Higher-Order Azimuthal Anisotropy of $\Lambda + \bar{\Lambda}$ hyperons in PbPb Collisions at $\sqrt{s_{NN}} = 2.76$ TeV measured by ALICE at LHC

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Front page: (Color online) Initial-state energy distribution calculated at pseudorapidity \( \eta = 0 \) and proper time \( \tau = 1 \, fm/c \) for one simulated central collision (0-5\%) with the NeXuS code. The subsequent hydrodynamic evolution essentially transforms inhomogeneities into momentum-space anisotropy of the outflowing matter.

Abstract

Ultra-relativistic heavy-ion collisions are essential in probing strongly interacting matter at high temperatures and low baryon density. One of the important discoveries made at the Relativistic Heavy Ion Collider (RHIC), and more recently at the Large Hadron Collider (LHC), is the large elliptic flow $v_2$. Collective flow, as manifested by the anisotropic emission of particles in the plane transverse to the beam direction, known as the reaction plane, is characterized by a series of Fourier coefficients. The observed second order harmonic $v_2$ near the mid-rapidity region ($|y| < 0.5$), for not too large impact parameters between the colliding nuclei and low transverse momenta of the detected hadrons, agrees remarkably well with predictions made by relativistic viscous fluid dynamics. Most prominently, these predictions verify the experimentally extracted dependence of $v_2(p_T)$ on transverse momentum $p_T$ and hadron rest-mass. It is then of common belief that in these collision energies a Quark-Gluon Plasma (QGP) is formed, which thermalizes on a very rapid time scale and subsequently evolves as an almost ideal fluid with exceptionally low viscosity.

While earlier studies had focused on elliptic flow, most of the recent activity is concerned with the effect of fluctuations in the initial geometry. The conventional assumption of a smooth initial almond-shape profile has hindered full exploitation of the odd harmonics. Such fluctuations result in initial density profiles, which have no particular symmetry and new types of flow, such as the triangular flow $v_3$, are not required by definition to be zero. Flow harmonics (both of even and odd order) stem from a so-called eccentricity-driven hydrodynamic expansion of the matter in the collision zone, i.e. a finite eccentricity $\varepsilon_n$ drives uneven pressure gradients, hence the resulting anisotropic expansion leads to the anisotropic emission of particles about the reaction plane. The $v_n$ coefficients are sensitive to both the initial eccentricity and the ratio of the QGP shear viscosity $\eta$ to its entropy density $s$. It is expected that $v_n$ for identified particles will provide further constraints on the underlying models that treat initial conditions and the evolution of the hot plasma.

During the actual study, Fourier coefficients of order 2–4 have been measured as a function of transverse momentum $p_T$ in the $(0-60)\%$ most central $\text{PbPb}$ collisions at $\sqrt{s_{NN}} = 2.76\text{ TeV}$ for the case of $\Lambda + \bar{\Lambda}$ hyperons and compared to various species. The Scalar Product technique has been employed, which consistently produces the root-mean-square $\sqrt{\langle \varepsilon_n^2 \rangle}$ of the correlation between the identified hadrons under study and the reference-flow particles. Residual short-range correlations have been diminished by imposing a pseudorapidity gap between particles of interest and reference-flow particles, with the former having been reconstructed in the central barrel of the ALICE detector (TPC) and with unidentified particles having been recorded in the forward region (VZERO).

All three anisotropy coefficients, i.e. $v_2$, $v_3$, and $v_4$, have been observed to increase with $p_T$ up to about 3.5 GeV/$c$, then to saturate and decrease, a pattern persistent all over centralities. Elliptic and triangular flow are the dominant harmonics, and it seems to be driven mainly by the associated ellipticity $\varepsilon_2$ and triangularity $\varepsilon_3$. In the most central collisions, where the $\varepsilon_n$ moments are expected to be comparable due to event-by-event fluctuations in the initial geometry, the hadronic azimuthal anisotropy is found of similar magnitude across the different $n^{th}$-order harmonics. Interestingly, the less dominant quadrangular flow $v_4$ seems to receive contributions both from its corresponding $\varepsilon_4$ and the lower-order anisotropy $\varepsilon_2$.

The characteristic features of collective dynamics, i.e. the linear-$p_T$ dependence and the mass ordering at low $p_T \lesssim 2$ GeV/$c$, are clearly observed across harmonics of all the identified cases considered during the actual study. At variance with their low-$p_T$ behavior, these final-state coefficients exhibit a centrality-dependent crossing point at intermediate $p_T$. The enhanced flow of baryons over mesons at the intermediate $2.5 \lesssim p_T \lesssim 8$ GeV/$c$ window has been investigated in the realm of the coalescence mechanism. The constituent quark scaling of anisotropic flow $v_n(p_T) \approx n v_2 (p_T/n)$ found to barely hold at LHC energies. Last, a non-particular particle species dependence is evident for $v_2$ at $p_T \gtrsim 10$ GeV/$c$, consistent with expectations from a path-length driven emission of particles. Fluctuations might become unimportant for $v_3$ and $v_4$ in such a parton fragmentation dominated regime, whereas more data are undoubtedly needed before drawing a conclusive answer.
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1 High Temperature Quark Soup

“Asymptotic freedom and quantum chromodynamics: the key to the understanding of the strong nuclear forces”

NOBEL PRIZE IN PHYSICS, 2004

1.1 Learning from the Past

With the advent of quantum mechanics in the first decades of the previous century, it was realized that the electromagnetic field is quantized, with the relevant force being mediated by photons. Later, Quantum electrodynamics (QED) was developed as the relativistic quantum field theory (QFT) that describes photons and electrons, i.e. the electromagnetic force. In practice, QED was proven to be a renormalizable theory by preserving gauge invariance all over the renormalized quantum corrections. The gauge symmetry in QED, namely the arbitrariness in the local change of the phase in the electron wave function, is called abelian, since the underlying U(1) group is commutative.

As the electromagnetic force holds the atoms together, short-range nuclear forces should function within the nucleus; a (strong) force binding nucleons together, along possibly with their constituents. After the incompleteness and apparent failure of Yukawa theory, involving exclusively nucleons and pions with an effective coupling substantially greater than unity, much interestingly, a non abelian gauge theory was constructed. This has been achieved by Yang and Mills on the grounds of promoting the global isotopic spin (isospin) symmetry of the Yukawa theory to the local isospin group SU(2). However, their non-abelian QFT has been severely criticized due to having introduced a massless vector force mediator, which constituted a striking elusive particle from the discovered particle zoo. Not to mention that such a particle would mediate a force with an infinite range, albeit nuclear forces must be of short-range nature.

In the late 1960s, the renormalization scheme has been extended in view of the experimentally verified Bjorken scaling in deep inelastic scattering processes. It was argued that a physical theory consistent with such a scaling behavior should incorporate a negative β-function, with \( \beta = \frac{\mu \partial g}{\partial \mu} \); \( \mu \) is the (hidden) mass scale at which the renormalization terms are eliminated and \( g \) corresponds to the coupling constant. More specifically, the term asymptotic freedom was coined for such a theory, inasmuch as the force, i.e. the (running) coupling constant, increases the further the distance from the charge, while decreases the closer to it. Furthermore, it was realized that the asymptotic behavior of the coupling strength is uniquely governed by the \( \beta \)-function. Clearly, a negative \( \beta \)-function could not be intuitively understood, since virtual pair of particles and antiparticles anti-screen the charge.

In the early 1970s, with the development of the electroweak theory, which unified the weak nuclear and electromagnetic interaction into the common symmetry group \( SU(2) \times U(1) \), it became increasingly attractive to search for gauge theories for the description of all the fundamental interactions. It was only until 1972 when Gross, Wilczek and Politzer performed the pivotal study about a QFT for strong nuclear interactions, a non abelian gauge theory based on the \( SU(3) \) symmetry group for quarks along with massless vector mediators, the gluons. Thus, the solution to the problem of receiving a short range interaction is naturally given by the Quantum ChromoDynamics (QCD) theory, since it respects the property of asymptotically free interactions. QCD complemented the electro-weak theory, unifying three fundamental interactions into one non abelian gauge field theory of \( SU(3) \times SU(2) \times U(1) \) symmetry. This model is widely known as the Standard Model (SM) for particle physics, a consistent four-dimensional relativistic quantum theory.

Since the coupling constant depends on the characteristics of the interaction, the study of systems of quarks and gluons falls into the two broad categories of perturbative (short
wavelength) and non perturbative (long wavelength) QCD. Despite the apparent consistency of QCD with high energy experiments, the distinctive feature that renders QCD precision tests challenging is the peculiar behavior of its basic constituents. Quarks and gluons do not exist as free particles and thus cannot be directly detected in a collision experiment. Such an infrared slavery reads that the force between quarks grows with distance so that they are permanently bound together; once the strong coupling is large, a $q\bar{q}$ pair are as likely to exchange many gluons as they are to exchange just one. Though no definite mathematical proof has been feasible so far, effective field theories and non-perturbative techniques have been developed for the large-distance behavior of the system. The existence of these two dynamical limits in QCD implies a complex transition region at some intermediate energy scale $\Lambda_{QCD}$, in which excited color degrees of freedom dominate. This transition, commonly known as the color deconfinement phase transition, is of major interest in nuclear physics.

1.2 Facing the Future

1.2.1 Onset of deconfinement?

By colliding large nuclei at very high kinetic energies it is believed the nature of interactions between the QCD constituents to be revealed. Particularly, the amount of energy deposited into a minuscule volume containing nuclear matter would be sufficient enough for the formed medium having dissolved into deconfined colored degrees of freedom. An heuristic understanding of the possibility for forming a fireball is depicted in Fig. (1), where hadrons be viewed to exhibit well-defined bag-like boundaries. When the temperature of the matter is high, the medium changes from distinguishably hadronic to a quark-matter dominated phase with hadron boundaries having disappeared. Since color charges are free to move throughout the medium, the term Quark Gluon Plasma (QGP) was coined for such an abnormal nuclear matter phase, in analogy to conventional, i.e. abelian, plasmas.

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1A possible exception is said the $t$ quark, for its life time $\tau_t \sim 1/\Gamma_t \sim 10^{-25}$ sec as predicted in SM is smaller than a typical timescale for formation of QCD bound states $\tau \sim 1/\Lambda_{QCD} \sim 10^{-24}$ sec; it decays long before it can hadronize.

2A simple intuitive description of the competing forces that lead to a stable system is the balance between the inward bag pressure and the outward pressure from the kinetic energy of quarks. The difference in the energy density of the vacuum inside and outside the bag-like hadron gives rise to the bag pressure, while the outward directed pressure is associated to the quantum stress tensor coming from the spatial variation of the amplitude of the quark wave function.
Figure 1: Illustration of hadronic matter that transitions into deconfined matter as the temperature increases at zero baryon density, or equivalently as the baryon density increases at low temperature. Although quarks are bound in discrete hadrons (left), as the temperature or baryon density increases, it becomes less clear which quarks belong to which hadrons (middle). Finally, each quark is attracted by multiple quarks surrounding it (right), rather than being bound to one or two partners within a hadron. If gluons are confined in the bag (not depicted in the current illustration), then they should also be deconfined in the QGP phase.

The initial energy densities achieved in relativistic heavy-ion collisions exceed the energy density of atomic nuclei, which approximately equals to $150 \text{ MeV/fm}^3$ in their ground state, by at least one order of magnitude [10]. On one hand, at ultra-relativistic collision energies the system net baryon density equals approximately to zero, since the initial excess of $q$ over $\bar{q}$ is negligible compared to the total number of created particles. Such conditions are believed to be present in the early phase of the evolution of the universe, about $1\mu s$ after the Big Bang (e.g. [3]). On the other hand, when the temperature is low and the baryon density is large, there should be a point where the degenerate pressure exceeds the inward pressure. A QGP phase of non-zero net baryon density, exceeding a multiple of the normal nuclear matter density, has been also conjectured to be present in the interior of compact, dense stellar objects (e.g. [4]). For a system in between these two limits a phase diagram is typically constructed (Fig. (2)), i.e. a schematic diagram in the space of energy density (or temperature) and net baryon density (or baryo-chemical potential).

One of the primary objectives of heavy-ion physics is to explore the QCD phase diagram. Of particular interest is the QCD equation of state (EoS), quantifying the relationships between intrinsic quantities such as energy density and temperature. Since the nature of these relationships depends on the phase of the matter, it is worth focusing on the phase transitions. Locating a critical point in Fig. (2), beyond which a first-order phase transition takes places, requires lower temperatures and larger chemical potentials, i.e. lowering the collision energy [14]. In parallel, scanning the QCD phase diagram with variable collision energy would determine whether the QGP signatures, observed at the highest energies, have been turned off or not. In other words, such a beam energy scan would provide complementary, if not conclusive, proof about the existence of non-ordinary nuclear matter.
An overview of the QCD phase diagram; here, the temperature $T$ is plotted against the net baryon chemical potential $\mu_B$. One particular corner of special interest, i.e., the regime of small values of $\mu_B$ and high $T$, is experimentally accessible at ultra-relativistic heavy-ion collisions at RHIC and LHC. Currently for this region, lattice QCD theory predicts (e.g., [7]) no discrete phase boundary, namely a continuous crossover, between the hadronic medium and the QGP. Various experiments at lower energies, for instance RHIC II or the future FAIR facilities, aim to study the system at large densities. One of the most important objectives is to identify a possible critical (end)point on the phase diagram, beyond which the transition becomes first order. Detecting the presence of the critical point depends on the ability of experiments to create QGP matter above the critical temperature at continuously larger $\mu_B$. On the other hand, there exists the entire low-$T$ and high-$\mu_B$ regime, which is of great importance in the astrophysical context. At this scale, the Color Flavor Locked (CFL) phase (e.g., [8]) is an emerging phenomenon well explained, where the formation of quark-quark condensate leads to a new colour-superconducting phase of cold dense matter.

Extensive numerical calculations indicate that at low baryon densities the transition occurs at a fairly low temperature of $O(150)\text{ MeV}$, thus opening the path for the experimental observation of the QGP in the laboratory. However, direct observation of the medium is impossible, owing to the estimated time scale of $O(10^{-24})\text{ s}$ that surpasses by many orders of magnitude any state of the art detector resolution. Instead, experiments identify remnants of collisions and the directly emitted gamma rays that carry information about the microscopic phenomenon of the fireball formation.

Each and every relativistic heavy-ion collision system is described by its initial density profile, producing a fireball of highly excited state, whose constituents collide frequently to establish an equilibrium state. After the pre-equilibrium evolution stage, not lasting much longer than $\sim 1\text{ fm/c}$, the fireball is found in the QGP phase, a strongly-coupled plasma (sQGP, [9]) that flows like an almost minimally viscous liquid. Macroscopically, its evolution, i.e., the collective expansion, is properly described by viscous hydrodynamics. As the system expands, it cools down and the constituents, below a (crossover) temperature, are confined into hadrons. In the hadronic phase, the fireball further cools down via inelastic and elastic interactions until it eventually becomes non interacting. Certainly, the hydrodynamic description breaks down, whereas hadrons decouple at the freeze-out surface and freely stream towards the detectors (Fig. (3)).
Figure 3: In ultra-relativistic energies, achieved at top RHIC and LHC energies, the fireball formation and evolution contains multiple stages which are governed by different underlying physics [5]. Fluctuations result in lumpy initial conditions, the later represented here by the initial energy density. During the first $\sim 1 \text{ fm/c}$, the system achieves approximately local thermal equilibrium. The quarks and gluons that are produced after the collision form a strongly coupled plasma (sQGP), whose dynamics can be described by macroscopic viscous hydrodynamics; the viscous corrections parameterize the remaining deviations from thermal equilibrium. As the system expands and cools, it will smoothly crossover from the QGP phase to a hadron gas phase, whereas at hadronization, the quark-gluon fluid will convert into hadrons due to confinement. As the fireball continues to expand and cool, the collision rates between the hadronic resonances decrease, with the inelastic collisions between particles having firstly ceased, i.e. the system reaches chemical freeze out, and with particles having subsequently reached kinetic (thermal) freeze out. This is the surface that detectors can register. Rare electromagnetic observables, like photons (wavy lines) and/or dileptons (green wavy line), could provide constraints on the early dynamics of the fireball that are complementary to those obtained from the much more abundant hadronic observables; since they exclusively interact with the medium through the electromagnetic interaction, they are the cleanest penetrating probes for the heavy-ion collisions.

To avoid misunderstanding, the discovery of the QGP would not mean that its physical properties would have been quantitatively measured. In fact, it only signals a long-sought and well focused direction of research, which has been underway using generations of lower energy accelerators than at Relativistic Heavy Ion Collider (RHIC) and currently at Large Hadron Collider (LHC). A series of experimentally accessible observables cope with the task of constraining the properties of the QGP. Clearly, it is of utmost importance to construct observables that could discriminate among initial-state nuclear dynamics and, at the same time, would be proven the least distorted by uninteresting hadronic final interactions.

1.2.2 Towards a quantitative reliable description

Anisotropic flow studies constitute one of the most indispensable experimental tools, mainly affecting the bulk of the hadrons. Much interestingly, the long-wavelength modes (low transverse momentum) in the QGP are as maximally coupled as traces of initial profiles to survive the complete dynamics, while the dilution of the late-hadron gas phase is essentially eliminated. Parallel to its nearly perfect fluidity, the QGP additionally retains part of its QCD asymptotic freedom character. Briefly, the interaction of short-wavelength (high momentum) partons with the medium induces gluon radiation, which has been extensively studied in terms of perturbative QCD (pQCD). High-energy partons, whose trajectories are indicated by the blow arrows in Fig. 4, loose energy in the QCD medium via gluon-bremsstrahlung radia-
tation (curl lines) and elastic collisions with the constituents of the medium. Different pQCD formalisms assume that the interference of the multiple scatterings over the path length \( L \) is usually destructive [15], leading to a radiation rate that depends on \( L \) and results in a characteristic quadratic path-length dependence for the parton total energy loss, i.e. \( \Delta E \propto L^2 \).

Single-particle observables at high transverse momentum, such as the nuclear modification factor \( R_{AA} \), viz. the normalized ratio of particle yields in different collision systems, and the final hadron momentum anisotropy are sensitive to this path-length variation of energy loss. Finally, it has been argued that non-equilibrium non-perturbative processes at intermediate transverse momentum might be also present during the QGP evolution, thus being reflected in the hadron momentum anisotropy. Undoubtedly, anisotropic flow measurements offer a wide range of accessible QGP-related phenomena.

