure (a). So we can conclude that \( \theta_\lambda \) has no influence, which makes sense if we look at the potential, because it is in front of both \( a \) and \( A \), so we when we tweak it, it works on both angles at the same time, negating the effect on \( n_B \). In other words: we can absorb \( \theta_\lambda \) into the original \( n\theta \) in equation (7.42) or, when we calculate the phase-difference between \( \theta_a \) and \( \theta_A \) the \( \lambda \) cancels out since it is on both sides of the equation.

Figure 6.4: (a) In this plot we see the error between figure (6.3) (a) and (6.3) (b). From figure (6.3) we see that they are one period out of phase, so we add them to find the numerical error. (b) This is the angular dependence of \( n_B \) as function of \( \theta_\lambda \), it is even smaller than the numerical error in (a).

And finally in figure (6.5) we have a 3D scatter plot of \( n_B \) as a function of both \( \theta_a \) and \( \theta_A \). The resulting values of \( n_B \) show the exact same shape and we can conclude that only the difference between angles \( \theta_a - \theta_A \) is of importance. We see the sine, and we know from the theory that the angles can be absorbed into one another, meaning that if you take one constant the other one will give the sine shape and if we do it the other way around we get the same shape. We can conclude that a large difference \( \theta_a - \theta_A \) yields a larger baryon number \( n_B \) which makes sense, since the field has to roll farther in the same time-span.

Since the field starts out in the minimum there is no dependence on an initial phase, only on the difference between the phases.

We can conclude that the baryon density is proportional to

\[ n_B \propto \sin (\theta_a - \theta_A). \]  

(6.6)
6.2. Behaviour of the field

Figure 6.5: $n_B$ plotted against $\theta_a$ and $\theta_A$ simultaneously.

6.2.2 Conditions on the mass range

Continuing categorizing the behaviour of the system for its parameters in the next part, we have used

\begin{align}
  n &= 4, \ c = 1, \ m_\phi = 0.3 \text{ GeV}, \ m_{3/2} = 1 \text{ GeV}, \ M = 1 \text{ GeV}, \\
  a &= 0.8 + 0.2i, \ A = 0.8, \ \lambda = 0.8 + 0.2i
\end{align}

(6.7)

(6.8)

where the starting position and velocity were chosen at random and are of order $\mathcal{O}(1)$. We did add a starting velocity to induce a rotation since we have $a = A$. We chose these random initial values to see if the system would react differently than if we would just take the minimum as starting point, but it only mattered with very carefully chosen initial values when the masses are of the same order, $m_\phi \sim m_{3/2}$.

The results can be seen in figure (6.6).

We see that the field gets stuck in a minimum at $\phi \neq 0$ and stays there. Once the field is stuck in this minimum, the angular momentum will dissipate and the field will stop rotating. At some point in time, the local minima will still deform and disappear, but since masses differ it will be after $H \sim m_{3/2}$. As the minimum disappears, the field will roll towards the global minimum at the origin, but it will not have the angular momentum it would have gotten when $m_\phi > m_{3/2}$ since it has dissipated. Therefore it will just roll towards the middle without any angular momentum, resulting in a zero baryon number.

Moreover, this effect is amplified by the difference in the masses, see figure (6.9).

And when the mass difference becomes even a slightly bit bigger, with $m_\phi = 0.1\text{GeV}$ and $m_{3/2} = 1\text{GeV}$, so a factor of 10, the minimum is not lifted at all.
Chapter 6. Numerical solution of the Affleck-Dine field

Figure 6.6: (a) The field starts out somewhere on the unit circle in the first quadrant and ends up in an angular minimum. (b) An almost identical situation from the third quadrant.

Figure 6.7: (a) Once again the field rolls towards an angular minimum that would have disappeared if $m_\phi \gtrsim m_{3/2}$. (b) For well chosen initial conditions it is possible to end up in the centre, even though it is rare.
6.2. Behaviour of the field

Figure 6.8: With $m_\phi < m_{3/2}$ the field rolls towards a local minimum and stays there until the angular momentum has dissipated. When the angular minimum disappears at late times $\phi$ will start moving again, but without angular momentum, resulting in zero baryon number.

Figure 6.9: (a) Here $m_\phi = 1$ GeV, $m_{3/2} = 0.3$ GeV and the time it takes for the lifting to occur is from $t = 0.7$ GeV$^{-1}$ to $t = 2$ GeV$^{-1}$. (b) Here we have the masses $m_\phi = 0.3$ GeV, $m_{3/2} = 1$ GeV and the lifting time is from $t = 1$ GeV$^{-1}$ to $t \sim 20$ GeV$^{-1}$.
Figure 6.10: $m_\phi = 0.27 \text{ GeV}, m_{3/2} = 1 \text{ GeV}$. The time scale for the lifting is now respectively $t = 3 \text{ GeV}^{-1}$, $t = 5 \text{ GeV}^{-1}$, $t = 10 \text{ GeV}^{-1}$, $t = 10^{10} \text{ GeV}^{-1}$. So a tiny mass-difference can make the potential not lift at all.

In figure (6.10) we see that even on an exponential time scale the angular minimum is not lifted if the mass-difference is big enough. This makes sense because if the $Am_{3/2}$ term is larger than the $m_\phi^2$ term we always get a minimum below zero away from the origin. When the $H$ terms have disappeared the potential will be of the form

$$V(\phi, \theta) = m_\phi^2 |\phi|^2 + \frac{Am_{3/2} \lambda |\phi|^n \cos (n\theta + \theta_A)}{n M^{n-3}} + \frac{|\lambda||\phi|^{2n-2}}{M^{2n-6}}.$$  \hspace{1cm} (6.9)

For sufficiently large $m_{3/2}$ the $A$-term will always be larger than the $m_\phi$ term, and with the minus sign of the cosine, will always result in an angular minimum at every time scale.

$$m_\phi^2 > m_{3/2} |A||\lambda||\phi|^{n-2}.$$  \hspace{1cm} (6.10)

Which for small enough $\phi$ comes down roughly to (seeing as we take $A$ and $\lambda$ at order $O(1)$)

$$m_\phi^2 \gtrsim m_{3/2}.$$  \hspace{1cm} (6.11)

that must be satisfied if we want to have successful baryogenesis.

### 6.2.3 Influence of parameter $c$

When we look at the potential $V_{AD}$ we can also wonder what the influence is of parameter $c$. 

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6.2. Behaviour of the field

\[ V(\phi) = (m_\phi^2 - cH^2)|\phi|^2 + \left( \frac{(Am_{3/2} + aH)\lambda}{nM^{n-3}} \phi^n + \text{h.c.} \right) + |\lambda|^2 \frac{|\phi|^{2n-2}}{M^{2n-6}}. \]  

(6.12)

We first notice that \( c \) is only in the first part of the potential, so we know that it only affects \((m_\phi^2 - cH^2)|\phi|^2\). This only influences when the minimum of the potential is deformed, when the minimum disappears. Before we had deformation happening around \( H \sim m_\phi \) since \( c \) is usually taken of order \( \mathcal{O}(1) \). But now it becomes \( \sqrt{c}H \sim m_\phi \), so let’s take a look at the numerical evolution.

In the regular case the usual angular momentum build up is of the form:

\[
\dot{t} = t_{\text{osc}} \iff \dot{\phi}_x\phi_y - \phi_x\dot{\phi}_y \text{ is maximal.}
\]  

(6.13)

The plots in this section use the following parameters unless otherwise specified.

\[
n = 4, \ m_\phi = 10^5 \text{ GeV}, \ m_{3/2} = 10^2 \text{ GeV}.
\]  

(6.14)

The oscillation time is \( t_{\text{osc}} = 2 \times 10^{-5} = 2m_\phi^{-1} \).

Now we make \( c \) of order \( \mathcal{O}(10^2) \) and we get the curve of figure (6.12).

It is obvious that the angular momentum is again generated and conserved after a while, but we see that in the non-normalized graph the form changes slightly. Also \( t_{\text{osc}} \) becomes a little smaller, but is still around \( 2m_\phi^{-1} \). Note that the angular momentum is also 3 orders larger than in the situation where \( c \) was 1.
In our final example we have $c$ of order $O(10^4)$ and we find this:

From these plots we already notice a general trend that $t_{osc}$ is proportional to $m^{-1}$ like we would expect from the theory and equation (6.12). The next plot shows a more general behaviour over a large range of $c$.

Figure 6.14: The oscillation time $t_{osc}$ as a function of $c$ with $n = 4, m_\phi = 10^6$ GeV, $m_{3/2} = 10^4$ GeV. The decay is very gradual. Higher dimensions show the same curve at different orders of magnitude.
6.2. Behaviour of the field

We notice that $t_{\text{osc}}$ declines slowly as function of $c$, the fitted function is $t_{\text{osc}} \propto c^{-0.06}$.

From the figure (6.14) we also notice that the angular momentum stabilization has a quite discontinuous shape. The total angular momentum is also a factor 5 larger for $c$ of order $\mathcal{O}(10^4)$.

Next we look at $n_B$ itself as a function of $c$, inspecting the baryon density we get figure (6.15).

![Figure 6.15: The baryon density for $n = 4$ (blue), $n = 5$ (purple), $n = 6$ (yellow) at large times. The fits are all of the form $a^3 n_B \propto c^1$ and they match very well with the data.](image)

We can conclude that $c$ definitely has influence on $t_{\text{osc}}$ for values larger than $10^1$, but for order $\mathcal{O}(1)$ the difference in $t_{\text{osc}}$ seems to be very small, maybe even negligible. More importantly we note that increase in $c$ does increase $n_B$, more notably our fits show it is $n_B \propto c^1$. So concluding:

\[
\begin{align*}
    t_{\text{osc}} &\propto 2 m_{\phi}^{-1} c^{-0.06}, \quad (6.15) \\
    n_B &\propto c^1. \quad (6.16)
\end{align*}
\]

We usually take $c$ of order $\mathcal{O}(1)$ resulting in $n_B(c) \propto 1$ and $t_{\text{osc}} \propto 2 m_{\phi}^{-1}$, but we can use $c$ to increase the created $n_B$. The value of $c$ depends on the coupling of the field $\phi$ to the inflaton, and if the masses of $m_{\phi}$ and $m_{3/2}$ turn out to be exceedingly large, say $\gg$ TeV, then this is a way to make sure we generate enough asymmetry.
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Figure 6.16: (a) From left to right the graphs correspond to a mass $m_{3/2}$ of $10$ GeV, ..., $10^7$ GeV. (b) This plot shows every function normalized for $m_{3/2}$. They seem to be the same.

6.2.4 Mass dependence

In figure (6.16) we see a clear power law relation between the gravitino mass, the mass of the Affleck-Dine field and the net baryon number. The baryon number is proportional to the quotient $m_{3/2}/m_\phi$. This can now give us information on the bounds of the masses of the particles in our model, since we have an experimental value for the asymmetry parameter $\eta$.