![Figure 4](image)

**Figure 4:** Although, the medium itself is non-perturbative, the interaction between high-energy partons and the medium is perturbative. The high-energy parton-medium interaction has been treated by different dynamical approaches of the QCD medium and kinematics of the interactions. In pQCD descriptions of the parton-medium interaction the predicted parton energy loss should scale as the path length squared. Obviously, the energy of the fast partons is not lost, but redistributed inside the collision system [16].

Primarily, it is crucial to construct sensitive experimental probes to flow effects, thus making their physical interpretation unambiguous. What is really measured during flow studies is the azimuthal asymmetry of the measured final state spectra, usually characterized by a set of Fourier coefficients \( \nu_n \). To that end, the azimuthal part of the momentum distribution of hadrons is decomposed into a Fourier series,

\[
E \frac{dN}{d^3p} = \frac{dN}{p_Tdp_Tdyd\phi} (\phi) = \frac{dN}{p_Tdp_Tdy} \left\{ \frac{1}{2\pi} \left[ 1 + \sum_{n=1}^{\infty} \nu_n \cos (n\phi - n\Psi_n) \right] \right\} . \tag{1}
\]

The total yield of particles \( N \) is obtained via the triple integration over transverse momentum \( p_T \), rapidity \( y \) and azimuthal angle \( \phi \), with the later being measured relative to the direction of the reconstructed plane of symmetry \( \Psi_n \) (cf. section §A). The essence of the very last relation encapsulates the idea that in a given collision event particles are emitted according to an anisotropic probability distribution, probed by the Fourier terms \( \nu_n e^{in\Psi_n} = \langle e^{in\phi} \rangle \). Note that angle brackets denote an average value over outgoing particles. The underlying flow distribution is sensitive to the initial geometry (cf. section §2.2.1), the subsequent collective expansion (cf. section §2.2.2) and the freeze out of the system (cf. section §2.2.3). For avoiding confusions and closely following the literature, nomenclatures **Fourier coefficient**, likewise **Fourier harmonic**, are simultaneously reserved for the \( \nu_n \) Fourier term.

It should be noted that the harmonic spectrum is typically analyzed based on diverse techniques, which can be divided into two broad categories, i.e. two- and multi-particle correlation techniques. A typical example of results obtained with different methods is shown in Fig. (5). More specifically, the second order harmonic \( \nu_2 \), integrated over transverse momentum \( p_T \) and pseudorapidity \( \eta \), is plotted for the indicated methods as a function of centrality, i.e. percentile
of the geometric cross section (cf. section §F.4). The significant difference between two- and multi-particle correlations is caused by the different sensitivity of these methods to effects not related to the collectivity of the system, e.g. decays of resonances, and event-by-event fluctuations of the flow magnitude itself due to fluctuations in the initial participant region. Clearly, multi-particle correlations are proven a systematic way of suppressing such nonflow contributions [13]. Note that the effect of nonflow on two-particle estimate is more apparent in more peripheral collisions. Finally, the current study (cf. section §4) principally focus towards a slight variant of the event-plane method, viz. a two-particle correlation technique ([12] in section §4).

Figure 5: Heavy ions are extended objects and the system created in a head-on (central) collision is different from that in a grazing (peripheral) collision. The centrality in heavy-ion collisions is defined as a fraction of the total geometric cross section, with 0% denoting the most central collisions and 100% the most peripheral ones. True flow values should lie between these two bands; the upper band compiles two-particle correlation techniques, whereas the lower one corresponds to multi-particle methods [12].

Transverse dynamics is of special interest. This is because, before the collision of the two nuclei, the longitudinal phase space is filled by the beam particles, whereas the transverse phase space is initially empty. Therefore, the actual analysis exclusively focus towards the primordial study of the Fourier coefficients in Eq. (1). In early studies, the values of \( v_n (p_T) \) were presumed to be zero for odd harmonics due to the reflection symmetry that the system possesses across the transverse plane (cf. section §A.1). For instance, in a non-central collision, where the incoming nuclei partially coincide, the averaged initial density profile presents an almond shape. Such a spatial asymmetry is commonly characterized by the so-called elliptic eccentricity \( \varepsilon_2 \) (cf. section §B.1), the latter feeding the anisotropic flow of order 2, i.e. the elliptic flow quantified by \( v_2 \) (cf. section §5.1). However, it has been lately realized that event-by-event fluctuations (cf. section §B.1) in the initial transverse shape of the interaction region generate non-zero odd Fourier harmonics [17]. For example, fluctuations on the positions of the nucleons in the overlap region, yet model dependent (cf. section §B.2), gave rise to a triangular anisotropy \( v_3 \) in the azimuthal particle distribution (cf. section §5.2), as a consequence of a triangular anisotropy \( \varepsilon_3 \) in the initial density distribution ([15] in section §4 and references therein).

First measurements, in particular of \( v_3 \) and \( v_5 \), have been reported only recently. One key piece of evidence that the initial collision region has a rather complex geometry of convoluted symmetry planes of both even and odd harmonics, has been the di-hadron azimuthal correlation data. Briefly, a correlation function \( C(\Delta \phi, \Delta \eta) \) between two particles in relative azimuthal angle \( \Delta \phi \) and relative pseudorapidity \( \Delta \eta \) is constructed. Thus, the projection of the two-dimensional correlation function on \( \Delta \phi \) represents the distributions of relative azimuthal
angles between particle pairs consisting of the trigger and the associated partner at $p_T^T$ and $p_T^a$, respectively. This is illustrated in Fig. 5, where the correlation function is taken as the ratio of the same-event pair (foreground) distribution to the combinatorial pair (background) distribution, \( i.e. \ C(\Delta \phi) \propto \frac{S(\Delta \phi, \Delta \eta)}{B(\Delta \phi, \Delta \eta)} = \frac{N^{\text{pairs}}_{\text{trigger}}(\Delta \phi, \Delta \eta)}{N^{\text{pairs}}_{\text{mixed}}(\Delta \phi, \Delta \eta)} \). In that case, trigger-associated particle pairs within $2 < p_T^T < 2.5$ and $1.5 < p_T^a < 2$ GeV/c, \( i.e. \) within the bulk-dominated regime, and for the 2% most central PbPb events are considered. On top of that, they are pseudorapidity-separated for at least $|\Delta \eta| > 0.8$ units, in order to diminish short-range correlations, \( e.g. \) from jets and resonance decays. One recognizes then the intriguing feature at the away side $|\Delta \phi| = \pi$: a concave, doubly-peaked, correlation structure at $|\Delta \phi-\pi| \approx \pi/3$ is revealed.

These features are parameterized by the sum of both even and odd $n^{th}$-order harmonics, finite in magnitude up to approximately $n = 5$. The event averaged-square Fourier coefficients $v_{n,n} \left(p_T^T, p_T^a\right) = \langle \cos \left(n \Delta \phi\right)\rangle \propto \sum C(\Delta \phi) \cdot \cos \left(n \Delta \phi\right)$ are illustrated with solid lines separately up to the fifth order. The superposition $\sum_{n=1}^{5} 2v_{n,n} \left(p_T^T, p_T^a\right) \cos \left(n \Delta \phi\right)$ of these pair anisotropies, depicted by the dashed line, reproduces $C(\Delta \phi)$ with high accuracy, as shown in the ratio between the points and sum of the components. Much interestingly, $v_{n,n} \left(p_T^T, p_T^a\right)$ were found to approximately factorize into single-particle harmonic coefficients, \( i.e. \ v_{n,n} \left(p_T^T, p_T^a\right) \approx v_n \left(p_T^T\right) \cdot v_n \left(p_T^a\right) \). This is expected, if the azimuthal anisotropy of final state particles at large $|\Delta \eta|$ is induced by a collective response to initial-state collision geometry and its fluctuations. Overall, the correlation function $C(\Delta \phi)$ seems to reflect a mechanism that affects all particles in any given event and $v_{n,n} \left(p_T^T, p_T^a\right)$ to depend only on the single-particle azimuthal distribution with respect to the $n^{th}$-order symmetry plane $\Psi_n$ (Eq. (1)). Conclusively, the factorization of $v_{n,n} \left(p_T^T, p_T^a\right)$ is consistent with expectations from a collective response to anisotropic initial conditions, which provides a complete and self-consistent picture explaining the observed structure.

![Figure 6: Two-particle correlation function $C(\Delta \phi) = \frac{S(\Delta \phi, \Delta \eta)}{B(\Delta \phi, \Delta \eta)} = \frac{N^{\text{pairs}}_{\text{trigger}}(\Delta \phi, \Delta \eta)}{N^{\text{pairs}}_{\text{mixed}}(\Delta \phi, \Delta \eta)}$ for trigger-associated ($2 < p_T^T < 2.5$ and $1.5 < p_T^a < 2$ GeV/c) particle pairs, pseudorapidity-separated $|\Delta \eta| > 0.8$, in the 0-2% most central PbPb events. The averaged-square Fourier coefficients $dN^{\text{pairs}}/d\Delta \phi \propto 1 + \sum_{n=1}^{\infty} 2v_{n,n} \left(p_T^T, p_T^a\right) \cos \left(n \Delta \phi\right) = 1 + \sum_{n=1}^{\infty} 2v_n \left(p_T^T\right) v_n \left(p_T^a\right) \cos \left(n \Delta \phi\right)$ are separately indicated with solid lines up to the fifth order; if the anisotropy is driven by collective expansion, $v_{n,n} \left(p_T^T, p_T^a\right)$ should factorize into the product (square) of two single-particle harmonic coefficients $v_n \left(p_T^T\right)$ and $v_n \left(p_T^a\right)$. The sum of $v_{n,n} \left(p_T^T, p_T^a\right) = v_n \left(p_T^T\right) v_n \left(p_T^a\right)$ $n \leq 5$, shown as the dashed curve, consistently describes the revealed correlation structure (13) in section §5.

\[1\text{3}\]
References


2 Flowing Medium

In ultra-relativistic heavy-ion collision experiments, a fraction of the incoming kinetic energy is converted into new matter, whose compression and its subsequent expansion is followed by multi-particle production. One of the most spectacular experimental results is the striking azimuthal anisotropy of final state particles [1]. The results provide compelling evidence that the matter produced in these collisions behaves collectively. Generically, the definition of flow encompasses the long range space-momentum correlations attributed to the collective motion of the created system.

Typically, fluid dynamics is considered as legitimate candidate for consistently describing the system that emerges from a heavy-ion collision. At the energies available at RHIC and recently at LHC, the created transient matter behaves as a strongly coupled liquid and achieves a state of local equilibrium. The appearance of non-trivial patterns in the azimuthal distribution of hadrons could be then understood by the fluid dynamical response to hydrodynamic forces, i.e. pressure gradients (cf. section §D), which in turn are given by the anisotropic geometric shape of the initial state. The precise details of this response to initial spatial deformations seem to depend on the transport properties of the matter, as for instance the shear viscosity \( \eta \) (e.g. [2]). Overall, the prerequisite of a thermalized system remains the cornerstone of the prevailing theory that describes the collision dynamics, since the timescale of local thermal equilibrium is much smaller than any macroscopic dynamical behavior.

The efficiency in imprinting the reaction zone asymmetry, along with its fluctuating inhomogeneities (cf. section §2.2.1), to final momentum distributions increases with the coupling strength between the medium constituents and becomes maximal for an infinitely strongly coupled system. For a given initial spatial deformation of the collision zone, ideal fluid dynamics generates the largest transverse flow [3]. On the contrary, shear viscosity, whose lower limit is imposed by quantum mechanics [4], accounts for finite interaction cross sections. Although this reduces the amount of flow that can be generated from a given geometric deformation, a very small ratio of shear viscosity to entropy density \( \eta/s \) was found both at RHIC and LHC (e.g. [3] for a review). In fact, the experimental value accounts for few multiples of the lower conjectured limit of \( \eta/s \sim \frac{1}{4\pi} \).

2.1 The paradigm of a perfect-like liquid

The magnitude of the observed anisotropy in the azimuthal momentum distribution is found strongly correlated with the anisotropic shape of the initial nuclear overlap region. This correlation is typically expected in case the interactions among the constituents of the produced fireball are frequently enough to maintain a kind of local equilibrium. Subsequently, this leads to anisotropic thermal pressure, which acts against the surrounding vacuum and transforms the initial spatial asymmetry into the anisotropic momentum distribution of the outflowing matter. An illustrative example is shown in Fig. (7), where the azimuthal variation of the momentum distribution for the outflowing matter is drawn in peripheral (left) and central (right) collisions of two nuclei, the later represented by the dotted circles. The density of the arrows reflects the magnitude of flow as seen at a large distance from the collision in the corresponding azimuthal direction.

In a peripheral collision, where the initial region (shaded area) is asymmetric, flow is stronger into the direction of the highest acceleration. For an almond-shape initial geometry the highest acceleration occurs along the \( x \)-axis, or alternatively along the 2\textit{nd}-order symmetry plane \( \Psi_2 \), i.e. the plane spanned by the \( x \)-axis and the direction of the incident nuclei. The development of anisotropic pressure gradients results in more outgoing particles with greater velocity in the in-plane relative to the out-of-plane direction. Primarily, this relative
The assumption relying behind the hydrodynamical approach is that particles interact due to the small, compared to the system size, mean free path $\lambda$. Ideally, if the ratio of the mean-free path $\lambda$ relative to the system transverse size $R_A$, known as the Knudsen number \[5\], asymptotically approaches zero, instantaneous thermalization is achieved. Two extreme cases can be thought and they are depicted in Fig. (8); when $\lambda$ is much larger than the system size (left), particles move out in their original directions, interacting little or not at all. The initial anisotropy cannot be observed in the final particle distributions and the system is said to behave like a gas. For ideal hydrodynamics though, $\lambda$ is infinitively small (right), meaning that particles interact frequently, thus the initial anisotropy is conserved. The later is manifested in the final momentum anisotropy, quantified by a series of Fourier coefficients $\nu_n$; here, the second order $\nu_2$ coefficient, referred to as elliptic flow, is illustrated. Non-zero, but still sufficiently small, values for $\lambda$ account for viscous effects, extending the range of validity for the fluid description. Both ideal and viscous fluidity can be addressed by hydrodynamic calculations (cf. section §C).

\[1\] Note that there exists the implicit assumption of $\Psi_2$ lying in the x-direction, i.e. $\Psi_2 \equiv 0$, which is precisely the minor axis of the overlap ellipse (Fig. (10)). In practice, the $\Psi_2$ angle is unknown experimentally, thus only relative azimuthal angles $\Delta\phi$ can be measured, meaning that the elliptic flow effect literally results in a $\cos(2\Delta\phi)$ modulation (Fig. (6)).
Figure 8: For a large mean-free path $\lambda$ (left), far from the ideal hydrodynamical limit, the amount of flow that can be generated out of a given geometric configuration is limited. On the contrary (right), deformations of the initial spatial density distribution are most efficiently converted into momentum anisotropies of the hydrodynamic flow when $\lambda$ is infinitively small.

Relativistic hydrodynamics has been proven the most relevant framework to account for the bulk and transport properties of the fluid QGP, since it succeeds to directly connect the collective flow during the hot and dense state with its EOS, the later extracted from e.g. lattice QCD calculations. Quantitatively trustworthy results require a detailed understanding of the pre-thermal evolution of the fireball and its matching to the (viscous) hydrodynamic stage. In addition to the initialization of the fluid dynamics, hydrodynamic calculations require a freeze-out condition, where eventually collisions among the medium constituents will be so infrequent to maintain local equilibrium; the fireball eventually becomes too dilute and equilibrium breaks apart. The main stages that a heavy-ion collision is thought to pass through are summarized in the following section.

2.2 Modeling the dynamics of the hot droplet

As previously highlighted, the study of strongly coupled matter with hydrodynamics requires a set of initial conditions. Hydrodynamic evolution then converts this spatial asymmetry into an asymmetry of the final particle distribution, quantified by the Fourier coefficients $\nu_n$. Adequate for the description of relativistic heavy-ion collisions are the proper time $\tau = \sqrt{t^2 - z^2}$ and rapidity $y = \ln(\sqrt{(t+z)/(t-z)})$, which along with the transverse coordinates $x_\perp = (x, y) = (r \cos \phi, r \sin \phi)$, $r = (x^2 + y^2)^{1/2}$ and $\phi = \arctan(y/x)$, constitute the $(3 + 1)$-D continuum $(x, y, y, \tau)$ for solving the equations of motion in the pure fluid approach.

By definition, a small value for $y$ is associated with a small value of $z$ for a given proper time. Hence, the mostly studied central rapidity region, customarily called mid-rapidity, reflects the central spatial region around $z \sim 0$, where collisions have taken place. Typically, longitudinal boost invariance is assumed, meaning that initial conditions are calculated at mid-rapidity, while possible correlation effects in the forward/backward rapidity regions (e.g. [6]) are neglected. Last but not least, it should not be neglected that the initial time $\tau_0$ to the hydrodynamic description suffers from an ab initio treatment. For eliminating such an arbitrariness in the choice of the thermalization time, refinements have been attempted (e.g. [7])\footnote{The impact parameter dependent saturation (IP-model) model is considered as a promising candidate for an improved matching to the hydrodynamical description. The IP-model is a successor of previous formulations that accounted for the observation of the rapid rise in the $\gamma^* p$ cross section with increasing $\sqrt{s}$ in the deep inelastic...}, whereas systematic studies on $\tau_0$ have been launched by adjusting post facto the outcome
of the initial conditions to measured hadron spectra and multiplicities (e.g. [8]).

The proper time \( \tau \) separates the stages of the space-time evolution, which is schematically given in Fig. (9). After the state of local thermodynamic equilibrium has been quickly reached at some time \( \tau_0 \), which usually range between 0.5 and 2 \( fm/c \) [7], the system undergoes an hydrodynamic expansion due to the large pressure gradients in the medium. Interactions in the dense nuclear medium, the latter expected to exist both in the QGP and a late (\( \tau > \tau_c \)) color confinement (hadronic) phase, maintain equilibrium up to the point (\( \tau \sim \tau_f \)) where the expansion overwhelms the hadron-hadron correlations. The later implies that by the time particles have traveled a distance of one mean free path owing to their thermal motion, they will have collectively receded from each other by more than one mean free path.

\[
\frac{\varepsilon_2}{\varepsilon_3} \propto \frac{Q_s}{x} \\
\]

During simulations of the initial state, each moment \( \varepsilon_n \), is evaluated event-by-event relative to the direction of the largest density gradient, which on average points in the direction of the largest hydrodynamic acceleration. This is indicated by the blue arrows in Fig. (10); for the elliptically deformed profile, the participant plane \( \Phi_2 \) coincides with the minor axis of the scattering (DIS) region, i.e. \( x \equiv \frac{Q_s^2}{x} < 1 \), at the HERA experiments. This observation indicates that over a region of size \( \sim 1/Q_s^2 \) additional gluons are abundantly radiated at small \( x \), which was explained by the impact-parameter dependence of the saturation scale \( Q_s \). Note that in the realm of saturation models, the impact parameter refers to the transverse distance of the formed \( \gamma^* \rightarrow q\bar{q} \) pair relative to the proton.
ellipse, whereas $\Phi_3$ points to the sides for the triangular profile $[9]$. The driving force of non-zero eccentricities $\varepsilon_n$ and thus of the final state $\nu_n$ coefficients seems to be not only the average deformation (shape) of the initial distribution, but also the fluctuations in each event, e.g. the positions of the nucleons that participate in the collision. In other words, hydrodynamic collective flow should not be considered as a property of the event ensemble, but rather develops independently in each collision event.

Figure 10: Quantum fluctuations in the wave functions of colliding nuclei result in lumpiness of the initial stage of these collisions, viz. the presence of small-scale structures. There are two sources of such quantum fluctuations: fluctuations of positions of nucleons within the nucleus, and fluctuations at the subnucleonic level. A lumpy initial density profile, instead of a smooth and symmetric density distribution, controls the initial geometry on an event-by-event basis. Subsequently, the system response to the average geometry and the initial-state fluctuations is determined by the $n^{th}$-order spatial anisotropy $\varepsilon_n$ along with the corresponding symmetry planes $\Phi_n$. Directions and magnitudes of density gradients are indicated by the blue arrows. Accounting for the medium response to a initial geometry with no particular symmetry, harmonics of all possible orders are expected.

2.2.2 Relativistic Hydrodynamics in heavy-ion collisions

In principle, the idea that hydrodynamics could describe the outcome of hadronic collisions has a long history $[10]$. However, with the advent of heavy-ion experiments at RHIC, the interest in relativistic hydrodynamics revived, owing to the model simplicity. For the hydrodynamical description of the QGP, the complete dynamics of the system is compactly derived by the local energy-momentum $T^{\mu\nu}$ and current $j^\mu_i$ conservation (e.g. $[11]$),

$$\partial_\mu T^{\mu\nu} = 0 ,$$

$$\partial_\mu j^\mu_i = 0 , \mu, \nu = 0, 1, 2 \text{ and } 3 ,$$

with $\mu, \nu = 0$ corresponding to the time-like component in the $(3 + 1)$-D continuum $(x, y, y, \tau)$ and $i = 1 \ldots N$ measuring currents of conserved charges (cf. section §C). A typical example of conserved current is the electric charge or the (net) baryon number. Note that there are $4 + N$ equations, but $10 + 4N$ independent unknown variables. $T^{\mu\nu}$ gives 10 independent components as a rank 2 tensor, while each out of $N$ conserved current $j^\mu_i$ introduces 4 degrees of freedom.

Therefore, the additional assumption to close the set of equations is provided by the EOS of the matter, most commonly taken in the form of connecting the system pressure $p_0$ to the energy density and number densities $n_i$ of charges. It is important to note here that the explicit form of the equation of state is completely unrestricted, hence anomalies like phase transitions are not forbidden. In principle, the EOS may be taken in the most sophisticated form as delivered by lattice QCD calculations (e.g. $[12]$), meaning that hydrodynamic calculations form a link between the QCD first-principle calculations and the dynamic properties of the expanding fireball.

For a state that reaches an approximate equilibrium to be described, the space of thermodynamic variables has to be extended relative to the case of ideal hydrodynamics. Among
the introduced dissipative coefficients, the shear viscosity appears to be the most relevant in heavy-ion collisions. Much interestingly, the analysis of elliptic flow alone is not sufficient to constrain models both for the initial state and of the QGP shear viscosity over entropy density ratio $\eta/s$. Event-by-event hydrodynamic simulations highlighted the increasing sensitivity of $v_n$ on $\eta/s$ with increasing $n$ (e.g. [13]).

An indicative example is given in Fig. 11. The $p_T$-differential $v_n$ of order 2 to 5, and in particular the RMS $\sqrt{\langle v_n^2 \rangle}$ of the flow distribution, is plotted as calculated within the realm of $(3 + 1)$-D event-by-event viscous hydrodynamics separately for $\eta/s = 0.08$ (upper left) and $\eta/s = 0.16$ (upper right). Experimental data at $\sqrt{s_{NN}} = 200$ GeV (markers) have been compiled from the Pioneering High Energy Nuclear Interaction eXperiment (PHENIX) at RHIC. The studied flow harmonics were found to depend increasingly strongly on the value of specific viscosity. To make this point more quantitative, authors in [13] plotted the ratio of the $p_T$-integrated $v_n$ from viscous as compared to ideal calculations as a function of the order of the harmonic $n$ (bottom). While $v_2$ is suppressed by $\sim 20\%$ when having used $\eta/s = 0.16$, $v_5$ is suppressed by as much as $\sim 80\%$. The Monte Carlo Glauber (MCG) model (cf. section §B.2) has been used to determine the initial conditions, a chemically equilibrated EOS has been employed, whereas the kinetic freeze out attained via the Cooper-Frye freeze-out emission of constant surface temperature, including viscous corrections to the distribution function (cf. section §E).

![Figure 11](image)

Figure 11: The utilization of higher $n > 2$ flow harmonics is thought a much more sensitive probe, since diffusive processes smear out finer structures corresponding to higher $n$ (e.g. [14]).

Since the expansion of matter triggers the rapid growth of the mean free path, a transition from a collision dominated (strongly coupled) system to a collision free (weakly coupled) one is expected. Therefore, proper model calculations should determine the point (temperature) at

---

3This can be intuitively understood as in the following. Due to the boost-invariance along the beam direction (Fig. 9), the initial anisotropic expansion occurs longitudinally than transversely to the beam direction. Shear viscosity tends to equalize the expansion rates along different directions, by building up a shear viscous pressure tensor (in the fluid local rest frame) that reduces the longitudinal and increases the transverse pressure.

4For easiness, viscosity is scaled with the entropy density $s$, resulting in the dimensionless specific viscosity.
which such a decoupling stage occurs, along with the strength of the transverse flow.

2.2.3 Freeze-out Conditions

The surface of the last scattering is usually referred to as the freeze-out surface. Since scattering could be both inelastic, where particle identities change, and elastic, where particle identities remain unaltered, it is possible to have two distinct freeze-out stages, namely, chemical and kinetic (thermal) decoupling. In general, freeze out could be a complicated process involving duration in time and a hierarchy where different types of particles decouple at different times. In addition, reactions with lower cross sections switch-off at higher temperature, \textit{viz.} earlier in time, compared to reactions with higher cross sections. Therefore, the chemical freeze out is expected to occur earlier in time compared to the kinetic freeze out. Interestingly, the separation between chemical and kinetic freeze-out temperatures seems to increase towards higher energies, indicating increasing hadronic interactions between the two freeze-out stages at higher energies (\textit{e.g.} [15]).

In practice, there exist a series of freeze-out models with hydrodynamically inspired parameterizations, whose assumptions are cross-checked by simultaneous fit to hadron spectra (\textit{e.g.} [16]). The transition between fluid mechanics to the momentum distribution of outgoing particles is typically performed on the basis of the Cooper-Frye freeze-out ansatz; a sudden freeze-out approximation that encompasses the two dynamical extremes, \textit{i.e.} the strongly and weakly coupled regime [17]. More specifically, it is assumed that the fluid towards the end of the hydrodynamic expansion behaves like an ideal gas (ensemble of independent particles), thus the momentum distribution of hadrons is essentially taken as the momentum distribution of particles within the fluid (cf. section §E).
References


3 ALICE performance

3.1 A dedicated Heavy-Ion Program at LHC

ALICE (A Large Ion Collider Experiment) is one out of the four major detectors at LHC designed to study heavy-ion collisions (Fig. (50)). It is located at the second LHC interaction point (IP2) [1]. ALICE makes use of a right-handed Cartesian system. The beam direction defines the $z$-axis, with the ends of the detector being labeled A and C for the positive and negative $z$ direction, respectively. The $x$-axis is horizontal and points towards the center of the LHC apparatus, while the $y$-axis is vertical and points upwards. The main part of the detector fully covers the central region from $45^\circ$ to $135^\circ$ ($|\eta| \lesssim 0.8$) and is almost azimuthally symmetric.

As a specialized heavy-ion experiment, ALICE has first to cope with the extreme conditions associated with the large density of primary charged particles, measured as high as $dN_{ch}/d\eta \approx 1600$ in central collisions in mid-rapidity $|y| \lesssim 0.5$ [2]. In addition, the hadronic spectra should be measured in detail (e.g. [3]), thus it is essential that excellent spatial resolution and particle identification (PID) abilities are maintained. Finally, good momentum resolution is required over a wide range of momenta, since the relevant experimental probes are not limited in a specific $p_T$ region (e.g. [4]).

The majority of detectors are located in the central barrel (Fig. (50)), embedded within the large L3 magnet [5] which develops a magnetic field of $0.5 \, T$ and allows momentum measurement through the curvature of the particle trajectory. Excellent vertex reconstruction and energy-loss measurements are performed by the Inner Tracking System (ITS, cf. section §3.2.2) and the Time Projection Chamber (TPC, cf. section §3.2.1) detectors. Further particle identification is offered by time-of-flight measurements recorded in the Time Of Flight (TOF) detector [6]. The lead-scintillator Electromagnetic Calorimeter (EMCAL) [7] along with the lead-tungsten crystal Photon Spectrometer (PHOS) [8] and the Transition Radiation Detector (TRD) [9] can discriminate between electrons and photons. The ring imaging Cherenkov module HMPID (High Momentum Particle IDentification, [10]), designed to enhance the PID capability of ALICE beyond the momentum range allowed by the time-of-flight measurements, adds to the central PID detectors. Muons are detected in the muon arm located at large negative pseudorapidity $-4 < \eta < -2.5$ [11].

Multiplicity counters, the Forward Multiplicity Detectors (FMD) [12], T0 [13] and VZERO (cf. section §3.3.1) complete the overall event characterization (e.g. centrality determination), simultaneously providing with triggering to identify events of interest. Finally, the quartz fiber sampling Zero Degree Calorimeters (ZDC) [14], two sets of neutron (ZNA and ZNC) and proton (ZPA and ZPC) calorimeters located at distances ($z \approx \pm 116 \, m$) where spectator protons and neutrons are spatially separated by the LHC magnetic field, assist with centrality determination by recording the energy of such noninteracting nucleons.

In the following, TPC, ITS and VZERO detectors (Fig. (12)) are discussed in greater detail owing to their relative importance during the current analysis.
3.2 Central barrel tracking

In ALICE, the full event reconstruction could be partitioned to four generic steps [15]:

- **Primarily**, the preliminary position of the primary vertex(-ices) is(are) determined using the clusters in the silicon pixel detectors, which constitute the two innermost layers of the ITS.
- **Subsequently**, the influence of the magnetic field and interactions with the detector material are taken into account during the tracking reconstruction steps (cf. section §F.2).
- **In addition**, by referring to the combined TPC and ITC tracking information, the vertex parameters are improved.
- **Finally**, determining secondary vertices, namely searching for photon conversions and decays of (multi-)strange hadrons, concludes the procedure.

Given that the outer diameter of the TPC is less populated as compared to parts more adjacent to the primary vertex, track seeds are built within the outer TPC regions characterized by lower densities, and are prolonged towards smaller TPC radii. Subsequently, the ITS
tracker tries to extrapolate the TPC tracks as close as possible to the primary vertex, while additional reconstructed ITS clusters are assigned, improving the track parameters (Fig. 13, left panel).

When the ITS tracking is completed, i.e. all the track candidates from the TPC have been assigned to clusters in the ITS, the tracking restarts and follows the track from the inner ITS layers outwards, now. Once the outer radius of the TPC is reached, tracks are simply extrapolated to the TRD, TOF, HMPID and PHOS detectors, since the precision of the estimated track parameters is sufficient (Fig. 13, middle panel). In other words, detectors at a radius larger than that of the TPC have not been used to update the measured track kinematics (track parameters and their covariance matrices), albeit their information is stored for PID purposes. Finally, the tracking process is reserved for one last time and all tracks are refitted from the outside inwards, in order to obtain the values of the track parameters at or nearby the primary vertex (Fig. 13, right panel).

3.2.1 Time Projection Chamber

Figure 14: The ALICE cylindrical TPC is divided by the central high-voltage electrode into two drift regions of 250 cm, while 2 × 18 MWPC are mounted into 18 trapezoidal sectors in each end-plate.

The Time Projection Chamber (cf. section §F.3 [16]) has a cylindrical design that extends longitudinally over −2.5 < z < 0 and 0 < z < 2.5 m, while transversally over 0.85 < r < 1.3 and 1.3 < r < 2.5 m (Fig. 14). The tracking efficiency, namely the ratio between the reconstructed tracks and the generated primary particles in simulation mode, including strong resonance decays, is depicted in Fig. 15. The drop below transverse momentum of $O(0.5)$ GeV/c is caused by the substantial energy-loss in the detector material, whereas the characteristic shape at larger $p_T$ is determined by the loss of reconstructed clusters owing to the $p_T$-dependent fraction of the track trajectory projected on the dead zone between readout sectors, in which case no cluster is expected. However, the efficiency is almost independent of the occupancy in the detector, indicated by different centrality classes (filled circle against open rectangular) and collisional systems (markers against the line).
Figure 15: The ALICE TPC track-finding efficiency for primary particles in both pp and PbPb collisions. The efficiency does not depend on the detector occupancy [15].

Having identified tracks, the $p_T$ of a particle, given by the curvature of the trajectory in the 0.5 $T$ magnetic field, is determined with a finite resolution. Using information recorded by TPC alone, ALICE reaches a momentum resolution of $\sim 0.8\%$ for momenta around 1 GeV/c and of $\sim 0.2\ (6.5\%)$ for intermediate(higher) momenta. This can be further improved by incorporating information from the SPD tracking system (Fig. (16)).

Figure 16: The inverse $\sigma_{1/p_T} \left((GeV/c)^{-1}\right)$ resolution for TPC alone and ITS-TPC matched tracks with the track seeding successively tried with and without constraint to the preliminary vertex. The vertex constrain significantly improves the resolution of TPC standalone tracks, whereas no effect is found for ITS-TPC tracks. The relative $\sigma_{p_T}/p_T$ is trivially obtained via $\sigma_{p_T}/p_T = p_T \cdot \sigma_{1/p_T}$. Although ITS-TPC tracks provide with the best estimate of track parameters, track reconstruction suffers from gaps in the ITS acceptance. In particular, in the innermost two SPD layers, up to 20% of the modules were inactive during 2010 [15].

Although the TPC is the main tracking detector in ALICE, it additionally provides information for particle identification over a wide momentum range (Fig. (17)). The ALICE TPC demonstrates a clear separation between the different particle species; at low-momenta $p \lesssim 1$ GeV/c particles can be identified on a track-by-track basis, while higher-momenta particles can still be separated on a statistical basis via a sum of four Gaussians, whose means and widths have been constrained from the $\langle dE/dx \rangle (\beta\gamma)$ and $\sigma (\langle dE/dx \rangle)$ parameterizations separately for $\pi^\pm$, $K^\pm$, $p$ ($\bar{p}$) and $e^\pm$ [17]. ALICE achieves an overall TPC $\langle dE/dx \rangle$ resolution of around 6.5% with the highest-$p_T \sim 20$ GeV/c resolved regime to be limited by statistical precision, at
the moment.

Figure 17: (left) Distributions of the measured energy-loss signal \( \langle dE/dx \rangle \) in the TPC as a function of momentum \( p \). The lines show the Allison-Cobb (ALEPH) parametrization of the expected mean energy loss (Eq. (F.3)). Unique identification on a track-by-track basis is possible in the low-\( p_T \) region, where the different bands are clearly separated from each other [15]. (right) In the relativistic rise, particle identification is still possible based on statistical unfolding using a multi-Gaussian fit. Here, the ionization energy-loss distributions relative to parameterized pion energy-loss \( \langle dE/dx \rangle_\pi \) in the TPC for the \( 8 < p_T < 9 \) GeV/c interval is depicted [17].

### 3.2.2 Inner Tracking System

In ALICE, close to the interaction point, six cylindrical layers of silicon semiconductor detectors are used, grouped per two in distinct sub-detectors [24]. For collisions not further than \( \sim 10 \) cm to the nominal point, ITS layers offer full tracking capability for particles with \( |\eta| < 0.9 \), while the coverage in azimuth by design should be found uniform in \( 2\pi \).

Starting with the two innermost layers, situated at \( r = 4 \) and \( 7 \) cm where the overall particle multiplicity is the highest, the Silicon Pixel Detector (SPD) is deployed. The SPD provides with complementary primary vertexing information, whereas it detects secondary vertices, crucial for discriminating between primary and secondary particles. Notice that the first layer of the SPD, together with the FMD, gives a continuous coverage in \( |\eta| < 2 \) region to aid multiplicity measurements. In addition, it has a fast read-out allowing to act as a trigger for events of interest. Pixels on SPD measure \( 50 \times 424 \) \( \mu \)m, offering excellent spatial resolution to distinguish between neighboring tracks. However, even with such resolution, in the dense LHC environment there exist instances of conflict between two prolongation candidates in the involved tree (e.g. Fig. (51)). In other words, shared clusters are found, thus the tracking algorithm cannot unambiguously resolve to which candidate the registered deposition of energy corresponds.
The next two layers are Silicon Drift Detectors (SDD), located at $r = 15$ and 24 cm. The particle density has dropped about a factor 10 relative to the SPD proximity, thus larger drift detectors with active area $70 \times 75 \text{ mm}^2$ seem sufficient. They offer particle resolution of $35 \mu \text{m}$ in the $r\phi$ direction, and $25 \mu \text{m}$ longitudinally. Finally, the remaining 2 outermost layers, at $r = 39$ and 44 cm respectively, are Silicon Strip Detectors (SSD) with a $r\phi$ resolution of around $20 \mu \text{m}$, for precision matching of tracks between the ITS and TPC. Since the reconstruction efficiency in the TPC sharply drops at low transverse momentum (Fig. (15)), the four outer layers of the ITS have an analog readout to measure the deposited charge, thereby providing with $\langle dE/dx \rangle$ measurements (ITS standalone tracking).

![Figure 19](image)

Figure 19: (left) ITS-TPC matching efficiency as a function of track $p_T$ for real data (solid) and Monte Carlo (open) in PbPb collisions with two different requirements of ITS layer contributions. (right) Distribution of the measured truncated mean energy loss values $\langle dE/dx \rangle$ as a function of momentum $p$ in PbPb collisions at $\sqrt{s_{NN}} = 2.76 \text{ TeV}$; both $\langle dE/dx \rangle$ and $p$ were measured by the ITS alone [15]. The lines show the parametrization according to the PHOBOS parametrization of the Bethe-Bloch curves convoluted with a polynomial [25].