The power law relation of the masses and $n_B$ is obvious from figure (6.16). The relation between $m_3$, $m_\phi$ and $n_B$ is quite obvious from the raw data, as all points at the top of the plot have the exact same value. And only the ratio of masses $m_{3/2}/m_\phi$ determines $n_B$. Fits of the curves give us the following behaviour:

$$n_B \propto \frac{m_{3/2}}{m_\phi} \quad (6.17)$$

6.2.5 Dimension $n$

One important parameter we have not looked at is the dimension of the flat direction $n$. When one thinks about this parameter, what can it influence? It changes the slope of the potential since a higher $n$ makes the $\phi^{2n-2}$ term in the potential more dominant, but the denominator contains a $M^{2n-6}$ which would suppress it more. In general a higher $n$ means the flat direction is flatter since it is lifted at higher order. The result is a different velocity of the field and a therefore a different baryon number $n_B$. On top of that, having more angular minima makes the maximum angle that can be used for the kick of the field smaller, resulting in a lower baryon number as it will rotate slower.
6.2. Behaviour of the field

This is a quite complicated problem, and results are given in figure (6.17) without a fit.

Figure 6.17: The baryon number as function of operator dimension \( n \) for different mass ratios. Blue: \( m_{3/2}/m_\phi = 10^{-1} \), red: \( m_{3/2}/m_\phi = 10^{-2} \), yellow: \( m_{3/2}/m_\phi = 10^{-3} \), green: \( m_{3/2}/m_\phi = 10^{-4} \).

As we can see the baryon density is not a linear function in this plot and that it grows slower for every increase of \( n \). This means that the relation is not a polynomial, but grow slower than that. Unfortunately we have not enough data to make a proper conclusion about the full relation between dimension \( n \) and baryon number density \( n_B \).

Combining all results

Putting it all together we can find the baryon number as function of the parameters and is as follows.

\[ n_B \propto c \frac{m_{3/2}}{m_\phi} \sin (\theta_a - \theta_A) \]  

(6.18)

6.2.6 Physical values

Now that we have established some basic facts about the behaviour of the system we can study the evolution when we plug in physical values, such as \( m_\phi = 2 \times 10^4 \) GeV, \( m_{3/2} = 10^4 \) GeV and \( M = M_{pl} = 10^{18} \) GeV, which means we will also have to adjust our time-scales, since \( H \sim m_{3/2} \) means that

\[ \frac{2}{3t} \sim 10^4 \text{ GeV} \Rightarrow t \sim \frac{2}{3 \times 10^4 \text{ GeV}} \sim 10^{-29} \text{s} \]  

(6.19)

using the fact that \( 1 \text{ GeV}^{-1} \sim 10^{-25} \text{s} \).
In figure (6.18) we see the evolution of the Affleck-Dine potential using parameters

\[ m_\phi = 2 \times 10^4 \text{ GeV}, m_{3/2} = 10^4 \text{ GeV}, M = 10^{18} \text{ GeV}, \]

and the parameters \( c, a, A \) have all been chosen at random and are of order \( O(1) \).

Figure 6.18: Contourplot of the potential with \( m_\phi = 2 \times 10^4 \text{ GeV} \) and \( m_{3/2} = 10^4 \text{ GeV} \) shows the lifting of the minima over time. It starts at \( t = 10^{-4} \text{ GeV}^{-1} = 10^{-29} \text{s} \) and ends at about \( t = 1.03 \times 10^{-3} \text{ GeV}^{-1} \) with intervals of \( 10^{-6} \text{ GeV}^{-1} \).

In the contourplot\(^2\) we see that the minimum disappears extremely fast (in about \( 0.03 \times 10^{-29} \text{s} \)), below we see the actual evolution of the field through \( \phi \)-space. Inflation ends around \( t \sim 10^{-33} \text{ s} \) for certain models, so this gives us an upperbound on \( m_\phi \). During inflation the Hubble friction term in the equations of motion supresses any angular momentum density generation and the AD mechanism starts when \( H \sim m_\phi \) so this gives an upper bound for the mass of the field: \( H > m_\phi \).

The boundary conditions in figure (6.19) were chosen close to the bounds from the LHC, so we took the masses in the TeV scale. This gives an allowed \( n_B \) and we used particle masses

\(^2\)When one makes a 3D plot of this potential it resembles the underside of a paprika (or bell pepper). After the mexican hat, we now also have the paprika potential.
6.2. Behaviour of the field

Figure 6.19: $m_\phi = 2 \times 10^4$ GeV, $m_{3/2} = 1 \times 10^4$ GeV, $\theta_a = \pi/4$, $\theta_A = 0$. (a) The comoving angular momentum. (b) The angular momentum density of the field. (c) The evolution of the field through $\phi$-space, the rotation is not very obvious, but in (b) we see a non-zero angular momentum. (d) The total energy (red), potential (blue) and kinetic energy (green) of the field as function of time, starting from $t = t_0$. 
that are not excluded yet. We take the angle such that it gives a the largest value, since it can be scaled down all the way to zero, and with the mass of the field at $2 \times 10^4 \text{ GeV}$ and the gravitino mass at $10^4 \text{ GeV}$, we are close to the allowed bounds. The resulting density and $H$ are

$$n_B(t_{\text{osc}}) = 2.2 \times 10^{24} \text{ GeV}^3, \quad H_{\text{osc}} = 6.3 \times 10^3 \text{ GeV}. \quad (6.21)$$

In figure (6.19) we see the numerical solutions for the evolution of the field. In (a) we see the movement through $\phi$-space, starting from the minimum $\phi_0(t_0)$ with velocity $\dot{\phi}_0(t_0)$ given by equation (5.63). The field starts rotating after passing close to the origin and then rolls inwards. (b) shows the build up of angular momentum. The angular momentum stars out at 0, but shortly after, the field gets a kick and the angular momentum is generated. The values are comoving to remove the dependence on the size of the universe (which is expanding) and it is evident that after the initial kick the momentum is conserved.

Figure (c) shows the angular momentum not normalized for the expansion of the universe and we see the exact same shape as in figure (6.2). (d) shows the total energy of the field, normalized, and we see that it is conserved so the condensate can decay at some point with this energy, creating an abundance of beryons over anti-baryons. (d) also shows the potential en kinetic energy of the field.

These plotted functions are given by

$$(a) \quad a(t)^3 L = t^2(\phi'_x(t)\phi'_y(t) - \phi_x(t)\phi'_y(t)) \quad (6.22)$$

$$(b) \quad L = \phi'_x(t)\phi_y(t) - \phi_x(t)\phi'_y(t) \quad (6.23)$$

$$(c) \quad (\phi_x(t), \phi_y(t)) \quad (6.24)$$

$$(d) \quad T = a(t)^3 \frac{1}{2} \dot{\phi}^2, \quad V = a(t)^3 V_{AD} \quad E = T + V.$$

### 6.3 Asymmetry parameter $\eta$

From the previous chapter we know that the asymmetry parameter is

$$\eta_{B, \text{measured}} = \frac{1}{7} \eta_X = \frac{n_X(t_{\text{osc}}) T_R}{28 H_{\text{osc}}^2 M_{\text{pl}}^2}. \quad (6.25)$$

With the values from the previous section,

$$n_B(t_{\text{osc}}) = 2.2 \times 10^{24} \text{ GeV}^3, \quad H_{\text{osc}} = 6.3 \times 10^3 \text{ GeV}, \quad (6.26)$$
6.3. Asymmetry parameter $\eta$

we get

$$\eta_X = \frac{n_X(t_{osc})}{28H_{osc}^2M_{pl}^2} \frac{T_R}{T_R^3}$$  \hspace{1cm} (6.27)

$$6.1 \times 10^{-10} = \frac{2.2 \times 10^{24} \text{ GeV}^3}{28 \times 3.9 \times 10^7 \text{ GeV}^2} \frac{T_R}{10^{38} \text{ GeV}^2} \hspace{1cm} (6.28)$$

Giving us the corresponding value of $T_R$,

$$T_R = 3.0 \times 10^{13} \text{ GeV.} \hspace{1cm} (6.29)$$

Which is 6 orders below the Planck temperature. Gravitino production gives an upper bound on the reheating temperature at $T < 10^9 \text{ GeV}$, making our $T_R$ 4 orders too large.\[24\] Fortunately we can decrease the angle between the minima and perhaps increase the parameter $c$ to get a temperature within the bounds.

We will now try to get an analytical expression to match that of the calculated expression of the previous section. We have the relation $n_X = \phi^2 \dot{\theta} = \phi_{osc}^2 \Delta \theta$ and since we have the maximum angle we get $\Delta \theta = \sin(\theta_a - \theta_A) = 1$. And we also know that $t_{osc} \propto 2m^{-1}$ or $H_{osc} = m_{\phi}/3$. Combined with the rough estimate for the minimum at oscillation time from equation (5.27) as $\phi_{osc} = M \left( \frac{H}{M} \right)^{\frac{1}{n-2}} = M \left( \frac{m_{\phi}}{M_{pl}} \right)^{\frac{1}{n-2}}$. With the previously chosen $m_{\phi} = 2 \times 10^4 \text{ GeV}$ and $n = 4$ this gives us

$$\eta_X = \frac{\phi_{osc}^2 T_R}{28H_{osc}^2M_{pl}^2} = \frac{9M_{pl}^2 \left( \frac{m_{\phi}}{3M_{pl}} \right)^{\frac{1}{n-2}}} {28m_{\phi}^2 M_{pl}^2} \frac{T_R}{3T_R} = \frac{3T_R}{28m_{\phi}M_{pl}^2}. \hspace{1cm} (6.30)$$

Resulting in

$$6.1 \times 10^{-10} = \frac{3T_R}{28 \times 2 \times 10^4 \text{ GeV} \times 10^{18} \text{ GeV}} \hspace{1cm} (6.31)$$

$$T_R = 1.14 \times 10^{14} \text{ GeV.} \hspace{1cm} (6.32)$$

So the analytical estimate is larger, and off by only a factor of about 4.
Chapter 7

Affleck-Dine in gauge mediated supersymmetry breaking

In this chapter we do a numerical analysis of the parameter space of the Affleck-Dine mechanism using a gauge mediated (GMSB) potential. In the previous chapter we had a gravity mediated (PMSB or Planck mediated) potential, meaning that supersymmetry was broken by gravity. Now we have susy breaking mediated by gauge interactions. The potential that we get from the non-renormalizable operators and the gauge mediation is

\[ V(\Phi) = m_s^2 M_m^2 \ln^2 (1 + |\Phi| M_m) - cH^2 |\Phi|^2 + \left( \frac{A\Phi^n + \text{h.c.}}{n M^{n-3}} \right) + \frac{|\Phi|^{2n-2}}{M^{2n-6}}. \] (7.1)

The main difference between this potential and the one in gravity mediation is the fact that our mass term is not constant. The logarithm enables that if we are at small \( \Phi \) we can recover the original mass term, since \( M_m \log (1 + \frac{|\Phi|}{M_m}) \approx |\Phi|, |\Phi| < 1 \) but at large \( \Phi \) the mass term is function of the field amplitude.

7.1 Equations of motion

We will take a good look at the equations of motion, we want to make them solvable, easy to inspect and dimensionless so that we can make general statements and inspect broad parts of the parameterspace.
7.1.1 Polar coordinates

In this section we are going to transform the equations of motion into polar coordinates and rescale them to be dimensionless.