### 3.3 Forward Region

#### 3.3.1 V-ZERO scintillation arrays

The VZERO system (Fig. (20)) is a pair of plastic scintillator arrays, VZERO-A and VZERO-C, which are placed asymmetrically ($2.8 < \eta_A < 5.1$ and $-3.7 < \eta_C < -1.7$) relative to the nominal collision point $z = 0$. The operational functionality of the ALICE VZERO system [18] extends from the necessary requirements for the lowest level triggers [19] and rejection of beam-induced background [20] to luminosity monitoring [21] and characterization of global event properties, e.g. collision centrality (cf. section §F.4.1). More specifically, the VZERO information arises out of the analog to digital converter (ADC) counts that the $2 \cdot (4 \times 8) = 64$ PMT channels register. A weighted time average results in the individual channel time resolution of the $O(1) \text{ ns}$ order for both arrays, independent of the colliding system. Parenthetically, during 2011 the VZERO system delivered, apart from minimum-bias (MB) triggering (cf. section §4.1), central and semi-central trigger signals, selecting up to the 10% and 50% most central collisions (e.g. [22]), respectively.
Figure 20: Sketches of VZERO-A (left, $2.8 < \eta_A < 5.1$) and VZERO-C (right, $-3.7 < \eta_C < -1.7$) scintillation arrays showing their $r\phi$-segmentation. Radii of 0-3 rings are scaled. Fictitious dashed lines simply emphasize the different technical designs, related to the integration constraints of each array [18].

The monotonic relation between the total collected charge and the number of primary charged particles emitted over an ensemble of collisions is exploited by the VZERO system. However, the response of each scintillator fluctuates on a run-by-run basis, meaning that the same ADC count in two different PMT channels does not necessarily correspond to the same number of charged-particle hits. Therefore, a run-dependent flattening or equalization of the ADC counts is necessary so as to ADC values to be comparable across the $32 + 32 = 64$ channels. An indicative example of such an ADC response equalization for VZERO is given in Fig. (20), adapted from reference [1] (Appendix). It is obvious that ADC values are unequalized across VZERO channels (upper left), so that equalization factors $w^{A(C)}_{ADC}$

$$w^{A(C)}_{ADC} = \frac{Counts^{A(C)}_{ADC}}{Counts^{A+C}_{ADC}}$$

are separately applied for each channel (upper right). In that case, the mean ADC value, i.e. $\overline{Counts^{A+C}_{ADC}}$, was calculated having considered all VZERO channels (64). The resulted equalized ADC values for each channel, i.e. after the ADC value of each channel has been divided by the corresponding equalization factor for that channel (Eq. (4)), are shown in the bottom panel.

Note that during the current analysis $w^{A(C)}_{ADC}$ have been accounted for each centrality class, whereas the mean ADC value has been separately calculated for VZERO-A and VZERO-C. The raw ADC counts have been scaled on a run-by-run basis and utilized for the current flow-related study. In practice, an extra gain equalization procedure should be followed, if multiple LHC periods were to be included; Eq. (4) should be further averaged over all the runs recorded in a given LHC period [23].
Figure 21: Illustrative example ([1], Appendix) of the ADC response equalization. (upper left) Un-equalized ADC values for each VZERO channel in a single run during 2010. (upper right) Equalization factors $w_{ADC}^{A(C)}$ for each channel in the VZERO for the same run. (bottom) Resulted equalized ADC values for each channel in the VZERO. The color-scale is given in arbitrary units. Note that during the current analysis $w_{ADC}^{A(C)}$ have been separately calculated for each centrality class and separately for VZERO-A and VZERO-C.
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[22] Xiaoming Zhang, on behalf of the ALICE Collaboration. Nuclear modification factor and elliptic flow of muons from heavy-flavour decays in Pb-Pb collisions at \( \sqrt{s_{NN}} = 2.76 \) TeV. J. Phys.: Conf. Ser. 509, 012045 (2014)

[23] P. Christakoglou. Charge dependent correlations in pp collisions at \( \sqrt{s_{NN}} = 7 \) TeV. ALICE Internal Note


Table 1: Outline of the selected data and the respective software packages used during the current study. Only Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ ATeV registered by the ALICE detector during the end of 2010 (2010h) are considered. The number of analyzed events having remained after the applied prerequisites, described in the text, amounts to $11 \cdot 10^6$.

<table>
<thead>
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<th>Type</th>
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<th>AliRoot</th>
<th>Number of Events</th>
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<td>v5-34-08-6</td>
<td>vAN-2014X</td>
<td>$11 \cdot 10^6$</td>
</tr>
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</table>

4 Analysis Techniques

4.1 Event Characterization

The current analysis makes use of Pb-Pb data which have been recorded by ALICE during November 2010 at a nucleon-nucleon center of mass energy $\sqrt{s_{NN}} = 2.76$ ATeV \[1\]. The main detectors used at hardware level (on-line) in ALICE for trigger requirements are the two forward VZERO scintillators and the two innermost layers of the ITS \[2\]. The events used for the current analysis have been filtered off-line during the Analysis Object Data (AOD) type creation with the Physics Selection framework developed within AliRoot.

More specifically, the off-line event selection replays the on-line trigger condition and is applied for attaining high efficiency in hadronic events, i.e. Minimum Bias (MB) interactions. Interactions arising out of beam with residual gas collisions in the accelerator ring are promptly discarded after the weighted time average over all channels is used for VZERO (cf. section \[3.3.1\]). Such a discrimination from machine-induced background is feasible on the grounds of the different arrival time for particles having been emitted from beam-beam and beam-gas interactions, respectively. Analogously, the SPD detector provided a prompt trigger signal, which is activated when at least one out of 1200 pixels was hit by a particle. In conclusion, the MB interaction trigger, used in this analysis, required at least two out of the following three conditions:

- two pixel chips hit in the outer layer of the SPD
- a signal in VZERO-A
- a signal in VZERO-C

Moreover, the primary vertex was required to be reconstructed such that the $z$-coordinate satisfied the condition $|z_{vtx}| < 10$ cm to ensure a uniform acceptance in the central pseudorapidity region $|\eta| < 0.8$. By using the $z_{vtx}$-cut, the machine-induced background due to parasitic collisions from debunched ions is also eliminated \[3\]. An additional requirement is that the difference in the $z$-coordinate of the separately reconstructed primary vertex by the SPD and the TPC detector has to be less than 5 $\text{mm}$.

The default centrality determination in ALICE is obtained from the measured multiplicity in the VZERO detectors, mapping the summed scintillation amplitudes onto percentages of the total geometric cross section through a Monte-Carlo Glauber model (cf. section \[8F.4\]). In addition, during the present analysis, centrality estimation extracted based on the multiplicity

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1The efficiency of the MB$_{OR}$ interaction trigger, the former defined as the ratio of events selected by a given condition to all simulated events, found 99.5% (19\) in section 4.

2The radio-frequency of the LHC (400 MHz) is such that there are 10 equidistant RF buckets within the 25 $\text{ns}$ time interval between two possible nominal bunch positions. However, ions can migrate into one of the neighboring buckets, meaning that collisions occur either among ions in the nominal buckets or between one or two ions displaced by one or more buckets, the later named as satellite bunches (e.g. \[3\]). This causes a displacement in the $z_{vtx}$ of $2.5 \text{ns}/2c \approx 40$ cm, well outside the fiducial region of $|z_{vtx}| < 10$ cm.
measured by the TPC detector was required not to fall beyond 5% relative to the centrality determined by the VZERO method. The analyzed data correspond up to the 60% most central collisions, while peripheral events are excluded owing to the sensitivity of collective flow to non flow effects with decreasing centrality (e.g. \cite{15}).

4.2 Reconstructing neutral particles

Figure 22: Principle of secondary vertex reconstruction for transverse radius $\text{Rad}_{XY} \geq 5\ cm$, based exclusively on the decay topology of $\Lambda$ ($\bar{\Lambda}$) candidates. The selection of decaying products proceeds considering I) the distance of their closest approach (DCA$_1$ and DCA$_2$) to the primary vertex $V_{\text{prim}}$, II) their apartness at the point of closest approach (PCA), III) the cosine of the pointing angle $\theta_p$, namely the angle between the reconstructed momentum $\vec{p}_{\text{pair}}$ and the line that connects the secondary and the primary vertex, while IV) the $V^0$ transverse decay length $\text{Rad}_{XY}$ is also regarded. For clarity, the decay points were placed between the first two ITS layers, ITS$_0$ and ITS$_1$ (radii are not scaled). The solid lines represent the reconstructed charged particle tracks, extrapolated to the secondary vertex candidates. Extrapolations to the primary vertex and auxiliary vectors are shown with dashed lines.

The identification of $\Lambda$ ($\bar{\Lambda}$) hyperons is based on the reconstruction of secondary vertices (Fig. 22). These particles are referred to as $V^0$s due to their distinctive decay topology. Since the initial particle is neutral, the decay shows up (Fig. 25) as a $V$-shaped vertex separated from the primary collision point, in tracking detectors embedded within magnetic field. During the current study, the analyzed $\Lambda$ ($\bar{\Lambda}$) candidates have been identified by taking advantage of their predominant weak decay to $p\pi^-$ ($\bar{p}\pi^+$) with branching ratio of 63.9% \cite{4}.

Assuming the nature of the decay remnants, henceforth the daughter tracks, and calculating the $\Lambda$ ($\bar{\Lambda}$) invariant mass, with the $V^0$ momentum $\vec{p}_{\text{pair}}$ as the sum of the daughter tracks momenta, a peak exactly at the $V^0$ mass, i.e. $1.116\ GeV/c^2$ \cite{4}, Fig. 22), is revealed. The latter constitutes a smoking gun for the presence of signal, while background can be found both over the two sideband regions and, though invisibly, underneath the peak. At this point, it should be noted that the majority of the background is rooted to combinatorial origin, namely randomly primary pions and/or protons which happen to pass close to each other or even cross
over. Suppressing the level of background proceeds by adopting the *hard cut* approach performed in [5], where either the signal over background ratio or the signal significance has been optimized. Alternatively, *multivariate* techniques could offer the advantage to consider simultaneously multiple properties of the system under study, exploiting at most the available information through *machine learning* techniques (e.g. [6]).

### 4.2.1 $\Lambda$ ($\bar{\Lambda}$) candidate selection

More specifically, $\Lambda$ ($\bar{\Lambda}$) were reconstructed within the $|y| < 0.5$ rapidity window, in order to suppress effects coming from the lower reconstruction efficiency close to the edges of the detector acceptance. Moreover, the cosine of the pointing angle (in the transverse plane) between the $\Lambda$ ($\bar{\Lambda}$) candidate momentum vector and the vector defined by the the interaction point and the secondary vertex was selected to be at least $\cos\theta_p > 0.998$, since the ideal value of $\cos\theta_p = 1$ would indicate a $V^0$ pointing directly back to the primary vertex. The search for secondary vertices was deployed at least $\text{Rad}_{XY} \geq 5\ cm$ away from the collision point in the transverse plane, for not being hampered by the high track occupancy that characterizes the first ITS layer. Finally, having determined the particle transverse decay length $\text{Rad}_{XY}$ (as measured in the laboratory), the background behavior can be further distinguished by referring to the $\Lambda$ ($\bar{\Lambda}$) rest-frame lifetime, i.e. $c\tau \sim m_{0,\Lambda(\bar{\Lambda})} \cdot \text{Rad}_{XY}/p_T$; accepting only $\Lambda$ ($\bar{\Lambda}$) candidates with lifetime $c\tau \leq 3 \cdot c\tau_0 = 3 \cdot 7.89 \ cm$ [4], the background contribution, now arising out of secondaries and interactions with the detector material, was strongly reduced.

### 4.2.2 Daughter track selection

For the reconstruction of all charged particles in the ALICE detector, the ITS and TPC detectors are used as the main tracking devices. A schematic idea about the reconstruction efficiency and the momentum resolution for tracks both for the combined ITS-TPC and the TPC *standalone* procedure could be found in section §3.2.1. Pertaining to the actual study, tracks having been reconstructed *only* by the TPC are used. They are required to have at least 70 space points out of the maximum 159 in the TPC, since a large number of clusters signifies more precise tracking information (e.g. finer momentum resolution, cf. section §2.3). For the TPC reconstruction quality, the overall $\left< \chi^2 \right>$ (per TPC space point) per number of degrees of freedom –with 2 degrees of freedom per cluster– was selected less than 2; a low $\left< \chi^2 \right>$ means a better tracking fit, hence the probability of having tracks with wrongly associated clusters (Fig. (51)) is suppressed. Furthermore, the option for the track parameters being updated by the ITS information, during the last stage of the three-step tracking reconstruction process (Fig. (13)), was disabled (ITSrefit = false). Such a choice is motivated on the grounds of the innermost possible radius for the Kalman filter (cf. section §5.2) expected in the case of secondary tracks.

Respectively to their originators, the decay products, protons, pions and their $CP$ conjugates, were required to lie within the tracking acceptance range of $|\eta| < 0.8$, while having a minimum $p_T$ of 0.6 $GeV/c$. On one hand, the lower boundary in the transverse momentum is imposed due to the tracking capabilities of the ALICE detector, which significantly deteriorate approximately below 100 $MeV/c$ (Fig. (15)), mostly because such low-$p_T$ tracks do not succeed to enter the TPC volume. On the other hand, the specific requirement about the transverse decay length, combined with the lifetime cut $c\tau \leq 3 \cdot c\tau_0$, equates to removing all $\Lambda$ ($\bar{\Lambda}$) candidates, which had decayed before one lifetime has passed at $p_T \approx 0.6\ GeV/c$. The latter removed the majority of low-$p_T$ $\Lambda$ ($\bar{\Lambda}$), hence rendering quite challenging, if not impossible, their reconstruction below $p_T \approx 0.6\ GeV/c$ simply following the regular *hard cut* approach.

An additional preliminary requirement was placed pertaining to the distance of the two
tracks forming the candidate at the point of their closest approach (PCA \(<\ 1\ cm\));\ the\ smaller
the\ PCA,\ the\ greater\ the\ probability\ of\ daughter\ tracks\ having\ emerged\ out\ of\ a\ common
mother\ particle.\ Excluding\ tracks\ which\ could\ have\ originated\ from\ the\ primary\ vertex\ is
achieved\ by\ requiring\ the\ distance\ of\ the\ closest\ approach\ between\ each\ of\ the\ daughter\ track
and\ the\ primary\ vertex\ (DCA)\ to\ be\ greater\ than\ 0.1\ cm.\ Finally,\ since\ at\ low-\pT\\ to\ dis-
tinguish\ \Lambda (\bar{\Lambda})\ from\ the\ combinatorial\ background\ is\ particularly\ difficult,\ the\ TPC \langle dE/dx \rangle\ in-
formation\ is\ applied\ to\ identify\ \p (\bar{\p})\ from\ the\ \Lambda (\bar{\Lambda})\ weak\ decay;\ \p (\bar{\p})\ of\ the\ \Lambda (\bar{\Lambda})\ candidate
with\ \pT \lesssim 1.2\ \text{GeV/c}\ are\ required\ to\ fall\ within\ \ 3\sigma\ of\ the\ \langle dE/dx \rangle\ parameterization\ (cf.\ sec-
tion\ §3.2.1).

The\ total\ number\ of\ retained\ events\ are\ given\ in\ Table\ 1,\ whereas\ the\ kinematic\ and\ topo-
logical\ requirements\ applied\ on\ tracks,\ which\ have\ been\ used\ to\ build\ the\ \Lambda (\bar{\Lambda}) \rightarrow \p \pi^- (\bar{\p} \pi^+ )
candidates,\ are\ summarized\ in\ Table\ 2.\ In\ principle,\ it\ is\ not\ expected\ for\ the\ reconstruction
efficiency\ to\ be\ centrality\ dependent,\ whereas\ a\ strong\ \pT\-dependence\ is\ anticipated,\ with\ its
peak\ value\ \sim 30\%\[3\]\ to\ be\ found\ at\ intermediate\ transverse\ momentum\ \pT \lesssim 5\ \text{GeV/c}\ (Fig.
(23)).\ Last\ but\ not\ least,\ analysis\ in\ [5]\ revealed\ little\ difference\ between\ the\ optimal\ selec-
tion\ criteria\ for\ different\ centrality\ intervals,\ thus\ the\ very\ same\ requirements\ are\ considered
throughout\ the\ centrality\ (0-60) % classes.

<table>
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<td>(p_T &gt; 0.6\ \text{GeV/c})</td>
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</table>

Table 2: Quality criteria for the decay products and topological selections for the \Lambda(\bar{\Lambda}) candidates.

Note that the 63.9\% branching ratio of the \Lambda (\bar{\Lambda}) \rightarrow \p \pi^- (\bar{\p} \pi^+)\ decay has been already included.
4.3 Eliminating experimental bias in flow measurements

Although several experimental methods are available for flow extraction [7], the current analysis exclusively employs the Scalar Product (SP) technique [8]. Historically, the most common way to access information related to the $\nu_n$ coefficients is the event-plane (EP) measurement [10, 12]. In both techniques, the flow coefficients are determined from two-particle correlations. During the actual study, correlations are established between unit momentum vectors $\bar{u}_n = \exp\left[\ii n \phi_{\Lambda(\bar{\Lambda})}\right]$ of $\Lambda(\bar{\Lambda})$ (particles of interest, POI) in a small phase-space window, i.e. a given $p_T$ bin, and flow vectors $\bar{Q}_n = \sum_{i \in \text{RFP}} w_i \exp\left[\ii n \phi_i\right]$ of a different set of unidentified particles (reference-flow particles, RFP); $\phi_{\Lambda(\bar{\Lambda})}$ and $\phi_i$ stand for the azimuthal coordinate of the POI and the $i^{\text{th}}$-RFP, respectively. Typically, in order for statistical uncertainties being diminished, a weight $w_i$ might be given, which, for segmented detectors, could amount to the signal observed in the $i^{\text{th}}$-reference detector element (e.g. the scintillation amplitude (ADC counts) in the $i^{\text{th}}$-reference VZERO channel [7]). Conclusively, the original idea behind the EP and the SP method is that the direction of the reference-flow vector provides an estimate of the corresponding (common) symmetry plane $\Psi_n$ (cf. section §A.1).

However, event-by-event fluctuations of $\nu_n$ render the EP method ambiguous (Fig. (24)), for the exact measured quantity strongly depends on the detector acceptance, even after a resolution correction due to the finite sample of particles is applied (cf. section §A.1). Fortunately, such ambiguity can be removed by referring to the SP method, since it has been proven [9] to consistently yield the root-mean-square $\sqrt{\langle \nu_n^2 \rangle}$ of the underlying flow distribution, irrespective of the analysis details.

Therefore during the current study $\sum_{i \in \text{RFP}}$ is taken as the summation over the 32 + 32 VZERO elements (channels) and thus $\phi_i$ corresponds to the azimuthal angle of each element.
sub-event correlation is attained via the substitution \( \langle v_n^2 \rangle \) since flow is a genuinely long-range effect. For the case of non-identical sub-events, the three RFP samples; contributions from short-range, i.e. almost one unit of pseudorapidity separation |\( \eta \rangle \)

\[ \langle u_n \cdot \frac{\bar{Q}^A_n}{M_A} \rangle = \langle u_n \cdot e^{-i\psi_n} \rangle \langle \frac{\bar{Q}^A_n}{M_A} e^{-i\psi_n} \rangle^* = \langle u_n \cdot \bar{Q}^A_n \rangle = \langle v_n v_n^A M_A \rangle \quad (5) \]

is constructed\(^5\) where \( v_n^A \) denotes the value of the underlying flow distribution \( v_n \) as measured in a reference detector \( A \), e.g. VZERO-A. In order to estimate \( v_n \), the correlation \( \langle \bar{Q}^A_n \cdot \bar{Q}^C_n \rangle = \langle v_n^2 A(C) M_A M_C \rangle = \langle v_n^2 A(A) M_C \rangle^* \) of \( \bar{Q}^A_n \) with at least one additionally defined \( \bar{Q}^C_n \), e.g. as given by VZERO-C, should be quantified. The SP measurement then gives

\[ v_{n,uncor} (SP) = \frac{\langle u_n \cdot \frac{\bar{Q}^A_n}{M_A} \rangle}{\sqrt{ \langle \frac{\bar{Q}^A_n \cdot \bar{Q}^C_n}{M_A M_C} \rangle }} = \frac{\langle v_n v_n^A \rangle}{\sqrt{ \langle v_n^2 \rangle}} = \sqrt{ \langle \frac{v_n^2}{\langle v_n^2 \rangle} \rangle } \quad (6) \]

where we have assumed that the mean number of reference particles in the two sub-events, \( \langle M_A \rangle \) and \( \langle M_C \rangle \), are identical and that \( \langle M_{A(C)} \rangle = \sqrt{ \langle M_{A(C)}^2 \rangle } \). Although, the later assumption is reasonable (e.g. \[ [18] \]), a priori there is no reason to expect that the reference particles are equally distributed between the two reference detectors.

Therefore, during the actual study, each event is divided over three (arbitrary) sub-events, i.e. particle subsets, let them be A, B and C, standing for particles registered by the two scintillator arrays (A,C) and TPC (B) detector, respectively. More specifically, RFP from \( \eta_C \in (-3.7,-1.7) \) are correlated with POI from \( \eta_{TPC} \in (-0.8,0.8) \). In turn, POI within \( \eta_{TPC} \in (-0.8,0.8) \) are correlated with RFP from \( \eta_A \in (2.8,5.1) \). The actual analysis employs then almost one unit of pseudorapidity separation |\( \Delta \eta \rangle = |\eta_{TPC} - \eta_{AC} | \rangle \geq 0.9 \) between POI and RFP samples; contributions from short-range, i.e. non-flow, correlations are diminished \[ [10] \], since flow is a genuinely long-range effect. For the case of non-identical sub-events, the three sub-event correlation is attained via the substitution \[ [9] \]

\[ \langle \frac{\bar{Q}^A_n \cdot \bar{Q}^C_n}{M_A M_C} \rangle \Rightarrow \frac{\langle \frac{\bar{Q}^A_n \cdot \bar{Q}^C_n}{M_A} \rangle}{\langle u_n \cdot \bar{Q}^A_n \rangle} \quad (7) \]

\[ \text{Figure 24: Model dependent evaluation of the ratio for the root-mean-square } \sqrt{ \langle v_n^2 \rangle } \text{ over the mean } \langle v_n \rangle \text{ of the flow distribution } v_n, \ n = 2,3. \text{ Depending on the detector performance, the result obtained with the EP method might lies anywhere between the rms and the mean value, hence yielding a not well-defined observable.} \]

\(^5\)Fluctuations in multiplicity are diminished by dividing the flow vectors, event by event, by their multiplicity.

\(^6\)Note that the \( v_{n,A} \) and \( v_n^A \) notations are equivalent.
in Eq. (6), which finally reads

\[ v_{n,uncor}(SP, |\Delta\eta| > 0.9) = \left[ \frac{\langle (\vec{u}_n \cdot \vec{Q}_{n,A}^+ \rangle)}{\langle (\vec{u}_n \cdot \vec{Q}_{n,C}^+ \rangle)} \left/ \sqrt{\frac{\langle \vec{Q}_{n,A}^+ \cdot \vec{Q}_{n,C}^+ \rangle}{\langle \vec{Q}_{n,A}^+ \cdot \vec{Q}_{n,A}^+ \rangle}} \right. \right]^{1/2} = \sqrt{\frac{\langle v_nv_A^\prime \rangle}{\langle v_nv_n^\prime \rangle}} = \sqrt{v_n^2}. \] (8)

For clarity, the double-average notation is preferred to be explicitly present in the nominator, in order to keep in mind that the inner angular brackets indicate an average over all reconstructed POI (\( \Lambda (\bar{\Lambda}) \)) and the outer angular brackets correspond to an average over all events in a given centrality.

The subscript \( uncor \) in Eq. (6) and (8) above stands for the acceptance uncorrected \( v_n \) values, since the non uniformity in the VZERO azimuthal efficiency may lead to spurious correlations [11]. In that case, acceptance corrections are directly determined from experimental data by applying the inverse of the \( event-averaged \) scintillation signal \( \text{ADC}_{A(C)} \) as a weight for every signal in the \( 32 + 32 \) VZERO segments (cf. section §3.3.1):

\[ Q_{n}^{A(C)} = \sum_{i \in \text{RFP}} \frac{\text{ADC}_{i,A(C)}}{\text{ADC}_{A(C)}} e^{\text{exp} \left[ i n \phi_{i,A(C)} \right]} \] (9)

In deed, the small non-uniform azimuthal coverage is visible in the distribution of the flow vector components before the re-weighting formula has been applied (Fig. 25). Such a detector-induced bias has been corrected, hence \( Q_{n,x} \) and \( Q_{n,y} \) are found aligned (flattened) to zero. For redundancy, any residual nonzero average values were further reduced by a re-centering procedure \( Q_{n,x} - \langle Q_{n,x} \rangle \) and \( Q_{n,y} - \langle Q_{n,y} \rangle \), i.e. by subtracting the \( event-averaged \) centroid position of each VZERO sector [19].

![Figure 25](image)

**Figure 25:** The \( Q_n \)-vector distribution does not have any preferred direction, since the orientation of the collision is completely random in the laboratory frame, meaning that \( \langle Q_{n,x} \rangle = 0 \). Here, the \( \langle Q_{2,x} \rangle \) (real) and \( \langle Q_{2,y} \rangle \) (imaginary) second order flow vector components, compiled for the \( 32 + 32 \) VZERO sectors, are separately calculated before and after the re-weighting in Eq. (9). The results are further accumulated for the (0-60) % centrality range and are presented on a run by run basis, i.e. as a function of the detector operating period during November 2010. For clear visibility, the points were manually shifted by -0.02 after calibration [17].

Last but not least, if both \( \langle (\vec{u}_n \cdot (p_T) \cdot \vec{Q}_{n,A}^+ \rangle \left/ \sqrt{\langle \vec{Q}_{n,A}^+ \cdot \vec{Q}_{n,A}^+ \rangle} \right. \rangle \) and \( \langle (\vec{u}_n \cdot (p_T) \cdot \vec{Q}_{n,C}^+ \rangle \left/ \sqrt{\langle \vec{Q}_{n,C}^+ \cdot \vec{Q}_{n,C}^+ \rangle} \right. \rangle \) terms in Eq. (8) were to simultaneously produce negative values, \( v_{n,cor} \), viz. the acceptance corrected coefficients, should be manually reversed to \(-v_{n,cor} \). Such a case would correspond to out-of-plane preferential emission, which could occur at the lower-\( p_T \) \( \lesssim 1 \text{GeV/c} \) region and for heavier, relative to pions, POI (cf. section §5.1). Hereafter, and simply for brevity, the \( cor \) subscript is dropped, while \( v_n \) \( (SP, |\Delta\eta| > 0.9) \equiv v_{n,cor} \).
4.4 Anisotropic flow of weakly decaying particles

The flow of unstable particles, as for instance that of Λ (¯Λ) hyperons, cannot be studied but through their decay products [13]. Therefore, a correlation should be first identified between the daughter tracks, like the one the invariant mass plot reveals (e.g. [14]), prior to extracting the dependence of such correlation on the azimuthal angle of the decaying particle.

For each sub-event B, pairs of pπ (¯pπ+) are sorted into narrow, relative to the detector invariant mass resolution, windows of m_{inv} = 1 MeV/c^2 over the range 1.09 ≤ m_{inv} < 1.17 GeV/c^2 and for (pair) transverse momenta 0.6 ≤ p_T ≤ 20 GeV/c, whose upper limit has been set due to significant loss in statistics. The first step is then to separate the invariant mass distribution into a correlated and an uncorrelated part, i.e. the peak N_{Λ(¯Λ)}(m_{inv}) centered at the expected Λ (¯Λ) mass and the background N_b(m_{inv}) respectively, meaning

\[ N_{pairs}(m_{inv}) = N_{Λ(¯Λ)}(m_{inv}) + N_b(m_{inv}) \quad (10) \]

More specifically, both the signal and background terms on the right hand side of Eq.(10) are determined by counting the number of pπ (¯pπ+) pairs through a regular fitting routine. The peak is fitted with the sum of two Gaussians (centered at the same mean), for consistently describing the tails on the peak, while a second order polynomial accounts for the rest of the invariant mass spectrum. This is illustrated in Fig. (27), where the cyan and red dashed lines represent separately the signal and background fits, while the solid magenta curve amounts to their summation. The observed mean mass μ ≈ 1.116 GeV/c^2 (Fig. 26, left panel) is consistent with the accepted values [4] for all transverse momenta and centrality intervals. The combined width σ ∼ O (2.5) MeV/c^2 > 1 MeV/c^2, i.e. the root mean square of the two standard deviations of the Gaussian functions weighted by their yields (Fig. 26, right panel), is constrained by the momentum resolution of the detector and gets progressively worsen for p_T ≥ 4.5 GeV/c.

Since flow coefficients are additive, v_{n, pair} (m_{inv}), i.e. the particle-pair flow coefficients, are decomposed according to

\[ N_{pairs}(\cdot) v_{n, pair}(\cdot) = N_{Λ(¯Λ)}(\cdot) v_{n, Λ(¯Λ), pair}(\cdot) + N_b(\cdot) v_{n, b, pair}(\cdot), \quad \cdot \equiv m_{inv} \quad (11) \]
where $v_{n}^{\text{pair}}(m_{\text{inv}})$ has been extracted in Eq. (8). Therefore, the decomposition in both Eq. (10) and (11) is simultaneously performed, by assuming smooth background behavior. The latter can be seen in the bottom panel of Fig. (27), where the background component $v_{n}^{b,\text{pair}}(m_{\text{inv}})$ is fairly approximated by a first order polynomial (modulated by the relative $N_{\Lambda(\bar{\Lambda})}/N_{\text{pairs}}$ and $N_{b}/N_{\text{pairs}}$ yields), while the smooth background interpolation under the peak in the top panel of Fig. (27) is supported by Monte-Carlo studies [5]. Once all terms in Eq. (11) have been determined, the $v_{n}^{\Lambda(\bar{\Lambda}),\text{pair}}$ parameter could be finally extracted.

**Figure 27:** (top panel) Invariant mass distribution of $p\pi^{-}(\bar{p}\pi^{+})$ pairs for $1.4 \leq p_{T}^\text{pair} \leq 1.6$ GeV/c. The peak is described by a double Gaussian, while the background is assumed to behave smoothly and fitted with a second order polynomial. (bottom panel) Fourier coefficient $v_{3}^{\text{pair}} \equiv v_{3}^{\text{tot}}(\text{SP}, |\Delta \eta| > 0.9)$ decomposed in signal $v_{3}^{\Lambda(\bar{\Lambda}),\text{pair}}$ and background $v_{3}^{b,\text{pair}}(m_{\text{inv}})$ components, the latter being described by a first order polynomial modulated by the relative signal and background fractions. The invariant mass method performs reasonably well, since a set of data points are used in the fit over a wide $m_{\text{inv}}$ region for signal and background. Data points far from the mass peak constrain $v_{3}^{b,\text{pair}}(m_{\text{inv}})$, because pure background is expected in this region, while under the peak $v_{n}^{\text{pair}}(m_{\text{inv}})$ is dominated by the signal distribution.

### 4.5 Systematic uncertainty evaluation

Initially, it was verified that the harmonic values of order 2-4 are statistically compatible, separately for particles and anti-particles (e.g. [20]), for all centralities and transverse momentum range. An example is illustrated in Fig. (28), where the second order harmonic coefficient $v_{2}^{\text{pair}}$ is plotted as a function of $p_{T}$, separately for $\Lambda$ and $\bar{\Lambda}$ in the (10-20) % centrality interval. Note that the symbols have not been placed at the center of the bin, albeit at the calculated average of the $p_{T}$ distribution around the $\Lambda (\bar{\Lambda})$ mass, for consistency with the $p_{T}$ spectrum (e.g. [21]).

A higher order polynomial (solid lines in Fig. (28)) has been selected to simply describe the trend of the $v_{2}^{\Lambda}(p_{T})$ distribution. The final comparison is performed based on the difference of $v_{2}^{\Lambda}(p_{T})$ relative to $v_{2}^{\Lambda,\text{int}}(p_{T})$, i.e. the latter was interpolated at $p_{T,\Lambda}$ based on the polynomial parameterization. Such a numerical difference is depicted on the bottom panel of Fig. (28), along with errors calculated according to the standard error propagation formula. The error component, which corresponds to the interpolated $v_{2}^{\Lambda}(p_{T})$ values, was found following the very
same procedure as in the case of the central values, hence the presence of three fitting curves on the top panel of Fig. (28).

The differences have been tested about whether they could point to significant deviations from the hypothesis of \( v_2^\Lambda \) and \( v_2^{\bar{\Lambda}} \) being statistically compatible. This is illustrated by the red solid line on the bottom panel in Fig. (28), which constitutes a zero order polynomial fit to \( v_2^\Lambda - v_2^{\bar{\Lambda}} \) over the \( 0.6 \leq p_T \leq 10 \text{ GeV/c} \) range. By consulting the inset, which includes the relevant statistical results, it has been verified that any observed differences could be totally ascribed to statistical fluctuations. Since the results for \( v_2^\Lambda \) and \( v_2^{\bar{\Lambda}} \) are found to be consistent, they have been combined for rest of this analysis. The same property, i.e. equivalence of anisotropic flow between particle and anti-particle species, hold for \( v_3^\Lambda(p_T) \) (Fig. (56)) and \( v_4^{\Lambda(\bar{\Lambda})}(p_T) \) (Fig. (59)) respectively.

Figure 28: (top panel) Dependence of the second order harmonic on transverse momentum \( p_T \) separately for \( \Lambda \) (filled cross) and \( \bar{\Lambda} \) (open cross). Solid lines are higher order polynomial parameterization of the \( v_2^\Lambda(p_T) \) behavior. (bottom panel) Numerical difference of the interpolated \( v_2^\Lambda_{\text{int}}(p_T) \) relative to the extracted \( v_2^\Lambda(p_T) \); deviations were found to be compatible with statistical fluctuations.

Further inspection of sources that could introduce systematic deviations was performed by varying the event and particle selection criteria. Differences, if were to be registered, would be quantified with respect to the nominal harmonic values \( v_n^{\text{def}} \), the later derived based on the selection requirements described in section §4.1 and 4.2 (Table 2). Contributions from different sources on \( v_n^{\text{def}} \) were estimated for the (10-20) % and (40-50) % most central events, solely for \( v_3 \) and \( v_4 \); inspection of variations in \( v_2^{\Lambda(\bar{\Lambda})} \) could be found in [9] in section §5. It should be noted that all non-nominal values \( v_n^{\text{var}} \) were analyzed by considering the very same event sample, thus \( v_n^{\text{def}} \) and \( v_n^{\text{var}} \) are expected to be highly correlated. Assuming that either technique (nominal or non-nominal) saturates the Minimum Variance Bound (MVB), the estimate for systematic deviations writes [16]

\[
\sigma_{\text{def-var}}^2 = |\sigma_{\text{def}}^2 - \sigma_{\text{var}}^2|, \tag{12}
\]

where \( \sigma_{\text{def}} \) and \( \sigma_{\text{var}} \) amount to the statistical uncertainties on \( v_n^{\text{def}} \) and \( v_n^{\text{var}} \) respectively.
Potential deviations stemming from the event selection requirements were investigated by altering the longitudinal position of the primary vertex from $\pm 10 \text{ cm}$ to $\pm 7 \text{ cm}$. Moreover, an additional event requirement was concerned with a parameterized compatibility in the number of tracks separately recorded in ITS-TPC and TPC standalone reconstruction. For $\Lambda (\bar{\Lambda})$, the resulting Fourier coefficients were consistent with values obtained with the default selection criteria.

In addition, the track quality prerequisites were varied for the two daughters of the decaying $\Lambda (\bar{\Lambda})$. In principle, the number of TPC space points, the $\chi^2$ per TPC space point (per degree of freedom) and the width of the band ($N_p(\bar{p})$) around the expected energy loss in the TPC introduced no systematic deviations in the values of the measured $\nu_n$. Further, to estimate possible variations that the selection of the $\Lambda (\bar{\Lambda})$ candidates might introduce, the range in the choice for the $\Lambda (\bar{\Lambda})$ rest-frame lifetime ($c\tau$), the radial position of the secondary vertex (Rad$_{XY}$), the cosine of the pointing angle ($\cos\theta_p$) and the distance of the closest approach of the decay products both to the primary vertex (DCA) and relative to each other (PCA) were modified. These variations did not significantly affect the measured result.

Systematic uncertainties originating from the signal and the background extraction (cf. section §4.4) were probed by calculating the relative yields using different functions to describe the signal (Breit-Wigner, Gaussian and Voightian) and background (polynomial of different orders) in the invariant mass distribution, respectively. In addition, the range of the default fitting function, i.e. the combination of a double Gaussian with a second order polynomial, has been (discontinuously) decreased and augmented.