Starting from the equations of motion:

\[
\ddot{\Phi} + 3H \dot{\Phi} + \frac{\partial V}{\partial \Phi} = 0.
\] (7.2)

We transform to complex (polar) coordinates

\[
\Phi = \frac{\phi}{\sqrt{2}} e^{i \theta}, \quad \bar{\Phi} = \frac{\phi}{\sqrt{2}} e^{-i \theta}.
\] (7.3)

The time derivatives are now given by

\[
\dot{\Phi} = \frac{d\Phi}{dt} = \frac{\partial \Phi}{\partial \phi} \frac{\partial \Phi}{\partial \phi} \frac{\partial \phi}{\partial t} + \frac{\partial \Phi}{\partial \theta} \frac{\partial \theta}{\partial t} = \frac{1}{\sqrt{2}} e^{i \theta} \dot{\phi} + i \phi \frac{1}{\sqrt{2}} e^{i \theta} \dot{\theta}
\] (7.4)

\[
\ddot{\Phi} = \frac{d}{dt} \left( \frac{1}{\sqrt{2}} e^{i \theta} \dot{\phi} + i \phi \frac{1}{\sqrt{2}} e^{i \theta} \dot{\theta} \right)
= \frac{1}{\sqrt{2}} e^{i \theta} (\ddot{\phi} + 2i \dot{\phi} \dot{\theta} + i \dot{\phi} \dot{\theta} - \phi \theta^2).
\] (7.5)

Now we need the potential part \(\frac{\partial V}{\partial \phi}\), which can go quite wrong if you don’t do it completely from scratch.

\[
\frac{\partial V}{\partial \phi} = \frac{\partial V}{\partial \Phi} \frac{\partial \Phi}{\partial \phi} + \frac{\partial V}{\partial \Phi} \frac{\partial \Phi}{\partial \phi} = \frac{\partial V}{\partial \Phi} \frac{1}{\sqrt{2}} e^{i \theta} + \frac{\partial V}{\partial \Phi} \frac{1}{\sqrt{2}} e^{-i \theta}
\] (7.6)

\[
\frac{\partial V}{\partial \theta} = \frac{\partial V}{\partial \Phi} \frac{\partial \Phi}{\partial \theta} + \frac{\partial V}{\partial \Phi} \frac{\partial \Phi}{\partial \theta} = \frac{\partial V}{\partial \Phi} i \phi \frac{1}{\sqrt{2}} e^{i \theta} - \frac{\partial V}{\partial \Phi} i \phi \frac{1}{\sqrt{2}} e^{-i \theta}
\] (7.7)

From this we can write

\[
\frac{\partial V}{\partial \phi} + i \frac{\partial V}{\partial \theta} = 2 \frac{\partial V}{\partial \Phi} \frac{1}{\sqrt{2}} e^{-i \theta}
\] (7.8)
7.1. Equations of motion

\[
e^{i\theta} \left( \frac{\partial V}{\partial \phi} + \frac{i}{\phi} \frac{\partial V}{\partial \theta} \right) = \frac{\partial V}{\partial \Phi}
\]

(7.9)

So now we get the full form

\[
\ddot{\Phi} + 3H \dot{\Phi} + \frac{\partial V}{\partial \Phi} = \frac{1}{\sqrt{2}} \left( e^{i\theta} (\ddot{\phi} + 2i\dot{\phi}\dot{\theta} + i\phi - \phi^2) + \frac{1}{\sqrt{2}} e^{i\theta} 3H(\dot{\phi} + i\phi\dot{\theta}) \right) + \frac{1}{\sqrt{2}} \left( \frac{\partial V}{\partial \phi} + i \frac{\partial V}{\partial \theta} \right) = 0.
\]

(7.10)

We get rid of the prefactors and knowing that both the real and imaginary parts of the equation have to be identically zero this yields the two equations

\[
\ddot{\phi} - \phi \dot{\theta}^2 + 3H \dot{\phi} + \frac{\partial V}{\partial \phi} = 0
\]

(7.11)

\[
\dot{\phi} \ddot{\theta} + 2 \dot{\phi} \dot{\theta} + 3H \phi \ddot{\theta} + \frac{1}{\phi} \frac{\partial V}{\partial \theta} = 0.
\]

(7.12)

or after dividing equation (7.12) by \( \phi \)

\[
\ddot{\theta} + 2 \frac{\dot{\phi}}{\dot{\theta}} + 3H \ddot{\theta} + \frac{1}{\phi^2} \frac{\partial V}{\partial \theta} = 0.
\]

(7.13)

Now if we treat everything in the coordinates \((\phi, \theta)\) we can use these equations and do not have to worry about imaginary and real parts, we have the equations of motion in polar coordinates.

7.1.2 Dimensionless equations of motion

Now we want to make these dimensionless so we can inspect more general solutions to the equations.

To do this we reparametrize as such
\[ \Phi = \alpha X, \quad \bar{\Phi} = \bar{\alpha} \bar{X}, \quad t = \beta \tau \] (7.14)

for some parameters \( \alpha, \beta \).

So the derivative of a field \( \Phi \) becomes

\[ \frac{\partial \Phi}{\partial t} = \frac{\partial \alpha X}{\partial \beta \tau} = \frac{\alpha}{\beta} \frac{\partial X}{\partial \tau} = \frac{\alpha}{\beta} X' \] (7.15)

and the second derivative is

\[ \frac{\partial}{\partial t} \frac{\partial \Phi}{\partial t} = \frac{\partial}{\partial \beta \tau} \frac{\alpha}{\beta^2} \frac{\partial X}{\partial \tau} = \frac{\alpha}{\beta^2} X''. \] (7.16)

Moreover, the Hubble parameter which we usually take for inflaton matter domination as \( \frac{2}{3t} \) in our model becomes

\[ H(t) = \frac{2}{3t} = \frac{2}{3\beta \tau} = \frac{1}{\beta} h(\tau). \] (7.17)

Now that we have the basics, we can start substituting it into our equations of motion (4.4), this yields

\[ \frac{\alpha}{\beta^2} X'' + \frac{\alpha}{\beta^2} 3h(\tau)X' + m_s^2 M_m \ln (1 + \frac{|\alpha|}{M_m} |X|) \left( \frac{1}{1 + \frac{|\alpha|}{M_m} |X|} \frac{\alpha X}{M_m |X|} \right) \]

\[ - \frac{c}{\beta^2} h(\tau)^2 \alpha X + \frac{A \bar{\alpha}^{n-1}}{M^{n-3}} \bar{X}^{n-1} + \frac{(n - 1)|\alpha|^{2n-4} \alpha}{M^{2n-6}} |X|^{2n-4} X = 0. \] (7.18)

Where we used \( \bar{\alpha} \) for generality. Rewriting this equation by multiplying with \( \beta^2/\alpha \)

\[ X'' + 3h(\tau)X' + \]

\[ \frac{\beta^2 m_s^2 M_m}{\alpha} \ln (1 + \frac{|\alpha|}{M_m} |X|) \left( \frac{1}{1 + \frac{|\alpha|}{M_m} |X|} \right) \frac{\alpha X}{M_m |X|} - ch(\tau)^2 X \]

\[ + \frac{\beta^2 A \bar{\alpha}^{n-1}}{\alpha M^{n-3}} \bar{X}^{n-1} + (n - 1)\beta^2 \frac{|\alpha|^{2n-4}}{M^{2n-6}} |X|^{2n-4} X = 0. \] (7.19)

Now we define the parameter \( \frac{\beta^2 m_s^2 M_m}{\alpha} = 1 \). Or \( \alpha = \beta^2 m_s^2 M_m \).
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\[ X'' + 3h(\tau)X' + \ln (1 + |\beta^2 m_s^2 X|) \left( \frac{1}{1 + |\beta^2 m_s^2 X|} \right) \frac{|\beta^2 m_s^2 X|}{|X|} X + \ln (1 + |\beta^2 m_s^2 X|) \left( \frac{1}{1 + |\beta^2 m_s^2 X|} \right) \frac{|\beta^2 m_s^2 X|}{|X|} \]

\[-ch(\tau)^2 X + \frac{A\beta^{2n-2} m_s^{2n-2} M_m^{n-1}}{m_s^2 M_m M^{n-3}} X^{n-1} +
\]

\[+(n - 1) \beta^2 |\beta|^{4n-8} \frac{m_s^{2n-8} M_m^{2n-4}}{M^{2n-6}} |X|^{2n-4} X = 0. \quad (7.20)\]

It becomes obvious that we have to define \( \beta = \frac{1}{m_s} \) to simplify this equation, and now we also get \( \alpha = M_m \), therefore we find

\[ X'' + 3h(\tau)X' + \ln (1 + |X|) \left( \frac{1}{1 + |X|} \right) \frac{X}{|X|} - ch(\tau)^2 X
\]

\[+ A\beta^{2n-2} m_s^{2n-2} M_m^{n-1} X^{n-1} + (n - 1) \frac{M_m^{2n-4}}{m_s^2 M_m^{2n-6}} |X|^{2n-4} X = 0. \quad (7.21)\]

Defining the dimensionless constant \( b \) as

\[ b^{n-2} \equiv \frac{m_s M_m^{n-3}}{M_m^{n-2}} \quad (7.22)\]

and for \( A \) we define

\[ A = A_0 \quad (7.23)\]

The resulting eq. of motion is

\[ X'' + \frac{2}{\tau} X' + \ln (1 + |X|) \left( \frac{1}{1 + |X|} \right) \frac{X}{|X|} - \frac{4}{9\tau} X
\]

\[+ A_0 b^{2-n} X^{n-1} + (n - 1) b^{4-2n} |X|^{2n-4} X = 0, \quad (7.24)\]

and it only has parameters \( b, c, d \) and \( A_0 \) but \( c \) is usually set to 1. And of course we also have its conjugate.

This can also be rewritten as into numerically integrable form as
\[ X''_x + 3h(\tau)X'_x + \ln(1 + |X|) \left( \frac{1}{1 + |X|} \right) \frac{X_x}{|X|} - ch^2(\tau)X_x \\
+ A_0b^{2-n}|X|^{n-1}\cos((n-1)\theta) + (n - 1)b^{4-2n}|X|^{2n-4}X_x = 0. \quad (7.26) \]

\[ X''_y + 3h(\tau)X'_y + \ln(1 + |X|) \left( \frac{1}{1 + |X|} \right) \frac{X_y}{|X|} - ch^2(\tau)X_y \\
+ A_0b^{2-n}|X|^{n-1}\sin((n-1)\theta) + (n - 1)b^{4-2n}|X|^{2n-4}X_y = 0. \quad (7.27) \]

### 7.1.3 Dimensionless radial equations of motion

We still want to make them dimensionless, so we will have to transform it once more, like in the previous section, but this time, we already know how to do it!

\[
\begin{align*}
\phi & \to M_m \chi, \quad \theta(t) \to \theta(\tau) \\
t & \to \frac{\tau}{m_s}, \quad \partial_t \to m_s \partial_\tau, \\
H & = \frac{2}{3\ell} \to m_s \frac{2}{3\tau} = m_s h(\tau), \quad \frac{A}{m_s} \to A_0
\end{align*}
\]

(7.28) (7.29) (7.30)

are the transformations that will remove the dimensionality from our eoms.

Let’s start with the potential.

\[ V(\Phi) = m_s^2 M_m^2 \ln^2 \left( 1 + \frac{|\Phi|}{M_m} \right) - cH^2|\Phi|^2 + \left( \frac{A_0 \Phi^n + \text{h.c.}}{n M^{n-3}} \right) + \frac{|\Phi|^{2n-2}}{M^{2n-6}} \]

(7.31)

becomes

\[ V(\phi, \theta) = m_s^2 M_m^2 \ln^2 \left( 1 + \frac{\phi}{\sqrt{2} M_m} \right) - cH^2\frac{\phi^2}{2} + \left( \frac{A\phi^n e^{in\theta}}{n M^{n-3} \sqrt{2}^n} + \text{h.c.} \right) + \frac{\phi^{2n-2}}{\sqrt{2}^{2n-2} M^{2n-6}} \]

(7.32)

and has the derivatives
7.1. Equations of motion

\[
\frac{\partial V}{\partial \phi} = 2m_s^2 m_m^2 \ln(1 + \phi) \left(1 + \frac{1}{\sqrt{2} M_m}\right) - c H^2 \phi
\]

\[
+ \left( \frac{A_0 \phi^{n-1} e^{i n \theta}}{M_m^2 \sqrt{2} M_m^n} + \text{h.c.} \right) + (2n - 2) \frac{\phi^{2n-3}}{\sqrt{2} M_m^{2n-2} M_m^{2n-6}}
\]

\[
= \sqrt{2} m_s^2 M_m \ln(1 + \phi) \left(1 + \frac{1}{\sqrt{2} M_m}\right) - c H^2 \phi
\]

\[
+ \frac{|A_0| \phi^{n-1}}{\sqrt{2} M_m^{n-3}} \sqrt{2} \cos(n \theta + \theta_A) + (2n - 2) \frac{\phi^{2n-3}}{\sqrt{2} M_m^{2n-2} M_m^{2n-6}}. \quad (7.33)
\]

\[
\frac{\partial V}{\partial \theta} = \frac{i A_0 \phi^{n-1} e^{i n \theta}}{\sqrt{2} M_m^{n-3}} - \frac{i \bar{A}_0 \phi^{n-1} e^{-i n \theta}}{\sqrt{2} M_m^{n-3}} = \frac{|A_0| \phi^{2n} \sin(n \theta - \theta_A)}{\sqrt{2} \chi M_m^{n-3}} \quad (7.34)
\]

Equations \((7.11)\) and \((7.13)\) have now become

\[
\ddot{\phi} - \dot{\phi}^2 + 3H \dot{\phi} + \sqrt{2} m_s^2 M_m \ln(1 + \phi) \left(1 + \frac{1}{\sqrt{2} M_m}\right) - c H^2 \phi
\]

\[
+ \frac{|A_0| \phi^{n-1}}{\sqrt{2} M_m^{n-3}} \cos(n \theta + \theta_A) + (2n - 2) \frac{\phi^{2n-3}}{\sqrt{2} M_m^{2n-2} M_m^{2n-6}} = 0. \quad (7.36)
\]

\[
\ddot{\theta} + 2 \frac{\dot{\phi}}{\phi} \dot{\theta} + 3H \dot{\theta} + \frac{|A_0| \phi^{2n-2} \sin(n \theta - \theta_A)}{\sqrt{2} \chi M_m^{n-3}} = 0. \quad (7.37)
\]

The radial equation transforms into

\[
M_m m_s^2 \chi'' - M_m m_s^2 \chi'' + 3M_m m_s^2 h \chi'
\]

\[
+ \sqrt{2} M_m m_s^2 \ln(1 + \chi) \left(1 + \frac{1}{\sqrt{2}} \chi\right) - c M_m m_s^2 h^2 \chi
\]

\[
+ \frac{|A_0| M_m^{n-2}}{M_m^{n-3}} \chi^{n-1} \cos(n \theta + \theta_A) + \frac{(2n - 2) M_m^{2n-3}}{M_m^{2n-6}} \chi^{2n-3} = 0. \quad (7.38)
\]

using the definition of \(b\) from eq. \((7.22)\), after dividing by \(M_m m_s^2\) the final form becomes
\[
\chi'' - \chi \theta'^2 + 3h \chi' + \sqrt{2} \ln \left(1 + \frac{\chi}{\sqrt{2}}\right) \frac{1}{1 + \frac{\chi}{\sqrt{2}}} - ch^2 \chi
\]
\[
+ |A_0| b^{2-n} \frac{\chi^{n-1}}{\sqrt{2}^{n-2}} \cos (n \theta + \theta_A) + (2n - 2)b^{4-2n} \frac{\chi^{2n-3}}{\sqrt{2}^{2n-2}} = 0. \tag{7.39}
\]

The angular equation of motion becomes
\[
m_s^2 \theta'' + 2m_s^2 \frac{\chi'}{\chi} \theta' + 3m_s^2 h \theta' - \frac{M_{m}^{n-2}}{M_{m}^{n-3}} \frac{\chi^{n-2}}{\sqrt{2}^{n-2}} |A_0| \sin (n \theta - \theta_A) = 0. \tag{7.40}
\]

and once again dividing and replacing the mass terms for a factor involving b we get
\[
\theta'' + 2 \frac{\chi'}{\chi} \theta' + 3h \theta' - b^{2-n} \frac{\chi^{n-2}}{\sqrt{2}^{n-2}} |A_0| \sin (n \theta - \theta_A) = 0. \tag{7.41}
\]

And we have found the dimensionless equations of motion in polar coordinates. They only depend on parameters \(n, b\) and \(A_0\) and of course \(\tau\). (NB: I’ve removed \(c\) because it’s usually set to one.)

\[
\chi'' - \chi \theta'^2 + \frac{2}{\tau} \chi' + \sqrt{2} \ln \left(1 + \frac{\chi}{\sqrt{2}}\right) \frac{1}{1 + \frac{\chi}{\sqrt{2}}} - \frac{4}{9 \tau^2} \chi
\]
\[
+ |A_0| b^{2-n} \frac{\chi^{n-1}}{\sqrt{2}^{n-2}} \cos (n \theta + \theta_A) + (2n - 2)b^{4-2n} \frac{\chi^{2n-3}}{\sqrt{2}^{2n-2}} = 0, \tag{7.42}
\]
\[
\theta'' + 2 \frac{\chi'}{\chi} \theta' + \frac{2}{\tau} \theta' - b^{2-n} \frac{\chi^{n-2}}{\sqrt{2}^{n-2}} |A_0| \sin (n \theta - \theta_A) = 0. \tag{7.43}
\]

Where the potential is given by
\[
\tilde{V}(\chi) = \frac{V}{m_s^2 M_m^2} \tag{7.44}
\]

with \(V\) the potential from equation \((7.32)\).
7.2 Numerical integration

Now all the theoretical work we did in the previous section uses the assumption that the asymmetry generation is instantaneous, which is not the case. Using the numerics we can see that the generation is not instantaneous, although it does go quite fast. We know how to calculate \( n_x a^3 = \phi^2 \dot{\theta} a^3 = M^2 m_s \chi^2 \theta' a^3 \), and we have to plug this into \( \eta \).

After \( n_x a^3 \) is conserved it we know what the value is at reheating, for which we also know the entropy, \( S_R = \frac{8 \pi^2}{45} g_R T^3 R \). Combining the results with \( H_R = \frac{\sqrt{2} T_R^2}{M_{pl}} \) and \( H = \frac{2 m_s}{3 \tau} \) yields

\[
\eta = \frac{M^2 m_s \chi^2 \theta' a^3}{\frac{8 \pi^2}{45} g_R T R} = \frac{M^2 m_s H_R^2}{\frac{8 \pi^2}{45} g_R T R} \frac{H^2}{H_R} \chi^2 \theta' = \frac{M^2 m_s g_R T R^4}{\frac{8 \pi^2}{45} g_R T R} \frac{M^2}{M_{pl}^2} \frac{\chi^2 \theta'}{(m_s \frac{2}{3 \tau})^2} \quad (7.45)
\]

\[
\eta = \frac{9 M^2 T_R}{\frac{8 \pi^2}{45} m_s M^2 \chi^2 \theta' \tau^2} \quad (7.46)
\]

So now we are going to calculate \( \chi^2 \theta' \tau^2 \) and try to fit it to a function of \( n, b, A_0 \) and \( \theta_i \). Every time a figure is made for \( n_x a^3 \) we have calculated and plotted \( \chi^2 \theta' \tau^2 \).

7.2.1 Initial conditions

Now we are going to solve equations (7.42) and (7.43) numerically, so we will have to choose certain initial conditions to find a solution. A lot of these initial conditions are chosen from analytical considerations, because in this way the chosen numbers are in the right order of magnitude.

The minimum of the potential, where the field is found after inflation, is found from \( \frac{\partial}{\partial \phi} \phi = 0 \). This is at early times when \( H \) is large. It can be approximated by

\[
\frac{\phi_{\min}}{\sqrt{2}} = M \left( \frac{H}{M} \right)^{\frac{1}{n-2}} = (M^{n-1} H)^{\frac{1}{n-2}} = \left( M^{n-1} \frac{2}{3 t} \right)^{\frac{1}{n-2}} \quad (7.47)
\]

\[
\frac{\dot{\phi}_{\min}}{\sqrt{2}} = \frac{-1}{n-2} \left( M^{n-1} \frac{2}{3 t} \right)^{\frac{1}{n-2}-1} \left( M^{n-1} \frac{2}{3 t^2} \right) = \frac{-1}{n-2} \frac{1}{t} \phi_{\min} \quad (7.48)
\]

In dimensionless coordinates this becomes
\[
\chi_{\text{min}} \sqrt{2} = \frac{M}{M_{m}} \left( \frac{m_{s} h}{M} \right)^{\frac{1}{n-2}},
\]
(7.49)

\[
\chi'_{\text{min}} \sqrt{2} = \frac{-1}{n-2} \frac{\chi_{\text{min}}}{\sqrt{2}},
\]
(7.50)

Now we can find the minimum of the potential numerically, and we immediately have a starting velocity associated with it. So the initial conditions for position and velocity are known. We explicitly do not use an angular velocity, because we want to see if the system in itself can create an angular momentum, \(\theta'(\tau_{\text{start}}) = 0\).

We know that the potential has \(n\) angular minima and that only the \(A\) term results in such a minimum. This term is quite small compared to all the radial parts of the potential. The field starts off with initially with a random phase. Since the angular minimum is small compared to the Hubble term it will only settle in a radial minimum and not an angular one. If it settles in the angular minimum nothing will happen since in gauge mediation there is only one angular term. Therefore we take the numerically found minimum for the radial coordinate, \(\chi(\tau_{\text{start}}) = \chi_{\text{num}}\) and for the angular coordinate we take it to be the maximum such that we get the largest possible \(n_{X}\): \(\theta(\tau_{\text{start}}) = \pi/n\). Since we can make it as large as we want, in practice we take the maximum so that we get an upper bound on \(n_{X}\) where \(\tau_{\text{start}}\) is the numerical starting time.