Overall, no significant impact on either $\nu_3$ or $\nu_4$, i.e. $|\nu_n^{\text{def}} - \nu_n^{\text{var}}| \gtrsim 3 \cdot \sigma|^{\text{def-var}}$, both for (10-20) % and (40-50) % centrality intervals was found. Conclusively, deviations were compatible with zero, hence could be completely accounted for statistical fluctuations. Probably, this could indicate that the aforementioned variations did not capture all types of discrepancies that might affect the analysis. On the other hand, such cross-checks strengthen the consistency of the quoted results, since they validate the robustness of the detector response. The ranges of each and every attempted variation are summarized in Table 3 and depicted in Fig. (57)-(58) for $\nu_3$ and in Fig. (60)-(61) for $\nu_4$ (cf. section §H). Varying the transverse position of the reconstructed secondary vertex and the cosine of the pointing angle produced the largest deviation relative to the nominal selection criteria for $\nu_3$ (Fig. (29)) and $\nu_4$ (Fig. (30)) in the (40-50) % most central collisions respectively.
Table 3: Summary of cross-checks for systematic deviations on $v_3(p_T)$ and $v_4(p_T)$

<table>
<thead>
<tr>
<th>Systematic Cross-Check</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
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</thead>
<tbody>
<tr>
<td>$v_3^\Lambda - v_3^\bar{\Lambda}$</td>
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<td>-</td>
</tr>
<tr>
<td>Event Selection</td>
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<td>Track Multiplicity</td>
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</tr>
<tr>
<td>Tracking Quality</td>
<td>n. TPC Clusters</td>
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<tr>
<td></td>
<td>$\chi^2$/ndf</td>
<td>1.5 ($v_3$)</td>
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<tr>
<td></td>
<td>TPC $N_T^{p(\bar{p})}$</td>
<td>2</td>
</tr>
<tr>
<td>Topological Variables</td>
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<td>1.5 · $c\tau_0$</td>
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<tr>
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<td>$\text{Rad}_{XY}$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$\cos\theta_p$</td>
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</tr>
<tr>
<td></td>
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<tr>
<td></td>
<td>PCA</td>
<td>0.2 cm</td>
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</tr>
<tr>
<td></td>
<td>Background Function</td>
<td>-</td>
</tr>
<tr>
<td>Fitting Range</td>
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<td>$(\mu - 13\sigma &lt; \mu &lt; \mu + 20\sigma)$</td>
</tr>
</tbody>
</table>

Figure 29: (upper) The $p_T$-differential evolution for $v_3^\Lambda$ over $p_T = 0.6-6$ GeV/c for the (40-50) % most central collisions separately for the nominal selection criteria (Table 2) and the transverse radius variation at $\text{Rad}_{XY} = 8$ cm. (lower) The $p_T$-differential evolution for $v_3^{var}/v_3^{def}$ over $p_T = 0.6-6$ GeV/c for the same centrality class. (bottom) The $p_T$-differential evolution for $v_3^{var}/v_3^{def}$ over $p_T = 0.6-6$ GeV/c for the same centrality class. Binomial errors are considered for calculating the uncertainty on $v_3^{var}/v_3^{def}$. 
Figure 30: (upper left) The $p_T$-differential evolution for $v_4^{\Lambda(\bar{\Lambda})}$ over $p_T = 0.6-6$ GeV/c for the (40-50)% most central collisions separately for the nominal selection criteria (Table 2) and the cosine of the pointing angle at $\cos\theta_P = 0.996$ cm. (lower left) The $p_T$-differential evolution for $v_4^{\text{var}} - v_4^{\text{def}}$ over $p_T = 0.6-6$ GeV/c for the same centrality class. (bottom left) The $p_T$-differential evolution for $v_4^{\text{var}} / v_4^{\text{def}}$ over $p_T = 0.6-6$ GeV/c for the same centrality class. (upper right) The $p_T$-differential evolution for $v_4^{\Lambda(\bar{\Lambda})}$ over $p_T = 0.6-6$ GeV/c for the (40-50)% most central collisions separately for the nominal selection criteria (Table 2) and the cosine of the pointing angle at $\cos\theta_P = 0.999$ cm. (lower right) The $p_T$-differential evolution for $v_4^{\text{var}} - v_4^{\text{def}}$ over $p_T = 0.6-6$ GeV/c for the same centrality class. (bottom right) $p_T$-differential evolution for $v_4^{\text{var}} / v_4^{\text{def}}$ over $p_T = 0.6-6$ GeV/c for the same centrality class. In both cases, binomial errors are considered for calculating the uncertainty on $v_4^{\text{var}} / v_4^{\text{def}}$. 
References


[3] P. Cortesea, on behalf of the ALICE Collaboration. Neutron emission from electromagnetic dissociation of Pb nuclei at $\sqrt{s_{NN}} = 2.76$ TeV measured with the ALICE ZDC. EPJ Web Conf. 70, 00073 (2014)


5 The fate of initial Geometry and its Fluctuations

5.1 Second order harmonic

The $p_T$-differential second order harmonic is plotted for different centrality intervals in Fig.(31). It is clearly seen that the values of $v_2(p_T)$ are positive for the majority of the momentum range and centrality classes. Positive elliptic flow means that the anisotropic flow dominantly develops along the 2nd order symmetry plane (cf. section §A.1), hence more particles with greater velocity are found along the $\Psi_2$ direction than perpendicular to it. Moreover, $v_2(p_T)$ progressively increases moving from central to peripheral collisions up to the (40-50)$\%$ centrality class. Such a centrality ordering complies with the final-state momentum anisotropy being driven by the geometry of the collision (cf. section §B), which is typically parameterized by the initial-state eccentricities $\varepsilon_n$ (e.g. [1]) of the reaction zone. Last but not least, the nonzero $v_2$ magnitude for the 5$\%$ most central PbPb collisions (filled red circles), where by definition the ellipticity $\varepsilon_2$ vanishes for a complete overlap, has been attributed mainly to fluctuations in the positions of the participating nucleons [2].

![Figure 31](image.png)

Figure 31: The $p_T$ differential $v_2$ of $\Lambda+\bar{\Lambda}$ over 0.6-20 GeV/$c$ for the centrality classes given on the legend. A magnified version of the intermediate momentum $3 \lesssim p_T \lesssim 5$ GeV/$c$ region is illustrated in the inset, along with the calculated $v_{2,max}$ values (open points). See text for details.

However, for the (50-60)$\%$ centrality class (open diamonds) such an increase is hardly reproduced. In principle, the realization that collective flow patterns could be ascribed to an almost ideal fluid dynamical response of the fireball forced the scientific community to a paradigmatic shift (e.g. [3]) by accepting the notion of a strongly coupled and flowing plasma [4]. In practice though, the successful description of the data by viscous hydrodynamical models is restricted to low transverse momentum regions, approximately up to $p_T \approx 1-1.5$ GeV/$c$, and up to semi-central event classes (e.g. [5]). The latter hints to the gradual breakdown of the pure hydrodynamical approach in smaller collision systems and has been argued to signal the onset of dissipative effects during the late hadronic stage (e.g. [34]).

Interestingly, a similar pattern for the transverse momentum dependence of $v_2$ is reproduced in all centrality classes. The rise up to $p_T \sim 4$ GeV/$c$, almost linear up to $p_T \sim 2$ GeV/$c$, is followed by a saturation and then a gradual decrease. The position of the maxima seems to weakly vary with centrality, apart from the most peripheral class considered here, i.e. the (50-60)$\%$ centrality class. Since the current $p_T$-binning is coarse enough for readily resolving
whether there exists a significant shift between centralities in the position of the maximum
values of $v_2\ (p_T)$, viz. $v_{2, \text{max}}^{\Lambda(\bar{\Lambda})}$, a finer analysis was launched.

More specifically, and separately for each centrality up to the (40-50)% class, the point
where the maximum value of $v_2$ occurs, i.e. around the expected $3.5 \lesssim p_T \lesssim 4 \text{ GeV/c}$ region,
was determined. First, the $v_2\ (p_T)$ error statistics are treated as Gaussian, i.e. independent
and identically distributed (i.i.d.). Therefore, they are taken under consideration by uniformly
re-generating $v_2\ (p_T) \pm \sigma$ values, with $\sigma$ the statistical uncertainty. All resulting distributions
($10^6$ per centrality) are then parameterized by a higher-order polynomial function. The latter
allows to numerically calculate their local maxima around the $3.5 \lesssim p_T \lesssim 4 \text{ GeV/c}$ window. For
this purpose the fast-converging Brent method is used [6]. Finally, the solutions along with
their associate $p_T$ are plotted and separately fitted, as illustrated in Fig. (32).

![Figure 32: (left) Distribution of $v_{2, \text{max}}$ for $\Lambda+\bar{\Lambda}$ in the (20-30)% centrality class fitted with a three parametric
Gumbel distribution. (right) Distribution of $p_T$ where $v_{2, \text{max}}^{\Lambda(\bar{\Lambda})}$ occurs in the very same centrality class. A sum of two
Gaussian functions (solid magenta) on top of the smooth pedestal (dashed red) is used.

For the former, a three parametric Gumbel distribution [7] was applied owing to its po-
tential applicability to represent the distribution of maxima in case that the underlying data
sample is i.i.d., according to the formulated extreme value theory. The distributions of $p_T$
for which the $v_{2, \text{max}}^{\Lambda(\bar{\Lambda})}$ occurs are fitted with a five parametric double Gaussian function,
whose weighted mean $\mu = f \mu_1 + (1 - f) \mu_2$ is assigned with $\sigma_1$ and $\sigma_2$ asymmetric errors, while
the fifth parameter $f$ regulates the relative admixture. Note that $\mu + \sigma_1$, $\mu_1 > \mu_2$, otherwise
$\mu + \sigma_2$, $\mu_1 \leq \mu_2$, and a second order polynomial describes the pedestal. The results, drawn in the
inset of Fig. (31), and separately in Fig. (33), hint at the weak centrality dependence of the
$v_{2, \text{max}}^{\Lambda(\bar{\Lambda})}\ (p_T)$ up to the 30% most central collisions, whereas, for the rest of semi-central classes,
the trend of $v_2\ (p_T)$ being saturated at lower $p_T$ with decreasing centrality (e.g. [5]), is more
pronounced.
Figure 33: The $v_{2, \text{max}}^{\Lambda(\bar{\Lambda})}$ values seem aligned up to the 30% most central collisions, whereas $v_2(p_T)$ for $\Lambda(\bar{\Lambda})$ has saturated earlier for more peripheral classes.

Let us now elucidate the crucial features of the $p_T$-dependence of the second order harmonic. This is illustrated in Fig.(34), where four classes in the (10-50)% centrality range have been merged using as a weight their statistical uncertainty. Hydrodynamical calculations predict a clear mass-ordering of elliptic flow (e.g. [8]), meaning for instance that $v_2^{p(\bar{p})}$ (blue circles) is greater than $v_2^{\Lambda(\bar{\Lambda})}$ (magenta squares) in the low-$p_T$ region. Such an hierarchy is believed to be a consequence of the interplay of radial, i.e. the axially symmetric, flow with the azimuthally anisotropic transverse flow, both driven by the pressure-gradient expansion (cf. section §D).

Radial flow gives a boost to all particles that are formed in the system. By pushing them radially outward, it shifts low-$p_T$ hadrons to higher transverse momenta, affecting more efficiently heavier particles. This mass-dependent flow component introduces then a depletion in the low-$p_T$ particle yield. In the case of an azimuthally asymmetric system such a depletion becomes maximum along the direction of the symmetry plane, where the pressure gradients are the strongest. Therefore, the relative in-plane against out-of-plane difference of the particle yield, primarily quantified by $v_2(p_T)$, is regulated by two opposing trends, i.e. the radial (diminishing effect) and the azimuthally anisotropic transverse flow (enhancement effect). The interplay between these two components depends on their relative magnitudes and the particle mass.

Since the transverse momentum spectra of, in particular, heavier particles are influenced by radial flow, the relative depletion is expected stronger for increasing particle mass. In deed, heavier particles exhibit smaller $v_2(p_T)$ at fixed transverse momentum approximately below $p_T \lesssim 2$ GeV/$c$. In the extreme case, i.e. in the most central collisions where radially symmetric pressure gradients are dominant, this depletion effect might be large enough that there would be less small-$p_T$ particles moving along in-plane relative to out-of-plane direction. The latter would give rise even to negative values of $v_2(p_T)$ at the low-$p_T$ region (e.g. [9]). There exists an indication for $v_2^{\Lambda(\bar{\Lambda})}(p_T) < 0$ in the most (0-5)% central events (filled red in Fig.(34)) around $p_T \approx 0.6$ GeV/$c$, though the quoted result found compatible with zero within the current uncertainty.

At variance with the low-$p_T$ part, the $v_2$ behavior of high-$p_T$ particles cannot be reproduced by hydrodynamical calculations [10]. Any deviation from hydrodynamical expectations typically reflects a decreased amount of interactions between the constituents at an early time.
in the system evolution, preventing the matter from being thermalized in the reaction zone. Schematically, high-$p_T$ hadrons did not follow this collective dynamics and had escaped the fireball long before having suffered a sufficient number of secondary interactions to thermalize their momenta. Contrary to the bulk particle production, it has been realized that fragmentation of hard partons would dominate hadron production at high transverse momenta (e.g. [11] [12]).

Again, it is the geometry of the collision that results in a positive and finite elliptic flow at high-$p_T$, but the underlying mechanism radically differs from collective hydrodynamic flow, now. Schematically, it is expected that high momentum partons, traveling along the shorter axis of the overlap region, to have smaller probability of being completely absorbed by the medium or, at least, their energy being less attenuated relative to partons emitted, for instance, perpendicular to the reaction plane direction. Collisional and radiative energy loss mechanisms, which try to properly treat parton traversal through a dense medium of deconfined color charges (e.g. [13]), establish a path length dependent energy loss. Since the path length depends on the azimuthal emission angle with respect to the reaction plane, an azimuthal anisotropy of particle emission is also introduced at high-$p_T \gtrsim 8$-10 $GeV/c$. As Fig. (34) illustrates, the $p_T$-differential $v_2$ of $\pi^+\pi^-$, $p+p$ ([19], [26]) and $\Lambda+\bar{\Lambda}$ for the (10-50) % most central events is significant at least up to $p_T \sim 20$ $GeV/c$ at LHC. Lower energy $v_2 (p_T)$ results for $\pi^0$ (inverted triangles, [29]) follow the same trend and are superimposed on the LHC values.

Figure 34: For clarity, pion (red) and proton (cyan) markers are slightly shifted along the horizontal axis at $p_T > 8$ $GeV/c$. Error bars (shaded boxes) represent the statistical (systematic) uncertainties. See text for details.

In principle, striking similarities have been registered for both bulk and hard observables at RHIC and LHC, by means of the observed differential elliptic flow (filled triangles in Fig. (34) and the measured nuclear modification factor $R_{AA}$ at high $p_T$. The later revealed a suppression of high-$p_T$ particles in nuclear collisions relative to lighter collision systems ([14] [15]). More specifically, when measured as a function of transverse momentum, the values of $v_2 (p_T)$ at the two colliders are close, as expected due to similar initial eccentricities and values of viscosity. Moreover, the $v_2 (p_T)$ coefficient is comparable for different species at high $p_T \gtrsim 10$ $GeV/c$. The fact that there is no apparent particle species dependence in $v_2 (p_T)$ can be connected to a similar effect observed in $R_{AA}$ measurements in the light flavor domain. Interestingly, $v_2 (p_T)$ remains still significantly different from zero even at transverse momentum much as high as $\sim 40$ $GeV/c$ at LHC [35], serving, together with the $R_{AA}$ measurements, as robust tests for energy loss mechanisms. An indicative example is represented by the dashed
line in Fig. (34), which corresponds to the WHDG model calculations for $\pi^0$ extrapolated to the LHC collision energy for the (20-50) % centrality range [30].

In practice, a transition from pressure gradient dependent expansion, driven by collective motion, to path length dependent attenuation, driven by jet quenching, is expected to occur at intermediate momentum region $2 \lesssim p_T \lesssim 6$-8 GeV/c. One of the early experimental observations reported at RHIC (e.g. [15]) indicated that particles tend to group based on their hadron type, i.e. a universal scaling was revealed, when both $v_2$ and $p_T$ were scaled by the number of constituent quarks (NCQ). Such an effect has been successfully reproduced by quark coalescence models [16], with the latter being considered as the dominant hadronization mechanism in this momentum range. However, deviations have been later reported in noncentral collisions [17], hence it is intriguing to examine whether quark degrees of freedom dominate the early stages of the system evolution. Violation of the $v_2$ NCQ is clearly visible also at LHC, as Fig. (35) illustrates for both central and semi-central collisions, while the scaling found least broken in the $1 \lesssim p_T/n_q \lesssim 1.5$ GeV/c window. It is obvious that a simple interplay between recombination and jet energy loss in the medium is not enough to describe the $v_2 (p_T)$ behavior over the intermediate momentum region (e.g. [32]), i.e. the saturation for all species, yet a bit later for baryons, and the inverted hierarchy relative to the low-$p_T$ part.

Figure 35: The $p_T/n_q$ dependence of $v_2/n_q$ for $\pi^+\pi^-$, $K^+K^-$, $p+\bar{p}$ ([11]) and $\Lambda+\bar{\Lambda}$ for the (0-5) % (left) and (20-30) % (right) centrality classes.

5.2 Higher order harmonics

5.2.1 $v_3(p_T)$

Until recently, the collective response to the initial geometric configuration has been argued to exclusively imprint a $cos(2\phi)$ modulation on the particle angular distribution. Interestingly though, the non trivial long range patterns in the azimuthal correlations, measured in heavy-ion collisions (e.g. [18]), seem to find an alternative explanation, than previously thought as response of the medium to the energy deposited by the quenched jets (e.g. [19]). Inasmuch as azimuthal correlations are driven by the global event anisotropy, the spectral decomposition of the pairwise $\Delta\phi$ distribution is consistent with higher-order flow buildup, which is possible only when initial conditions fluctuate (e.g. [33]). In the following, we try to disentangle whether higher-order modes could be, at least qualitatively, understood by the presence of
correlations between momentum anisotropy of the produced particles and asymmetry of the initial density distribution, on one hand, and medium viscosity, on the other hand.

The measured $v_3$ as a function of the transverse momentum $p_T$ is given in Fig. 36 up to the 60% most central events. Clearly, the presence of $v_3$ is significant over all centrality classes, but most interestingly, the $p_T$ differential evolution of the third order harmonic exhibits quite similar features to $v_2 (p_T)$; the strong $p_T$ dependence with a pronounced rise out to $\approx 4-4.5 \text{ GeV/c}$ is subsequently followed by a slower decrease out to high-$p_T$. Such a measurement for triangular flow indicates that higher order moments are also present in the fluctuating initial collision geometry, translating themselves into the final momentum anisotropies.