From the literature we have the estimated value for the time when the field starts oscillating. This is given in terms of the Hubble parameter: \(H/m_{s} = \frac{2}{3\tau}\) of \(H_{\text{osc}}^{-1}\). \(H_{\text{osc}}/m_{s} = 1\). By rearranging this we get a time \(\tau_{\text{osc}}^{-1}\) at which we can start the numerical integration. \(H_{\text{osc}}/m_{s} = h(\tau_{\text{osc}}^{-1}) = 1\) resulting in \(h(\tau) = 2/3\tau\) so \(\tau_{\text{osc}}^{-1} = 2/3\).

We start the numerical solution at \(\tau = \tau_{\text{osc}}^{-1}/10 = \frac{2}{30}\). Take note: for \(b > 1\) the literature predicts that \(H_{\text{osc}}^{-1}\) decreases, meaning that \(\tau_{\text{osc}}^{-1}\) should increase. This is why we will start all the solutions at \(\tau = 2/30\), seeing as this is a kind of lower bound on the time coordinate. The time at which the calculation stops is chosen as \(\tau_{\text{end}} = 10^{3}\tau_{\text{start}}\), which turned out to always be long enough for the CP-violation to occur.

Then we still have the parameters of the system itself, \(b\), \(A_{0}\) and \(n\). From literature we know that \(5 < n < 9\) in our model, but we usually choose \(n = 5\) since higher powers result in more numerical instabilities. When it is necessary we check certain results for the entire range of possible \(n\)'s. \(A_{0}\) and \(b\) are usually chosen around order \(\mathcal{O}(1)\) but we will see later on in section (7.3) that it is actually better to choose them smaller than 1 when you want to check an entire range of values.
### 7.2. Numerical integration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Starting value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_{\text{start}}$</td>
<td>$\frac{2}{30}$</td>
</tr>
<tr>
<td>$\tau_{\text{end}}$</td>
<td>$10^5 \tau_{\text{start}}$</td>
</tr>
<tr>
<td>$\chi(\tau_{\text{start}})$</td>
<td>Numerical minimum of $V$</td>
</tr>
<tr>
<td>$\chi'(\tau_{\text{start}})$</td>
<td>$-\frac{1}{n-2} \tau_{\text{start}} \chi(\tau_{\text{start}})$</td>
</tr>
<tr>
<td>$\theta(\tau_{\text{start}})$</td>
<td>$\pi / n$</td>
</tr>
<tr>
<td>$\theta'(\tau_{\text{start}})$</td>
<td>0</td>
</tr>
<tr>
<td>$A_0$</td>
<td>$\mathcal{O}(1)$</td>
</tr>
<tr>
<td>$b$</td>
<td>$\mathcal{O}(1)$</td>
</tr>
<tr>
<td>$n$</td>
<td>$5, ..., 9$ but usually $n = 5$</td>
</tr>
</tbody>
</table>

#### 7.2.2 A general example of the behaviour

The initial values for the field $\chi$ are such that it starts out in a minimum $\chi_0$ with a velocity of $\dot{\chi}_0(\tau) = -\frac{1}{n-2} \frac{1}{\tau_{\text{start}}} \chi_0(\tau)$ and no angular velocity (of course).

![Figure 7.1](image.png)

Figure 7.1: (a) The general path a field takes through the potential. The field gets an angular momentum around $t = t_{\text{osc}}$. (b) The same plot as in (a), but zoomed in to show the rotation of the field.

In this figure we see the field starting out with a vev $\neq 0$. It starts rolling towards the global minimum and gets an angular momentum around $t = t_{\text{osc}}$, which is conserved at long time scales.

In figure (7.2) the angle is constant at first but after a while it gets a kick and we see that $\dot{\theta}$ becomes non-zero and converges to a finite number. The field is rotating and has a constant angular momentum.

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Chapter 7. Affleck-Dine in gauge mediated supersymmetry breaking

Figure 7.2: Figure (a) shows the angle $\theta$ and (b) the angular velocity $\dot{\theta}$.

Figure 7.3: (a) The charge density associated with the field $\phi$. (b) The charge associated with the field $\phi$. This figure shows that the field gets a kick and the charge density $j^0 = \phi^2 \dot{\theta}$ stabilizes and $Q = a^3 j^0$ is conserved after some time.

7.3 The critical value of $b(A)$

During our investigation of the potential and the way the field rolls through it we encountered moments where the field would roll inwards, but get stuck at some point $\chi \neq 0$. It turns out that for certain values of $A_0$ and $b$ the potential has angular momenta that do not disappear after a reasonably ($O(\tau_{osc})$) short time. Sometimes they vanish after a long time, such as $O(\tau_{end})$ or even longer and there were occasions that it did not disappear at all. So there is some phase-transition in the potential for a certain $A_0$ and $b$ where the angular minima do not vanish and the field does not roll towards the global minimum at all.

When we inspect the potential

$$V(\phi, \theta) = \ln^2 \left(1 + \frac{\chi}{\sqrt{2}}\right) - c \ h^2 \chi^2 + \left(\frac{A_0 \chi^n e^{i n \theta}}{b^{n-2} n \sqrt{2^n}} + \text{h.c.}\right) + \frac{\chi^{2n-2}}{b^{2n-4} \sqrt{2^{2n-2}}}.$$  (7.51)
7.3. The critical value of $b(A)$

We know that for large $\tau$ the $h$ term becomes zero, and that the last term dominates for large $\chi$. The log-term and the angular term have to balance each other, if we want to get a kick that results in a non-zero angular momentum. But when $A_0$ is very large and $b$ very small, the angular term becomes much larger than the log which, combined with the minus sign of the cosine, means we get a minimum that does not vanish.

7.3.1 Analytical analysis

We are curious to know the values of $A_0$ and $b$ for which the local minimum of the potential does not disappear at large time scales. If we know $b_{\text{crit}}$ we know when the field will not get an angular momentum and so does not generate a non-zero particle number $n_X$.

The potential is

$$V(\chi) = \ln^2 \left(1 + \frac{\chi}{\sqrt{2}}\right) - b^{2-n}A_0 \frac{\chi^n}{n\sqrt{2}} + b^{4-2n} \frac{\chi^{2n-2}}{\sqrt{2}^n} = 0,$$

(7.52)

where the Hubble induced term is set to zero since we are at late times.

When we have a minimum the derivative is zero

$$V'(\chi) = \ln \left(1 + \frac{\chi}{\sqrt{2}}\right) \frac{\sqrt{2}}{1 + \frac{\chi}{\sqrt{2}}} - b^{2-n}A_0 \frac{\chi^{n-1}}{\sqrt{2}^n} + b^{4-2n} \frac{\chi^{2n-3}}{\sqrt{2}^n} (2n - 2) = 0.$$  

(7.53)

At the critical point, the function has exactly one minimum. When we notice that this is a quadratic function in $y = b^{2-n}$ we now know that the discriminant will be zero if we are at a critical point.

$$y^2 \frac{\chi^{2n-3}}{\sqrt{2}^n n - 2} + y(-A_0 \frac{\chi^{n-1}}{\sqrt{2}^n}) + \ln \left(1 + \frac{\chi}{\sqrt{2}}\right) \frac{\sqrt{2}}{1 + \frac{\chi}{\sqrt{2}}} = 0.$$  

(7.54)

The discriminant is given by

$$D = A_0^2 \frac{\chi^{2n-2}}{2n} - 4 \frac{\chi^{2n-3}}{\sqrt{2}^n (2n - 2)} \ln \left(1 + \frac{\chi}{\sqrt{2}}\right) \frac{\sqrt{2}}{1 + \frac{\chi}{\sqrt{2}}}.$$  

(7.55)

We have two situations, $D = 0$, which is the point where we have a critical value of $b$ and the angular minimum is gone, and we have $D > 0$, in which case we have a minimum.
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The function \( \ln \left( 1 + \frac{\chi}{\sqrt{2}} \right) \frac{\sqrt{2}}{1 + \frac{\chi}{\sqrt{2}}} \) is bounded from above for some \( \chi \), this bound is around 0.5. This means that we can make \( A_0 \) as large as we want and at some point there is no \( b_{\text{crit}} \) for which the angular minimum in \( V_{AD} \) will vanish so it is independent of \( b \) and \( \chi \).

We therefore have two cases, \( D > 0, D = 0 \).

- \( D > 0 \) means there is no value of \( b \) such that (7.54) can not be solved in terms of \( b^{2-n} \), or there is always a minimum at \( \chi \neq 0 \) independent of the value of \( b \).

\[
A_0^2 \frac{\chi^{2n-2}}{2^n} - 4 \frac{\chi^{2n-3}}{\sqrt{2}^{2n-2}} (2n - 2)0.5 > 0 \tag{7.56}
\]

\[
A_0^2 > 8(n - 1) \Rightarrow A_0 > 2\sqrt{2}(n - 1)^{1/2}. \tag{7.57}
\]

And we have a minimum at \( \chi \neq 0 \) independent of \( b \) and \( \chi \).

When the \( A_0 < 2\sqrt{2}(n - 1)^{1/2} \) \( D \) becomes such that \( D \leq 0 \) for a \( b < b_{\text{crit}}(A) \).

- \( D = 0 \) yields the following expression

\[
\frac{A_0^2}{2^n} \frac{2^{n-1}}{8(n - 1)} = \frac{1}{\chi} \ln \left( 1 + \frac{\chi}{\sqrt{2}} \right) \frac{\sqrt{2}}{1 + \frac{\chi}{\sqrt{2}}}, \tag{7.58}
\]

\[
A_0^2 = \frac{16}{n - 1} \frac{\ln \left( 1 + \frac{\chi}{\sqrt{2}} \right)}{\chi/\sqrt{2}}. \tag{7.59}
\]

Solving equation (7.54) as function of \( b^{2-n} \) gives us

\[
b^{2-n} = \frac{A_0}{\chi^{n-2}} \frac{\sqrt{2}^{n-6}}{n - 1}, \tag{7.60}
\]

from which we can distil

\[
b_{\text{crit}} = \frac{\chi}{A_0^{n/2}} \frac{(n - 1)^{1/2}}{\sqrt{2}^{n/2}} \tag{7.61}
\]
7.3. The critical value of \( b(A) \)

for some \( \chi \) which is the local minimum of the potential. Now we rewrite this to

\[
b \left( \frac{A_0 \sqrt{2^{n-6}}}{n - 1} \right)^{\frac{1}{n-2}} = \chi
\]  
(7.62)

and plug it into (7.59) to get an expression that relates \( b \) and \( A \) that is still analytical and can be solved numerically.

\[
\frac{A_0^2}{n - 1} = \frac{16}{b} \left( \frac{4(n - 1)}{A_0} \right)^{\frac{1}{n-2}} \log \left( \frac{1 + b \left( \frac{A_0}{4(n-1)} \right)^{\frac{1}{n-2}}}{1 + b \left( \frac{A_0}{4(n-1)} \right)^{\frac{1}{n-2}}} \right).
\]  
(7.63)

**Large \( \chi \) approximation**

From equation (7.61), we have \( b \) as a function of \( A \) and \( \chi \), so we need to find \( \chi \) so that we can get \( b \) as a pure function in \( A \).

We can do a large \( \chi \) approximation, \( \chi \gg 1 \), because in general the minimum \( \chi_0 \) is larger than \( b \). From equation (7.59) we get

\[
\frac{16}{\chi} \ln \left( 1 + \frac{\chi}{\sqrt{2}} \right) \approx \frac{16c}{\chi^2/2} = \frac{32c}{\chi^2},
\]  
(7.64)

\[
A_0^2 = \frac{32(n - 1)}{\chi^2} \Rightarrow A_0 = \frac{4\sqrt{2}(n - 1)^{\frac{1}{2}}}{\chi}
\]  
(7.65)

\[
\chi = \frac{4\sqrt{2}(n - 1)^{\frac{3}{2}}}{A_0}
\]  
(7.66)

Resulting in the form

\[
b_{\text{crit}} = \frac{4\sqrt{2}(n - 1)^{\frac{1}{n-2}} (n - 1)^{\frac{1}{2}}}{\sqrt{2^{\frac{n}{n-2}} A_0^{\frac{1}{n-2}}} \frac{4^{n-1}(n - 1)^{\frac{n}{n-2}}}{A_0^{\frac{n}{n-2}}}} = \frac{4^{n-1}(n - 1)^{\frac{n}{n-2}}}{A_0^{\frac{n}{n-2}}}
\]  
(7.67)

\[
b_{\text{crit}} \propto \frac{1}{A_0^{\frac{n}{n-2}}}
\]  
(7.68)
7.3.2 Numerical findings on \( b_{\text{crit}} \)

Below we see \( b_{\text{crit}} \) calculated numerically as a function of \( A_0 \). The critical point for \( b \) is the value when the potential shifts from having no local minimum at \( \chi \neq 0 \) to having a local minimum. This means that the derivative of the potential in the radial direction is zero, since it becomes a saddle point before becoming a true minimum. Therefore we solved the equations

\[
\frac{\partial V}{\partial \chi} = 0, \quad \frac{\partial^2 V}{\partial \chi^2} = 0
\]

simultaneously over the range \( 10^{-2} < A_0 < 10^1 \).

\[
\log_{10} b_{\text{crit}}(A_0) = \alpha A_0 + \beta
\]

We then took the logarithm of every data point and fitted the data to a function of the form \( \alpha A_0 + \beta \) for some \( \alpha \) and \( \beta \). The resulting form is then given by \( b_{\text{crit}} = e^\beta A_0^\alpha \) or
7.4 Field values at onset of oscillations

\[ b_{\text{crit}}(A_0, n) = \frac{32}{A_0^6} \]  

(7.70)

where \( \kappa \) are given in the table below along with numerically found plots of \( b_{\text{crit}}(A_0) \). The 32 is the numerical fit we found.

<table>
<thead>
<tr>
<th>( n )</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \kappa )</td>
<td>1.45</td>
<td>1.37</td>
<td>1.31</td>
<td>1.28</td>
</tr>
</tbody>
</table>

which does not correspond well with the function found in equation (7.68). It does however match very well with numerical solutions to equation (7.63), so we are quite sure that our approximation in the previous paragraph is incorrect.

Also notice the fact that the fitted functions deviate from the data points at high \( A_0 \) and seem to go to zero. This is the limit case where \( A_0 > 2\sqrt{2}(n-1)^{1/2} \) and there is a minimum at \( \chi \neq 0 \) independent of \( b \), meaning there is no \( b_{\text{crit}} \).

7.4 Field values at onset of oscillations

In figure (7.6) we see the values of \( \chi_{\text{osc}} \) and \( H_{\text{osc}}/m_s \) as a function of \( b \). These quantities are defined similarly:

\[ \chi_{\text{osc}} = \chi(t_{\text{osc}}), \quad H_{\text{osc}} = H(t_{\text{osc}}). \]  

(7.71)

In this instance we have taken \( t_{\text{osc}} \) as the time one quarter phase before the minimum in the amplitude. We have done so because we want to know the amplitude of the field when it starts to roll, but when we take the first minimum of the amplitude we would always have a minimal value for \( \chi \). With this definition we will have a bigger value for \( \chi \) and \( H \).

So we find \( \phi_0 \) and the associated original \( t_{\text{osc}}^{\text{original}} \). Then we subtract a quarter of the phase from this time to get a new \( t_{\text{osc}} \). Then we use that to evaluate \( \chi \) and \( H \).

In figure (7.6) we see the oscillation amplitude at time \( t = t_{\text{osc}} \). The fits are given by

\[ \chi_{\text{osc}} \propto b, \quad H_{\text{osc}}/m_s \propto b^0 \quad b < 1, \]  

(7.72)

\[ \chi_{\text{osc}} \propto b^{0.8}, \quad H_{\text{osc}}/m_s \propto b^{-0.7} \quad b \gg 1. \]  

(7.73)

With proportionality constants
Figure 7.6: (a) The oscillation amplitude at $t = t_{\text{osc}}$ for $n = 5$. The fits are given by $0.49b^{-0.99}$ on the left and $0.41b^{-0.79}$ on the right. (b) The value of the Hubble parameter at time $t = t_{\text{osc}}$ for $n = 5$. The fits are $0.038b^{-0.0078}$ on the left and $0.726b^{-0.715}$ on the right.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$b &lt; 1$</th>
<th>$b \gg 1$</th>
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<tbody>
<tr>
<td>5</td>
<td>0.49</td>
<td>0.57</td>
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<tr>
<td>6</td>
<td>0.59</td>
<td>0.55</td>
</tr>
<tr>
<td>7</td>
<td>0.64</td>
<td>0.59</td>
</tr>
<tr>
<td>8</td>
<td>0.67</td>
<td>0.61</td>
</tr>
</tbody>
</table>

And in figure (7.7) (a) corresponds almost precisely with figure 2 from [1201.2200], while (b) has a similar shape, the numbers are off by a factor 2.5 for small $b$ and off by a factor of $10^{-1}$ for large $b$.

Figure 7.7: (a) The oscillation amplitude at $t = t_{\text{osc}}$ for dimensions $n = 5$ (blue), $n = 6$ (red), $n = 7$ (yellow), $n = 8$ (green). (b) The value of the Hubble parameter at time $t = t_{\text{osc}}$ for dimensions $n = 5$ (blue), $n = 6$ (red), $n = 7$ (yellow), $n = 8$ (green).

7.4.1 Numerical results for the parameter space of $n_X$

We expect from the theory three sectors of which we will consider two: $b < 1$ and $1 < b < b_{\text{crit}}(A_0, n)$
7.4. Field values at onset of oscillations

<table>
<thead>
<tr>
<th>n</th>
<th>$b_{\text{crit}}(A_0 = 0.01)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>24950</td>
</tr>
<tr>
<td>6</td>
<td>16334</td>
</tr>
<tr>
<td>7</td>
<td>12872</td>
</tr>
<tr>
<td>8</td>
<td>11112</td>
</tr>
</tbody>
</table>

The fits we make are made twofold: one fit is applied to the raw data as-is and one fit is done on the logarithm of every data point in the data set, so that the weight of the points is removed. This is done because the last few points in every data set skew the fit a bit in the favor of these points because they are larger and more far apart, which we want to avoid.

The values of $a^3 n_X$ are evaluated at late times, after baryon number is conserved, and we use the function

$$a(\tau) = \left(\frac{\tau}{m_s}\right)^{2/3} \Rightarrow a^3(\tau) = \frac{\tau^2}{m_s^2}$$

(7.74)

for the normalization of the calculated results, to get it in a comoving volume.

The predicted proportionalities are

$$n_X \propto q_X \frac{\sin(n\theta - \theta_A)}{2} \left(\frac{A}{m_s}\right) b^2 \left(\frac{M_{m}^2 T_R}{m_s M_{\text{pl}}^2}\right), \quad b < 1$$

(7.75)

$$n_X \propto q_X \frac{\sin(n\theta - \theta_A)}{2} \left(\frac{A}{m_s}\right) b^{4(n-1)/n-2} \left(\frac{M_{m}^2 T_R}{m_s M_{\text{pl}}^2}\right), \quad 1 < b < b_{\text{crit}}.$$  

(7.76)

**Parameter range** $b < 1$

The asymmetry parameter is theorized to go as equation (7.75) or just $n_X \propto \sin(n\theta - \theta_A) \left(\frac{A}{m_s}\right) b^2$.

**Angular parameters**

We give the field an initial angle $\theta_i$ and see how it responds to this. The results are shown below.
Chapter 7. Affleck-Dine in gauge mediated supersymmetry breaking

Figure 7.8: (a) The comoving asymmetry parameter at late times for $n = 5$ and $\theta_A = 0$ and (b) $n = 6$ and $\theta_A = \pi/3$.

We find an obvious relation between the initial angle of the field $\theta_i$ and the baryon number, meaning $n_X \propto \sin(n\theta_i - \theta_A)$ for $n = 5, ..., 8$.

Parameter A

Figure 7.9: (a) The comoving asymmetry parameter at late times for $n = 5$ as function of $A/m_s$ and (b) the logarithmic fit.

For the behaviour of $n_X$ as function of $A$ we have excluded points near $A = 1$ because around those values the theory breaks down since the system is then close to the critical value at which the minimum does not disappear. The fits found are

<table>
<thead>
<tr>
<th>$n$</th>
<th>fit $A_0^5$ (log)</th>
<th>fit $A_0^5$ (regular)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>6</td>
<td>0.99</td>
<td>0.97</td>
</tr>
<tr>
<td>7</td>
<td>0.99</td>
<td>0.95</td>
</tr>
<tr>
<td>8</td>
<td>0.98</td>
<td>0.94</td>
</tr>
</tbody>
</table>

Which seems to be reasonably close to the theoretical value of $A/m_s$. 

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Parameter $b$

Figure 7.10: (a) The comoving asymmetry parameter at late times for $n = 5$ as function of $b$ and (b) the logarithmic fit.

The first fit of figure (7.10) gives a relation $n_X \propto b^{2.08}$ and the second fit is $n_X \propto b^{2.01}$ which is very close to the theoretical value of $b^2$. The complete table of values is given below.

<table>
<thead>
<tr>
<th>$n$</th>
<th>fit $b^x$ (log)</th>
<th>fit $b^x$ (regular)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2.07</td>
<td>2.44</td>
</tr>
<tr>
<td>6</td>
<td>2.08</td>
<td>2.47</td>
</tr>
<tr>
<td>7</td>
<td>2.08</td>
<td>2.43</td>
</tr>
<tr>
<td>8</td>
<td>2.07</td>
<td>2.40</td>
</tr>
</tbody>
</table>

Best fit

Putting this all together we find

<table>
<thead>
<tr>
<th>$n$</th>
<th>fit $A_0^2$</th>
<th>fit $b^x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.99</td>
<td>2.07</td>
</tr>
<tr>
<td>6</td>
<td>0.99</td>
<td>2.08</td>
</tr>
<tr>
<td>7</td>
<td>0.99</td>
<td>2.08</td>
</tr>
<tr>
<td>8</td>
<td>0.98</td>
<td>2.07</td>
</tr>
</tbody>
</table>

which is fairly close to the analytical estimate $^{17}$ of

$$n_X \propto \sin (n\theta - \theta_A) \left( \frac{A}{m_s} \right)^2 b^2, \quad b < 1, n = 5, \ldots, 8$$  \quad (7.77)
Chapter 7. Affleck-Dine in gauge mediated supersymmetry breaking

Parameter range $1 < b < b_{\text{crit}}$

Angular parameters

Figure 7.11: (a) The comoving asymmetry parameter at late times for $n = 7$ and $\theta_A = 0$ and (b) $n = 8$ and $\theta_A = \pi/4$.