Figure 36: The $p_T$ differential $v_3$ (left panel) and $v_2$ (right panel) of $\Lambda+\bar{\Lambda}$ over 0.6-8 GeV/c for the centrality classes given on the legend.

However, elliptic and triangular flow seem not to share the same centrality dependence, since the centrality splitting might not be profound enough for $v_3 (p_T)$. This could be further manifested, if the $p_T$-integrated (cf. section §A.1) $v_3$ were to be plotted as a function of centrality [20]. In that case, a weak centrality variation is attained, consistent with the alleged nature of $v_3$ owing to the initial-state effects. The centrality ordering for the third Fourier coefficient could be also attributed to the centrality dependence of its corresponding triangularity $\varepsilon_3$. The latter increases as a function of the impact parameter [21], since fluctuations are naturally more pronounced in smaller systems. Again, $v_3 (p_T)$ slightly decreases for more peripheral (40-60) % collisions indicating the break down of pure hydrodynamical, i.e. both ideal and viscous, approach. The translation of initial fluctuations to final state momentum anisotropy becomes more copious, possibly due to the late dissipative hadronic stage.

Apart from the evident centrality ordering both for $v_2 (p_T)$ and $v_3 (p_T)$, an inverse hierarchy is registered with respect to the harmonic coefficients, meaning that higher values are seen for $v_2 (p_T)$. The fact that triangular flow becomes less dominant implies that the fluctuation-dominated eccentricity coefficients, like $\varepsilon_3$, are generically smaller than the geometry-dominated ones, e.g. $\varepsilon_2$. Furthermore, the dominance of elliptic over triangular flow in most of the centrality classes possibly indicates that the transfer of coordinate-space to momentum-space anisotropy is more efficient for lower harmonics or, stated alternatively, more suppressed for the higher moments [25]. Viscosity further reduces the correlation be-
between momentum anisotropy of the produced particles and asymmetry of the initial density distribution, meaning that experimental data on triangular flow could constrain $\eta/s$ better than elliptic flow data.

Contrary to the majority of the centrality classes, there exists a tiny window in the 5% most central collisions, where the magnitude and the shape of $v_3 (p_T)$ are similar to $v_2 (p_T)$. In most central collisions, event-by-event fluctuations constitute the dominant contribution to non-vanishing $\varepsilon_n$ and $v_n$, thus leading to initial- and final-state coefficients of roughly the same size across different harmonics. On top of that, since the $\varepsilon_n$- and $v_n$- associated angles, i.e. $\Phi_{n \geq 2}$ and $\Psi_{n \geq 2}$, are mainly fluctuation driven, they should be found essentially uncorrelated with the reaction plane in most central collisions (cf. section §B.1, [22]).

To investigate further the hydrodynamic origin of the triangular flow, the $p_T$-differential $v_3$ for $\Lambda (\bar{\Lambda})$ is plotted in Fig. (37), and compared to results measured for $\bar{p}$, $\pi^\pm$, $K^\pm$. It is verified that the same mass splitting pattern extracted in elliptic flow and persisted over all centralities is clearly observed in triangular flow as well, i.e. lighter particles have larger $v_3 (p_T)$ than heavier particles for $p_T$ below $\sim 2$ GeV/$c$. Such a scaling is inverted after mesons and baryons have crossed a point at intermediate-$p_T$, which is centrality dependent.

As previously highlighted (section §5.1), in the intermediate momentum $2 \lesssim p_T \lesssim 6-8$ GeV/$c$ region, the collective flow scaling with the number of constituent quarks $n_q$, viz. $v_n^{\text{hadron}} \approx n_q v_n (p_T/n_q)$, has been interpreted as a sign of the addition of valence quark momenta at hadronization; this is possible in a quark recombination process for particle production [23]. The NCQ scaling of $v_3$ is tested in Fig. (38) and Fig. (40) for different centrality and $p_T$ regions with identified particles. More specifically, the third order coefficients of $\pi^\pm$, $K^\pm$, $\bar{p}$ and $\Lambda (\bar{\Lambda})$ are scaled with $2 (3)$ for mesons (baryons) and plotted as a function both of the transverse momentum $p_T/n_q$ and the transverse kinetic energy $K E_T / n_q = (m_T - m_0) / n_q$ per quark, with

![Figure 37: The $p_T$ differential $v_3$ of $\pi^\pm$, $\bar{p}$, $K^\pm$ and $\Lambda (\bar{\Lambda})$, gradually moving from the most central (0-5%), upper left to more peripheral (50-60%), bottom middle) collisions up to the intermediate-$p_T$ range. Statistical and systematic uncertainties are added in quadrature and illustrated by the shaded areas.](image-url)
\( m_T = \sqrt{p_T^2 + m_0^2} \) and \( m_0 \) the particle rest-mass. Instead of \( p_T \), the \( KE_T \) variable, introduced in [24], is used for better representation of the NCQ scaling behavior at lower \( p_T \) values.

Overall, the data points illustrate a clear deviation from a universal behavior, evident in the whole centrality range. There exists a window in between \( 1 \lesssim p_T/n_q \lesssim 1.5 \text{ GeV/c} \), where the scaling might hold. In order to examine whether such a statement might be plausible or not, the \( p_T/n_q \) dependence of the scaled coefficient for \( \bar{p} \) (filled blue) is fitted with a higher order polynomial function. The difference of the NCQ-scaled triangular flow \( v_3/n_q \) for the rest of the species is then calculated with respect to \( (v_3^\bar{p}/n_q)_{\text{fit}} \) and presented in Fig.(39).

It is seen that up to the (30-40)\% most central collisions, the NCQ scaling for \( v_3 \) exhibits the least deviation in this narrow \( p_T/n_q \) region, similar to \( v_2 \) (Fig. (35)). Interestingly, such \( p_T \) window corresponds to the region after the hydrodynamical rest-mass hierarchy has been inverted (Fig. (34) and Fig.(37)). This constitutes a relevant momentum region for extracting signs of quark-like degrees of freedom and collectivity at the parton level, owing to a simple coalescence scenario. Unfortunately, for the (40-50)\% and (50-60)\% most central classes, uncertainties do not allow for a conclusive answer.

The extension of the scaling to low transverse momenta in terms of \( KE_T/n_q \) exhibits more subtle differences. Once more, in order to quantify possible deviations in Fig. (40), the transverse kinetic energy dependence of \( v_3/n_q \) for each species is calculated relative to \( (v_3^\bar{p}/n_q)_{\text{fit}} \) and is shown in Fig.(41). The scaling violation is observed all over centrality classes and lower \( KE_T/n_q \lesssim 1-1.2 \text{ GeV/c}^2 \) regions, with the possible exception of the upper \( KE_T/n_q \gtrsim 1.2 \text{ GeV/c}^2 \) part. Uncertainties prohibit from drawing a decisive argument in the most peripheral (50-60)\% class.
Figure 38: The NCQ scaled $\nu_3/n_q$ as a function of the transverse momentum per quark $p_T/n_q$ for $\pi^\pm$, $\bar{p}$, $K^\pm$ and $\Lambda$ ($\bar{\Lambda}$), gradually moving from the most central (0-5 %, upper left) to more peripheral (50-60 %, bottom middle) collisions. Error bars (shaded boxes) represent the statistical (systematic) uncertainties.

Figure 39: The $p_T/n_q$ dependence of the relative $(\nu_3/n_q) - (\nu_3^n/n_q)_n$ difference for $\pi^\pm$, $\bar{p}$, $K^\pm$ and $\Lambda$ ($\bar{\Lambda}$). Error bars (shaded boxes) represent the statistical (systematic) uncertainties.
Figure 40: The NCQ scaled $\nu_3/n_q$ as a function of the transverse kinetic energy per quark $KE_T/n_q$ for $\pi^\pm$, $\bar{p}$, $K^\pm [31]$ and $\Lambda (\bar{\Lambda})$, gradually moving from the most central ((0 – 5) %, upper left) to more peripheral ((50 – 60) %, bottom middle) collisions. Error bars (shaded boxes) represent the statistical (systematic) uncertainties.

Figure 41: The $KE_T/n_q$ dependence of the relative $(\nu_3/n_q) - (\nu_3^p/n_q)_m$ difference for $\pi^\pm$, $\bar{p}$, $K^\pm [31]$ and $\Lambda (\bar{\Lambda})$. Error bars (shaded boxes) represent the statistical (systematic) uncertainties.
Finally, the $v_3$ measurement in lower and intermediate momentum region is complemented by high-$p_T$ results, which are presented in Fig. (42). Charged $\pi^\pm$ and $p$ ($\bar{p}$), along with $\Lambda$ ($\bar{\Lambda}$) and unidentified hadron, $v_3$ for a broad range of transverse momenta is drawn in the (10-50) % range. Noticeably, the finite and positive $v_3$ for $p$ ($\bar{p}$) and $\Lambda$ ($\bar{\Lambda}$) is higher than that of pions out to $p_T \approx 8$ GeV/c, meaning that the particle type dependence persists out to high-$p_T$, a profound effect also revealed in the second order harmonic (Fig. (34)). Hereinafter, the magnitude of the measured $v_3 (p_T)$ for all cases considered here is consistent with zero within the large uncertainties, indicating that the effect of the initial-state fluctuations on the final momentum anisotropies might be different when viewed at low- and intermediate-$p_T$. More specifically, in [26] it was argued that the effect of flow fluctuations in the initial collision geometry extends at least up to $p_T \approx 8$-10 GeV/c, where particle production is dominated by hydrodynamical flow and quark coalescence up to that range, whereas such fluctuations become unimportant in the parton fragmentation dominated regime, i.e. $p_T \gtrsim 10$ GeV/c. Clearly, more $p_T$-differential anisotropic flow data for identified cases are needed before drawing a conclusive answer for this regime.

Figure 42: The $p_T$ differential $v_3$ of unidentified hadrons $h^+h^-$, $\pi^+\pi^-$, $p+\bar{p}$ ([25]) and $\Lambda+\bar{\Lambda}$ for the (10-50) % centrality class over a broad $p_T$ range. For clarity, the black, red and cyan markers are slightly shifted along the horizontal axis at $p_T > 8$ GeV/c. Error bars (shaded boxes) represent the statistical (systematic) uncertainties.
5.2.2 $v_4(p_T)$

Although odd harmonics of order three and above had been overlooked until recently, higher even-order harmonics had garnered more attention (e.g. [27]). In order to further illuminate the detailed dynamical origin of the $v_4$ component, its relation to the initial conditions and the possible effects of viscosity, these studies are being extended, during the current analysis.

The measured $v_4$ as a function of the transverse momentum $p_T$ is given in Fig. (43) up to the 60% most central events, with the centrality classes having been grouped together according to their statistical uncertainty. The experimental data clearly exhibit a sizable $v_4$ component in the particle momentum anisotropy. The presence of $v_4$ is significant over all centrality classes, while it retains quite similar features with the $p_T$ differential evolution of both $v_2$ and $v_3$. Although the centrality ordering for the fourth order harmonic is also seen within uncertainties, the final state flow coefficients seem not to share the same centrality dependence (e.g. [26]). At first sight it appears though that higher order $v_n$ are characterized by a weak centrality dependence, suggesting reduced influence of the average geometry of the overlap region, as might be expected if initial-state fluctuations dominate their values.

![Figure 43: The $p_T$ differential $v_4$ of $\Lambda+\bar{\Lambda}$ over 0.6-8 GeV/c for four centrality classes, clustered according to their statistical uncertainty.](image)

The $p_T$ and centrality evolution of the flow coefficients of order 2-4 is further examined in Fig.(44) and Fig.(45) respectively. Results are shown as a function of $p_T$ in Fig.(44) for the (10-50) % centrality interval. Interestingly, $v_2$ clearly exhibits the strongest $p_T$ variation, since it rises more quickly from the origin both than $v_3$ and $v_4$ for $p_T \lesssim 4$ GeV/c, on one hand. For all harmonics, the qualitative expectation from fluid dynamics in the low-$p_T \lesssim 2$ GeV/c part that $v_n \propto p_T$ (cf. section §D) is well reproduced, as indicated by the dashed fitting lines. On the other hand, $v_3$ and $v_4$ seem to have flattened out already from $p_T \approx 3.5$ GeV/c. For high-$p_T \gtrsim 5.5$ GeV/c though, $v_2$ appears to exhibit a slower decrease than $v_3$, while the current statistics do not allow for a conclusive answer about the high-$p_T$ behavior of $v_4$.

Concerning the higher-order anisotropy $v_4$, it was found to follow equally well the $v_4(p_T) \propto v_2^2(p_T)$ trend at low $p_T \lesssim 2$ GeV/c, which is depicted by the solid green line in Fig.(44) with unity as the coefficient. Although the ideal fluid picture generally predicts $v_4(p_T) \propto 1/2v_2^2(p_T)$ (cf. section §D), the apparent discrepancy with the extracted scaling\footnote{Note that discrepancies with predictions from ideal fluid dynamics have been already reported both at RHIC} has not been attributed

\begin{align*}
5.2.2 &\quad v_4(p_T) \\
\text{Although odd harmonics of order three and above had been overlooked until recently, higher even-order harmonics had garnered more attention (e.g. [27]). In order to further illuminate the detailed dynamical origin of the } v_4 \text{ component, its relation to the initial conditions and the possible effects of viscosity, these studies are being extended, during the current analysis.} \\
\text{The measured } v_4 \text{ as a function of the transverse momentum } p_T \text{ is given in Fig. (43) up to the 60\% most central events, with the centrality classes having been grouped together according to their statistical uncertainty. The experimental data clearly exhibit a sizable } v_4 \text{ component in the particle momentum anisotropy. The presence of } v_4 \text{ is significant over all centrality classes, while it retains quite similar features with the } p_T \text{ differential evolution of both } v_2 \text{ and } v_3. \text{ Although the centrality ordering for the fourth order harmonic is also seen within uncertainties, the final state flow coefficients seem not to share the same centrality dependence (e.g. [26]). At first sight it appears though that higher order } v_n \text{ are characterized by a weak centrality dependence, suggesting reduced influence of the average geometry of the overlap region, as might be expected if initial-state fluctuations dominate their values.} \\
\end{align*}
to the breakdown of ideal hydrodynamics, but rather it was explained to originate from elliptic flow fluctuations \cite{37}. This suggests that the centrality dependence of $\nu_4$ could be proven as a sensitive probe of the underlying mechanism of flow fluctuations. However, it seems that $\nu_4$ receives strong contribution from $\nu_2$, meaning that such mode-coupling (cf. section §B.1) renders $\nu_4$ less promising candidate than $\nu_3$ for systematic studies of the QGP viscous effects.

A clear splitting for the $\nu_n$ Fourier coefficients is registered both in central and semi-central collisions (left and right panel in Fig.\(\text{(45)}\) respectively), even though for the former case such a splitting seems not to follow the same ordering, when compared to more peripheral classes. As previously underlined, this is attributable to pure fluctuations being the dominant source of spatial anisotropies for both even and odd geometrical moments in central collisions. Overall, recovering the full geometry of the initial state from measuring higher flow moments seems quite challenging for $n \geq 5$ (e.g. \cite{28}), as Fig. \(\text{(44)}\) delineates, where $\nu_5$ for $\Lambda$ ($\bar{\Lambda}$) is also shown with its value having measured close to, but different than zero.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure44.png}
\caption{The $p_T$ differential evolution for $\nu_2$-$\nu_5$ of $\Lambda$ ($\bar{\Lambda}$) over 0.6-10 GeV/c for the (10-50) \% centrality interval. Dashed lines represent first order polynomial fitting results, while the solid line simply illustrates squared elliptic flow values, i.e. $\nu_2^2$ ($p_T$) for the (10-50) \% centrality interval.}
\end{figure}

and at LHC (e.g. \cite{36}).
Figure 45: The $p_T$ differential evolution for $v_2$, $v_3$ and $v_4$ of $\Lambda$ ($\bar{\Lambda}$) over 0.6-6 GeV/c separately for (0-5) % (left panel) and (20-30) % (right panel) most central collisions. Note that the $v_2$ and $v_3$ values for $p_T \sim 5.5$ GeV/c in the left panel have been shifted simply for visibility.

Finally, since $v_n$ have the ability to provide additional information on the transport properties of the fireball, the hydrodynamical origin of the final fourth order momentum anisotropy is examined in the left panel of Fig.(46) for the (10-20) %. The $p_T$-differential $v_4$ for $\Lambda$ ($\bar{\Lambda}$) is compared to results measured for $\bar{p}$, $\pi^\pm$, $K^\pm$. Much interestingly, the same mass splitting pattern, extracted both in elliptic and triangular flow, is clearly observed in rectangular flow, as well. Once more, violations for the NCQ scaling of $v_4$ are seen (right, upper panel in Fig.(46)) and found least broken within the narrow momentum region $1 \lesssim p_T/n_q \lesssim 1.5$ GeV/c (right, bottom panel in Fig.(46)).

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2The experiment data for $\bar{p}$, $\pi^\pm$, $K^\pm$ are under internal review, thus not yet published.
Figure 46: (left) The \( p_T \) differential \( v_4 \) of \( \pi^\pm, p (\bar{p}), K^\pm \) and \( \Lambda (\bar{\Lambda}) \) up to the intermediate-\( p_T \leq 6 \) GeV/c range within the (10–20) % centrality class. (right, upper panel) The NCQ scaled \( v_4/n_q \) as a function of the transverse momentum per quark \( p_T/n_q \). (right, bottom panel) The \( p_T/n_q \) dependence of the relative \( (v_4/n_q - (v_4^\pm/n_q)) \). Results for \( \pi^\pm, p (\bar{p}), K^\pm \) under internal review.
References


6 Summary

In summary, azimuthally asymmetric flow measurements of order 2-4 as a function of transverse momentum \( p_T \) in the (0-60)\% most central PbPb collisions at \( \sqrt{s_{NN}} = 2.76 \) TeV have been performed for the case of \( \Lambda + \bar{\Lambda} \) hyperons and compared to various species. The employed method involved the Scalar Product technique, which consistently quantifies the root-mean-square \( \sqrt{\langle v_n^2 \rangle} \) of the correlation between the identified hadrons under study and the reference-flow particles. Anisotropic flow induces genuine azimuthal correlations between the produced particles with respect to the common symmetry planes. Residual short-range correlations, such as correlations from resonance or cascade-like decays and jets, are removed or diminished during the actual analysis by imposing a pseudorapidity gap between particles of interest and reference-flow particles. More specifically, \( \Lambda \) (\( \bar{\Lambda} \)) are reconstructed in the central barrel of the ALICE detector, whereas unidentified particles were recorded in the forward region, hence giving an almost one unit of separation in pseudorapidity. Still, it should be kept in mind that correlation methods are sensitive to initial-state fluctuations, i.e. two-particle correlation techniques are expected to produce biased (higher) values with respect to expectations without initial-state fluctuations.