Again we find the sinoidal relation between the initial angle and the baryon number, $n_X \propto \sin(n\theta - \theta_A)$ for $n = 5, \ldots, 8$.

Parameter $A_0$

Figure 7.12: (a) The asymmetry parameter with $n = 6$ over a parameter range $10^{-3} < A_0 < 10^{-1}$ and (b) the log-fit with $n = 6$.

For $A/m_s$ we find a relation that is close to the theory: $n_X \propto A_0^{0.93}$ for figure (a) with the normal plot- and fitting method and $n_X \propto A_0^{0.98}$ in figure (b) with the logarithmic plot values. For higher dimensions the fit values are bit lower.

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7.4. Field values at onset of oscillations

<table>
<thead>
<tr>
<th>n</th>
<th>fit $A_0^b$ (log)</th>
<th>fit $A_0^b$ (regular)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1.00</td>
<td>0.98</td>
</tr>
<tr>
<td>6</td>
<td>0.98</td>
<td>0.95</td>
</tr>
<tr>
<td>7</td>
<td>0.97</td>
<td>0.92</td>
</tr>
<tr>
<td>8</td>
<td>0.96</td>
<td>0.89</td>
</tr>
</tbody>
</table>

The values are quite close to $n_X \propto A_0^1$.

Parameter $b$

And now last but certainly not least, the dependence on $b$ itself: $n_X(b)$.

Figure 7.13: (a) $n_X(b)$ with $n = 8$ over a parameter range $10^1 < b < 10^3$ and (b) $n_X$ with $n = 8$ fitted using the log-fit.

Now the resulting behaviour of the exponent of $b$ differs slightly from the theory. The theory predicts

$$n_X \propto b^{\frac{4(n-2)}{n-1}}.$$  \hspace{1cm} (7.78)

We have found the following fit for the exponent of $b$.

<table>
<thead>
<tr>
<th>n</th>
<th>Theory $b^x$</th>
<th>fit $b^x$ (log)</th>
<th>fit $b^x$ (regular)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3.0</td>
<td>2.78</td>
<td>3.06</td>
</tr>
<tr>
<td>6</td>
<td>3.2</td>
<td>3.02</td>
<td>2.66</td>
</tr>
<tr>
<td>7</td>
<td>3.33...</td>
<td>3.07</td>
<td>2.67</td>
</tr>
<tr>
<td>8</td>
<td>3.43...</td>
<td>3.10</td>
<td>2.55</td>
</tr>
</tbody>
</table>

Notice how the last few points in figure 7.13, near $b = 10^3$, skew the fit in favor of these points. This is exactly what we try to avoid by using the log of the data.
Best fit

All together we get the exponents of our relation to be

<table>
<thead>
<tr>
<th>n</th>
<th>fit</th>
<th>$A_0^*$</th>
<th>$b^*$ (theory)</th>
<th>$b^*$ (numerical)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.99</td>
<td>3.0</td>
<td>2.78</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.98</td>
<td>3.2</td>
<td>3.02</td>
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</tr>
<tr>
<td>7</td>
<td>0.97</td>
<td>3.33...</td>
<td>3.07</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.96</td>
<td>3.43...</td>
<td>3.10</td>
<td></td>
</tr>
</tbody>
</table>

We have found that the relation works very well for $A/m_s$ and the angular part. For the exponent of $b$ we find a slight deviation from the predicted values.\[17\]

$$n_X \propto \sin (n\theta - \theta_A) \left( \frac{A}{m_s} \right)^b \lambda, \quad b > 1, n = 5, ..., 8 \quad (7.79)$$

where $\lambda$ is given in table.

In conclusion we have confirmed the relation between $n_X$ and the parameters of the model, $A/m_s, b, \theta_A$ and the initial angle $\theta_i$ for $b < 1$. For $b > 1$ the exponents of $b$ differ from the predicted values but the difference in $b$ is less than 10%. Nonetheless the relation with $A/m_s, \theta_A$ and $\theta_i$ holds. There are not many differences with gravity mediated supersymmetry breaking from the parametric viewpoint, meaning that perhaps the produced $n_X$ might be different, but the behaviour of the system is not. Unfortunately we can not say anything about the $A$ dependence in PMSB because we only checked the angular behaviour, and we would also have to keep in mind that $a$ also plays a role, although it might just change the oscillation time. Also the behaviour of $c$ in GMSB and $b$ in PMSB were not investigated and as such we can not make any statements about them.

However when we compare the equations of motion for PMSB and GMSB we see

$$X'' + 3h(\tau)X' + (1 - ch(\tau)^2)X = 0. \quad (PMSB)$$

$$X'' + 3h(\tau)X' + \ln (1 + |X|) \left( \frac{1}{1 + |X|} \right) \frac{X}{|X|} - ch(\tau)^2X = 0. \quad (GMSB)$$
7.4. Field values at onset of oscillations

We can also redefine $\Delta m_\phi/m_{3/2}$ in equation (7.80) to $A_1$ since the mass dimensions cancel each other. The field equations are now very similar, with the only difference being the logarithmic mass term in GMSB and the angular hubble term in PMSB. From this we expect $c$ and $A_1$ to give similar behaviour as in GMSB.

On a final note, the inherent difference is that in gravity mediation the field gets a kick from the shifting minimum while in gauge mediation the angular momentum comes from the fact that the initial angle is different than the angle of the local minimum.
Chapter 7. Affleck-Dine in gauge mediated supersymmetry breaking
Chapter 8

Conclusions and discussion

8.1 Conclusions

We have done a numerical analysis of the parameter space of both the gravity mediated (PMSB) and gauge mediated (GSMB) Affleck-Dine mechanism.

Gravity mediation
In the gravity mediated case (or Planck mediated) we saw a very clear linear dependence on $c$ and $\frac{m_{3/2}}{m_\phi}$. Also the angular dependence was clearly a sinoidal function. This combined gave us the proportionality of

$$n_X \propto c \frac{m_{3/2}}{m_\phi} \sin (\theta_a - \theta_A).$$

Unfortunately the $n$-dependence was quite complicated and we could not find a proper function to fit to. Then we evaluated the analytical formula for $\eta_B$ and found a reheating temperature of $T_R = 1.1 \times 10^{14}$ GeV. Of course we also did a numerical calculation for $\eta_B$, plugged in all the numbers and found a value of $T_R = 3.0 \times 10^{13}$ GeV. And we see that the difference between the numerical calculation and the analytical estimation is only a factor of 4.

Gauge mediation
In the gauge mediated case we had a much more general approach. We nondimensionalized the equations of motion and inspected the behaviour of $n_X$ as a function of the parameter $b$, among others. This parameter is a combination of the masses of the theory, and so you can go over quite a broad range of energies quite easily, by just going through a range of $b$. 

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Chapter 8. Conclusions and discussion

We tried to find a general function for the critical value of \( b \), at which the angular minimum of the potential does not disappear and although finding the numerical results for different \( n \) worked, we did not find one general function to fit them all simultaneously.

Then we compared the \( n_X \) numerically to recently published analytical results and found that they match very well. There was only one parameter that did not match, which is the parameter \( \lambda \) in the equation below. We found

\[
\begin{align*}
n_X \propto b^2 \left( \frac{A}{m_s} \right) \sin (n \theta + \theta_A), & \quad b < 1 \\
n_X \propto b^\lambda \left( \frac{A}{m_s} \right) \sin (n \theta + \theta_A), & \quad b > 1
\end{align*}
\]

for certain values of \( \lambda \) which can be found in section (7.4.1). These values deviate only about 10%, but a series for the values can not be found easily that lies close to the theoretical series.

We also found the values of the field at oscillation time, \( \chi_{osc} \) and \( H_{osc} \), and compared them to analytical solutions. We found that they do have the same general shape but the exact numbers are off by a factor of 10 and 2 respectively. This is probably because the definitions of oscillation time and normalization factors that might be different or approximated in literature, and sometimes we had to use our own definitions that are better suited for numerical solutions.

In conclusion, both ways to perform Affleck-Dine baryogenesis work quite well. We have seen that in PMSB the difference is only a factor of 4, and in GMSB we saw — even though we did no explicit calculation of \( T_R \) in GMSB — that it works quite well and that the functions found for \( n_X \) match very well with the expectations from analytical approaches.
8.2 Discussion

We have seen that the mechanism works, but we have not taken into account bounds from other parts of cosmology. For example we know from gravitino production bounds that the upper limit for the reheating temperature is \( T_R > 10^9 - 10^6 \) GeV. The temperature we found was \( 10^{13} \) GeV which is a few orders too large. One the other hand we can always make the angle difference in both cases as small as we like. This means we have a freedom in our system that allows us to always scale the produced asymmetry down to whatever we like, but we can not scale it upwards.

Unfortunately the \( c \) parameter in GMSB, the amplitude of \( A \) in PMSB were not investigated, but from looking at the field equations we expect them to behave the same. One thing that we noticed during the investigation of GMSB is that the field precesses, meaning that one of the focal points of the elliptical curve of the field rotates while the other focal point is stationary at the origin. In PMSB this was completely absent while in GMSB it turns up quite easily. Precession usually comes from nonlinear terms in the equations of motion of a system, so this probably comes from the logarithmic mass term in PMSB, but it could have other causes.

We considered a constant \( A \) but in GMSB the coupling constant \( A \) is field dependent, \( A(\Phi) \), which could complicate the interactions and the efficiency of AD.

On top of that, we have only considered one flat direction at a time even though there are many FDs in the MSSM. These directions could go through the AD mechanism simultaneously which is bound to result in some sort of intereference of the created asymmetries, making the dynamics much more complicated.
Chapter 8. Conclusions and discussion
Acknowledgements

First I would like to thank Marieke for her guidance and her open door policy, I believe that in the past year it occurred only once that she was not available when she was in her office. Second I would like to thank Kallia for all the help she gave me in understanding the theory.

Next I would like to thank Erica, Otto and Sabrina for all the fun we had in the office over the past year and Marco in particular for always lifting my spirits and of course the nice hat. Sander and Lisa inspired me with their respective opinions on academia and the whole theory group made sure it was always ”gezellig”.

Jiri’s advice about fitting non-linear models helped me greatly and Arthur’s tips and tricks in Mathematica made programming a lot smoother and faster. Of course I would also like to thank Wieger for all the nightly talks we had about cosmology when we were biking home from parties.

And last but not least I would like to thank my parents for supporting me throughout all these years while I was studying.
Appendix A

Conventions

We will be using "natural" units, eg.

\[ c = 1 \quad h = 1, \quad G = 1. \]  \hspace{1cm} (A.1)

We use the following notation for covariant four-vectors:

\[ x^\mu = (t, \vec{x}), \quad p^\mu = (E, \vec{p}), \quad \partial_\mu = (\partial_t, \vec{\nabla}), \]  \hspace{1cm} (A.2)

where greek indices \( \mu, \nu = 0, 1, 2, 3 \).

And in Minkovski space we use the "mostly positive" metric

\[ \eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1). \]  \hspace{1cm} (A.3)

When we’re discussing Cosmology we use the Friedman-Lemaitre-Robertson-Walker (FLRW) metric

\[ ds^2 = -dt^2 + a(t)\delta_{ij}dx^idx^j, \]  \hspace{1cm} (A.4)

where \( a(t) \) is the expansion factor of the universe. And the Hubble constant is the usual \( H = \frac{\dot{a}}{a} \).

The representation we use for Pauli- and Dirac matrices in the Weyl basis are given by:
Appendix A. Conventions

\[ \sigma^0 = \bar{\sigma}^0 = 1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma^1 = -\bar{\sigma}^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (A.5) \]

\[ \sigma^2 = \bar{\sigma}^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = -\bar{\sigma}^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (A.6) \]

\[ \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (A.7) \]

where the bar indicates the adjoint of the matrix. Now four-component Dirac spinors can be written as two component Weyl spinors. We also use this notation for supersymmetric objects, since they are also anti-commuting, like spinors. Right-handed particles are indicated by the index \( \alpha = 1, 2 \) while left-handed particles have a dotted index \( \dot{\alpha} = 1, 2 \).

So a given Dirac spinor is usually written as

\[ \Phi = \begin{pmatrix} \chi \\ \xi \end{pmatrix}, \quad (A.8) \]

but in our notation it becomes

\[ \Phi = \begin{pmatrix} \xi_\alpha \\ \chi^\dagger \dot{\alpha} \end{pmatrix}. \quad (A.9) \]

and the adoint spinor shows how the indices behave, namely

\[ \bar{\Phi} = \Phi^\dagger \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = (\xi^\alpha \chi^\dagger \dot{\alpha}). \quad (A.10) \]

So the upper index becomes a lower one, and the adjoint has a dotted index. This is because the lefthanded and righthanded spinors swap sides when one conjugates. We can also see this from the projection operators, defined by

\[ P_L = (1 - \gamma_5)/2, \quad P_R = (1 + \gamma_5)/2. \quad (A.11) \]

which yields the left- and the right-handed parts of the spinor

\[ P_L \Phi = \begin{pmatrix} \xi_\alpha \\ 0 \end{pmatrix}, \quad P_R \Phi = \begin{pmatrix} 0 \\ \chi^\dagger \dot{\alpha} \end{pmatrix}. \quad (A.12) \]
So we see that in this convention all left-handed fermion fields have lower indices and right-handed ones have upper indices. Now these spinor indices are not the same as Lorentz indices, and are lowered and raised by an antisymmetric symbol $\epsilon$ because they are fermionic.

$$\epsilon^{12} = -\epsilon^{21} = \epsilon_{21} = -\epsilon_{12} = 1, \quad \epsilon_{ii} = \epsilon^{jj} = 0 \quad (A.13)$$

So contracting spinors this "metric", or the symbol, gives us

$$\chi_\alpha = \epsilon_{\alpha\beta} \chi^\beta, \quad \chi^\alpha = \epsilon^{\alpha\beta} \chi_\beta, \quad \xi_\alpha^\dagger = \epsilon^\alpha_{\dot{\alpha}} \xi^\dot{\alpha}, \quad \xi^\alpha = \epsilon^{\dot{\alpha}\beta} \xi_{\beta} \quad (A.14)$$

And suppressing repeated indices yields

$$\chi \xi = \chi^\alpha \xi_\alpha = \chi^\alpha \epsilon_{\alpha\beta} \xi^\beta = -\xi^\beta \epsilon_{\alpha\beta} \chi^\alpha = \xi^\beta \epsilon^\beta_{\dot{\alpha}} \chi^\dot{\alpha} = \xi \chi, \quad (A.15)$$

and we see that the product of fermionic fields is bosonic, which is crucial for supersymmetry. The same holds for conjugate fields. Similarly we get

$$\chi^\dagger \bar{\sigma}^\mu \xi = \chi^\dagger_{\dot{\alpha}} (\bar{\sigma}^\mu)^{\dot{\alpha}} \xi_\alpha = -(\chi \sigma_\mu \xi^\dagger)^\dagger \quad (A.16)$$

$$\chi \sigma_\mu \bar{\sigma}^\nu \xi = \xi \sigma_\nu \bar{\sigma}^\mu \chi = (\xi^\dagger \bar{\sigma}^\nu \sigma_\mu \chi^\dagger)^\dagger = (\chi^\dagger \bar{\sigma}^\nu \sigma_\mu \xi^\dagger)^\dagger. \quad (A.17)$$

More explicitly a contraction is

$$\chi \chi = \epsilon^{\alpha\beta} \chi_\beta \chi^\alpha = \chi_2 \chi_1 - \chi_1 \chi_2 = 2 \chi_2 \chi_1 \quad (A.18)$$

which is non-zero because of the anti-symmetry, so the contraction is well-defined. We see that this contraction would give zero on bosonic fields because they commute instead of anti-commute and vice-versa for fermionic fields with regular contraction with a diagonal metric.

In other words, for a vector $\vec{v} \neq 0$ we have

$$v_1 v_1 + v_2 v_2 \neq 0 \text{ and } v_1 v_2 - v_2 v_1 = 0 \quad (A.19)$$

while for a fermionic field $\psi \neq 0$ we have

$$\psi_1 \psi_1 + \psi_2 \psi_2 = 0 \text{ and } \psi_1 \psi_2 - \psi_2 \psi_1 \neq 0. \quad (A.20)$$
Appendix B

Harmonic Oscillator in an Expanding Universe

A universe described by the FLRW metric \( ds^2 = -dt^2 + a(t) \delta_{ij} dx^i dx^j \) is a universe of which the size is changing. The function \( a(t) \) is called the scale factor because it describes how much the universe has expanded (or shrunk!) as it is in front of the spatial part of the metric.

The equation of motion of a field in such a space is

\[
\ddot\phi + 3H\dot\phi + \frac{\partial V}{\partial \phi} = 0 \quad (B.1)
\]

where we take \( H \) to be in a matter dominated universe with \( a(t) = t^{2/3} \), so

\[
H = \frac{\dot{a}}{a} = \frac{2}{3t} \quad (B.2)
\]

resulting in

\[
\ddot\phi + \frac{2}{t}\dot\phi + V(\phi)' = 0 \quad (B.3)
\]

Now we want to study the behaviour of the harmonic oscillator, to see how it differs from classical non-expanding flat space. We take the usual potential for the harmonic oscillator as in flat space
Appendix B. Harmonic Oscillator in an Expanding Universe

\[ V = \frac{\phi^2}{2}, \quad V' = \phi \]  \hspace{1cm} (B.4)

The full equation of motion is now \( \ddot{\phi} + \frac{2}{t} \dot{\phi} + \phi = 0 \). As ansatz for the solution we take \( \phi = (A + Bt + Ct^2 + Dt^3)e^{i\lambda t} \).

\[
\phi = e^{i\lambda t} \left( A + Bt + Ct^2 + \frac{D}{t^3} \right) \hspace{1cm} (B.5)
\]

\[
\dot{\phi} = i\lambda e^{i\lambda t} \left( A + Bt + Ct^2 + \frac{D}{t} \right) + e^{i\lambda t} \left( B + 2Ct - \frac{D}{t^2} \right) \hspace{1cm} (B.6)
\]

\[
\ddot{\phi} = -\lambda^2 e^{i\lambda t} \left( A + Bt + Ct^2 + \frac{D}{t} \right) + 2i\lambda e^{i\lambda t} \left( B + 2Ct - \frac{D}{t^2} \right) + \left( 2C + \frac{2D}{t^3} \right) e^{i\lambda t} \hspace{1cm} (B.7)
\]

The equation now becomes

\[
-\lambda^2 e^{i\lambda t} \left( A + Bt + Ct^2 + \frac{D}{t} \right) + e^{i\lambda t} \left( A + Bt + Ct^2 + \frac{D}{t} \right) + 2 \left( i\lambda e^{i\lambda t} \left( \frac{A}{t} + B + Ct + \frac{D}{t^2} \right) + e^{i\lambda t} \left( \frac{B}{t} + 2Ct - \frac{D}{t^2} \right) \right) + 2i\lambda e^{i\lambda t} \left( B + 2Ct - \frac{D}{t^2} \right) + \left( 2C + \frac{2D}{t^3} \right) e^{i\lambda t} = 0. \hspace{1cm} (B.8)
\]

Multiplying with \( e^{-i\lambda t} \) and collecting all the terms by exponents \( t, t^2, \ldots \) and using the condition that the entire equation is zero we find conditions for the constants to be

\[
-B\lambda^2 + B + 6Ci\lambda = 0 \hspace{1cm} (B.10)
\]

\[
-C\lambda^2 + C = 0 \hspace{1cm} (B.11)
\]

\[
2Ai\lambda - 2B + D\lambda^2 + D = 0 \hspace{1cm} (B.12)
\]

\[
-A\lambda^2 + A + 4Bi\lambda + 6C = 0. \hspace{1cm} (B.13)
\]

Yielding the result

\[
\lambda = \pm 1 \hspace{1cm} (B.14)
\]

\[
A = 0, \quad B = 0, \quad C = 0 \hspace{1cm} (B.15)
\]

\[
D \text{ is a free parameter.} \hspace{1cm} (B.16)
\]
So the solution is given by

\[ \phi = \frac{D_1}{t} e^{it} + \frac{D_2}{t} e^{-it}. \]  

(B.17)

This result can be extended to any dimension, but to be able to relate it to the Affleck-Dine field we use a 2D harmonic oscillator. Typical behaviour looks like this:

![Figure B.1: The field rolls towards the minimum of the potential at the origin.](image)

The field rolls to the minimum of the potential in the center, but the damping term \( \dot{\phi} \) becomes less important over time. If the potential starts out with symmetric boundary conditions such as \( x = y \neq 0 \) and \( x' = y' \neq 0 \) it will still roll towards the minimum, but the rotating effect will be lost since the field will be in the path \( x = y \). The damping will still happen but it will not be as obvious to the eye.

The behaviour of the energy density of this oscillator can be seen in the graph below. The red graph represents the total energy of the system while blue and green are respectively kinetic and potential energy.

We see that the energy of the system slowly dissipates in the beginning but after a (small) number of oscillations the system settles and converges to a constant energy. Even though it converges to this value, we still see an internal oscillation in the energy, which in the classical harmonic oscillator would not exist. This is because the universe is expanding, causing a harmonic oscillator to expand with it, and after a while it will want to move back to a lower energy state, after which this cycle repeats itself.
Figure B.2: The total energy (red) is damped at first, but after a few oscillations converges to a constant value.
Bibliography


[22] B. Ryden, San Francisco, USA: Addison-Wesley (2003) 244 p