All three anisotropy coefficients \( v_2 \), \( v_3 \) and \( v_4 \) have been observed to increase with \( p_T \) up to about 3.5 GeV/c, beyond which they saturate and finally decrease. The same qualitative behavior is present in their centrality evolution, hinting at the scenario that final-state anisotropic flow coefficients are driven by initial-state eccentricity \( \varepsilon_n \) moments. However, the detailed behavior seems different for different harmonics. The second-order anisotropy constitutes the response of the average almond-shaped deformation of the nuclear overlap zone in non-central collisions, whereas \( v_3 \) and \( v_4 \) are dominated by event-by-event fluctuations in the initial geometry. That could explain the reason why \( v_2 \) is the leading harmonic for the majority of centralities, due to smaller values of fluctuation-dominated as compared to geometry-driven eccentricities. In most central collisions and at the intermediate-\( p_T \) region though, \( v_3 \) surpasses \( v_2 \), owing to the similar eccentricity values that are mainly driven by fluctuations. The origin of \( v_4 \) is more complicated, since it seems to arise from both \( \varepsilon_4 \) and a mixing with the lower-order harmonic \( \varepsilon_2 \).

Similar to previously well-established experimental observations for \( v_2 \), the higher order harmonics \( v_3 \) and \( v_4 \) show particle-mass ordering at low \( p_T \) and baryon-meson difference at intermediate \( p_T \). More specifically, lighter particles exhibit larger \( v_n \) than heavier particles for transverse momentum approximately below 2 GeV/c. This hierarchy is reversed at intermediate \( p_T \), where baryons show larger \( v_n \) than mesons, a pattern which persists over all centralities and up to about 8 GeV/c. Although hydrodynamical calculations are believed to reproduce, at least qualitatively, the mass-ordering \( v_n \) dependence in the low-\( p_T \) region, they would fail to reproduce the baryonic flow dominance above 2 GeV/c. Deviations from ideal fluidity is believed to occur generally as a result of viscous effects that tend to smooth the flow distributions at slight higher \( p_T \gtrsim 1.5-2 \) GeV/c, although viscosity alone cannot explain the inversion of the rest-mass hierarchy for \( v_n \). On the other hand, the ordering of the final-state harmonics are in line with viscous effects, since they weaken the correlation between \( \varepsilon_n \) and \( v_n \) with increasing sensitivity on higher \( n \).

As an alternative, the quark-coalescence scenario has been investigated, since such hadronization process has attracted a lot of attention at lower collision energies by having succeeded to correctly reproduce, within the relevant model uncertainties, the scaling of \( v_n \) with the number of constituent quarks, customarily known as the NCQ scaling. Quark-coalescence models make a definite prediction for a simple scaling behavior between the final-state flow coefficients for mesons and for (anti-)baryons. Within the coalescence picture anisotropic flow of baryons is expected around \( 3/2 \) times larger than that of mesons, as a result of the final-state
hadrons having largely inherited from the anisotropy of quarks in a preceding quark-matter phase. According to the experimental findings of the present study, there exists a tantalizing evidence that the NCQ scaling barely holds at LHC energies. It seems that the conjectured anisotropy mechanism, which simply amplifies approximately by a factor two the $v_n$ for mesons and by a factor of three that for baryons, is not enough. Clearly, further measurements involving more identified species will be required to discriminate among alternative scenarios for the origin of transverse anisotropic flow at the intermediate-$p_T$ region, on one hand. On the other hand, understanding the limits of the recombination domain is important in relation to viscous hydrodynamics and the extraction of the $\eta/s$ ratio, as well as for developing a unified approach in describing energy loss in the high-$p_T$ region.

Last, the high-$p_T \gtrsim 8-10$ GeV/c behavior is more subtle for different Fourier harmonics. Particles with high transverse momentum do not follow the collective dynamics, but instead radiate more or less energy, depending on how much material they traverse on their way out of the fireball. As a result, these particles also acquire similar azimuthal anisotropy in their momentum distributions, giving rise to non-negative $v_2$ also at such high-$p_T$ regime. Elliptic flow is said then to be driven by a path-length-dependent energy loss of partons inside the coloured medium. The current statistics do not allow for unambiguously resolving the high-$p_T$ behavior of higher harmonics.
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A Harmonic flow coefficients

Since the azimuthal part of the measured, typically momentum, distribution of hadrons is a periodic quantity, it is natural to decompose it in a Fourier series (\[12\] in section §4, or

\[
\frac{dN}{ptdpdyd\phi} = \frac{dN}{ptdpdy} \left\{ \frac{x_0}{2\pi} + \frac{1}{\pi} \sum_{n=1}^{\infty} \left[ x_n \cos(n\phi) + y_n \sin(n\phi) \right] \right\}
\]

\[
= \frac{dN}{ptdpdy} \left\{ \frac{x_0}{2\pi} + \frac{1}{2\pi} \sum_{n=1}^{\infty} \left[ (x_n - iy_n) e^{in\phi} + (x_n + iy_n) e^{-in\phi} \right] \right\} , \quad (A.1)
\]

where the total yield of particles \(N\) is obtained via the triple integration over transverse momentum \(pt\), rapidity \(y\) and azimuthal angle \(\phi\).

In turn, \(x_n\) and \(y_n\) are the Fourier coefficients, namely integrals of \(dN/d\phi\) with weights proportional to \(\cos (n\phi)\) and \(\sin (n\phi)\),

\[
x_n = \frac{\int_0^{2\pi} d\phi \cos(n\phi) d\phi}{\int_0^{2\pi} d\phi d\phi} = \langle \cos(n\phi) \rangle
\]

\[
y_n = \frac{\int_0^{2\pi} d\phi \sin(n\phi) d\phi}{\int_0^{2\pi} d\phi d\phi} = \langle \sin(n\phi) \rangle ,
\]

while for the second row in Eq. A.1 the complex notation

\[
\cos (n\phi) = \frac{1}{2} \left( e^{in\phi} + e^{-in\phi} \right)
\]

\[
\sin (n\phi) = \frac{1}{2i} \left( e^{in\phi} - e^{-in\phi} \right)
\]

(A.3)

has been used. In case of finite number of particles, the integrals in Eq. A.2 simply degenerate into summation over each particle \(i\) found in the specific rapidity window \(dy\), or

\[
x_n = \sum_i \cos(n\phi_i)
\]

\[
y_n = \sum_i \sin(n\phi_i) \quad (A.4)
\]

For simplicity, each non zero \( (x_n, y_n) \) pair is combined and quantifies the so-called \(n\)th order harmonic term \(v_n\)

\[
x_n = |v_n| \cos (n\Psi_n)
\]

\[
y_n = |v_n| \sin (n\Psi_n) ,
\]

(A.5)

and

\[
|v_n| = \sqrt{x_n^2 + y_n^2} , \quad (A.6)
\]

or compactly

\[
v_n = |v_n| e^{-in\Psi_n} . \quad (A.7)
\]
The last relation introduces the characterization of the \( n \)th order anisotropy by its magnitude \( |\nu_n| \) and phase \( \Psi_n \). The angles

\[
\Psi_n \overset{(A.3)}{=} \tan^{-1} \left( \frac{x_n}{y_n} \right) / n \quad \text{and} \quad \nu_n \overset{(A.2)}{=} \tan^{-1} \left( \frac{(\sin(n\phi))}{(\cos(n\phi))} \right) / n ,
\]

are commonly referred to as the \( n \)th order symmetry planes (Fig. (47)). The picture underlying anisotropic flow measurements is completed by reforming Eq. A.1 in terms of \( \nu_n \); isolating the azimuthal part of the invariant distribution, the Fourier expansion gives

\[
dN \over d\phi = \frac{\nu_0}{2\pi} + \frac{1}{2\pi} \sum_{n=1}^{\infty} |\nu_n| \left[ e^{i\phi} + e^{-i\phi} \right].
\]

(A.9)

Since \( dN/d\phi \in \mathbb{R} \), the imaginary part in Eq. (A.9) is dropped, while the remaining real part writes

\[
dN \over d\phi = \frac{\nu_0}{2\pi} + \frac{1}{\pi} \sum_{n=1}^{\infty} |\nu_n| \cdot \cos \left( n (\phi - \Psi_n) \right).
\]

(A.10)

A.1 Combining practical details

A series of brief comments are welcomed at that point, in order to further illuminate how the momentum anisotropy is systematically studied by decomposing the invariant distribution. Since the Fourier expansion is performed over the triple-differential distribution \( dN/p_T/\over dy/d\phi \), the combination of the Fourier coefficients in Eq. (A.7) is a function of both transverse momentum and rapidity. At variance with the \( p_T \) - and \( y \) -differential (e.g. [1]) dependence, the momentum and rapidity integrated invariant distribution could be also expanded, hence giving rise to integrated \( \nu_n \) values (Fig. (6)). During the actual study, only the \( p_T \) dependence of the observed anisotropy is measured.

Furthermore, Eq. (A.10) contains the implicit assumption that the \( n \)th order reference angle \( \Psi_n \) is a global quantity that depends little on transverse momentum and rapidity [2]. Such an assumption is equivalent to introducing a plane of symmetry in the collision zone, e.g. spanned by the impact parameter (\( x \)-axis) and the beam (\( z \)-axis), which in literature is dubbed the reaction plane \( \Psi_{RP} \) (Fig. (47)). Therefore, the particle distribution is traditionally expanded either in terms of cosines and phases \( \Psi_n \) or leaving the harmonics, i.e. cosines and sines, unaltered. For smooth initial matter distribution in the colliding nuclei \( \Psi_{RP} \) coincides with \( \Psi_n \) and due to the spatial symmetry in the spherical nuclei with respect to the \( y \)-axis (N.B. in the absence of fluctuations), \( \sin(n\phi) + \sin[n(-\phi)] = \langle \sin(n\phi) \rangle = 0 \), thus making both means of expansion equivalent. As merely abstract notions, directions of neither the reaction nor symmetry planes are \textit{a priori} known.

In order to analyze the experimental azimuthal distribution \( dN/d\phi \rightarrow dN_{ch}/d\phi \) in Eq. (A.1), what needs to be empirically determined is either the reaction plane angle \( \Psi_{RP} \), i.e. the axis of the collision symmetry, or the reference angle \( \Psi_n \), i.e. the phase of \( \nu_n \) and direction of the greatest azimuthal density of outgoing particles. During the course of the present analysis, the fact that various experimental estimates of \( \nu_n \) have compact expressions, in terms of the flow vector \( \vec{Q}_n \) (cf. section §4.3), is being exploited. For an ideal system with infinite multiplicity, \( \vec{Q}_n \) would be aligned to \( \Psi_{RP} \), or respectively \( \Psi_n \), thus azimuthal distributions could be

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measured relative to the $\vec{Q}_n$ angle, the later colloquially known as the event-plane angle \[10\] in section §4.

In an actual experiment though, the multiplicity is finite, bearing two major effects. First, when the azimuthal angle of a particle is measured with respect to $\vec{Q}_n$, there exists a trivial autocorrelation in case that the same particle has been used for constructing the $\vec{Q}_n$ angle \[3\]. Given a recorded collision event, this trivial effect can be avoided by splitting the whole event sample in sub-samples, e.g. constructing the $\vec{Q}_n$ vector and subsequently measuring correlations with respect to it over separated phase-space regions (cf. section §4.3). For instance, rapidity gaps could be employed, diminishing in parallel sources of correlations, other than flow, concisely described as nonflow effects \[4\]. Strong resonance decays constitute an obvious paradigm, for their decay products azimuthal angles being highly correlated.

Moreover, because of statistical fluctuations, deviations $\Psi_{RP(n)} - \Psi_{\vec{Q}_n}$ between the true reaction, or equivalently $n^{th}$ order symmetry, plane and $\vec{Q}_n$ should be expected. In principle, each and every particle alone could serve as a $\vec{Q}_n$ vector. However, that would not be practical, since the more particles used to define the event plane the closer its orientation to that of $\Psi_{RP}$ or $\Psi_n$. The later assumption highlights the need both for a resolution factor $\mathcal{R}_n(b) = \left\langle \cos n (\Psi_{RP(n)} - \Psi_{\vec{Q}_n}) \right\rangle$ \[5\], which compensates for the finite multiplicity, and usage of multiple events. But at the same time, it raises consiously about the limitation of averaging over a large number of events, having assumed the very same underlying flow distribution. Overall, the measured anisotropies are always smaller than the true ones (in the absence of flow fluctuations), since they are smeared by the error $\Psi_{RP(n)} - \Psi_{\vec{Q}_n}$, with increasing sensitivity to higher harmonics \[5\].

### B Initial-state parameterizations

The Fourier expansion for the single particle distribution in Eq. A.1 could be alternatively written, in terms of $\varepsilon_n$, 

$$\frac{dN}{d\phi} = \frac{1}{2\pi} \left( 1 + 2 \sum_{n=1}^{\infty} \varepsilon_n \cos n (\phi - \Phi_n) \right),$$ \ \ (B.1)

meaning that eccentricity moments

$$\varepsilon_n e^{in\Phi_n} = \frac{\int d\mathbf{x}_\perp e^{in\phi} \omega(\mathbf{x}_\perp) \rho_s(\mathbf{x}_\perp)}{\int d\mathbf{x}_\perp \omega(\mathbf{x}_\perp) \rho_s(\mathbf{x}_\perp)}$$ \ \ (B.2)

could be characterized by their magnitudes and phases

$$\Phi_n = \frac{1}{n} \tan^{-1} \left( \frac{\int d\mathbf{x}_\perp \sin (n\phi) \omega(\mathbf{x}_\perp) \rho_s(\mathbf{x}_\perp)}{\int d\mathbf{x}_\perp \cos (n\phi) \omega(\mathbf{x}_\perp) \rho_s(\mathbf{x}_\perp)} \right).$$ \ \ (B.3)

Hence, the idea for modeling the initial transverse profiles, served as inputs in hydrodynamic simulations, becomes evident, since a numerical tractable means of calculating correlations between initial-state eccentricity moments $\varepsilon_n$ (and their associated deformation angles $\Phi_n$) with final state flow anisotropies $\upsilon_n$ (and their associated flow angles $\Psi_n$) is secured. It should be noted though that Eq. (B.2) and (B.3) are not unique, in the sense that several alternatives for initial-state eccentricities are present in the literature (e.g. \[6\]). Likewise, this is reflected in the presence of the weighting factors $\omega(\mathbf{x}_\perp)$, usually taken either as $\omega(\mathbf{x}_\perp) = r^2$ or $\omega(\mathbf{x}_\perp) = r^n$, with the later promoting increased sensitivity to the outer regions of the transverse density distributions (e.g. \[7\]). So far, the most commonly employed techniques
for generating such initial transverse profiles of emission sources \( \rho_s(x_\perp) \) are the Monte Carlo versions of the Glauber (MCG) and the factorized Kharzeev-Levin-Nardi (MC-fKLN) models (cf. section §B.2).

In order to avoid misapprehensions, the nomenclature event (momentum) plane (MEP) is exclusively meant for the calculated, or reconstructed in experiments, plane from the momentum distribution of the produced particles. At variance with the event plane, the spatial or participant plane (SEP) in Eq. (B.3) defines the direction of the steepest density gradient, as the presence of the minus sign \( -1 = e^{-i\pi} \) indicates (Fig. (47), right); for instance, for an elliptically deformed profile, the participant plane coincides with the minor axis of the ellipse, whereas it points to the sides for a squared-shaped profile. Parenthetically, the label participant is motivated by the fact that the initial density distribution reflects the transverse distribution \( \rho_s(x_\perp) \) of nucleons participating in the particle production process.

Figure 47: (left) Shapes of the initial elliptic, triangular and quadrangular deformations, along with their related, same order, eccentricities \( \varepsilon_n = |\vec{\varepsilon}_n|, \ n = 2, 3, 4 \) and their associated directions (symmetry planes). Dashed lines represent hard sphere radii of nuclei. (right) The transverse \( xy \)-plane for a simulated collision event; circles represent nucleons from two colliding nuclei, with the shaded ones indicative of the participating nucleons. For the given collision event, directions of the reaction (RP), and the second order spatial (SEP, \( \Phi_n, \ n = 2 \)) and momentum (MEP, \( \Psi_2, \ n = 2 \)) event plane are also given. Authors in \cite{23} (cf. section §5) have introduced by hand an additional rotation \( \Phi_n \rightarrow \Phi_n + \pi/n \), thus \( \Phi_2 \) and \( \Psi_2 \) are at odd angles. Event-by-event fluctuations result in \( \Phi_2 \) and \( \Psi_2 \) not being always aligned to the \( y \)- and \( x \)-direction respectively.
B.1 Single shot versus EbyE hydrodynamic input

Figure 48: Event-by-event correlation of the spatial or participant plane (PP, a) and event plane (EP, b) angles with the reaction plane (RP), as well as the correlation between participant and event plane angles (c), up to the fifth order \( v_n \) and \( \varepsilon_n \) harmonics, the latter weighted by the initial energy density, i.e. \( \rho_s (x_\perp) = \varepsilon (x_\perp) \) in (B.2). Initial profiles are generated based on the MC-fKLN model [6].

Until recently, initial-state calculations and thus their matching to the hydrodynamic expansion were performed using averaged initial density profiles, the later obtained from MCG calculations, for instance. However, with fluid dynamics simulations using averaged initial states, there is a well-known deficit of \( v_2 \) in central collisions. Generally, the explanation for the deficit has been thought to be the initial state density fluctuations, which had not been accounted for. Taking into account density fluctuations requires then careful handling of the reference plane with respect to which \( v_2 \), and in general \( v_n \), are computed; final-state flow coefficients are computed relative to the initial reaction plane or in the best case relative to the participant plane (Fig. (47)), but not relative to the event plane [37]. It becomes obvious that without event-by-event simulations, it is impossible to determine how closely the computational extracted participant plane corresponds to the physical measured event plane. In the following, it is argued that the participant plane seems to be a good approximation for the event plane.

Relevant to the present study results are exhibited in Fig. (48), where mixed-centrality event-by-event initial-state profiles are simulated for the first four nontrivial \( \varepsilon_n \), \( n = 2-5 \) harmonics (Fig. (48), left). Complementary, final state shapes, as quantified correspondingly by the same order flow coefficients \( v_n \), \( n = 2-5 \) (Fig. (48), middle), were preferred to be discussed in the current section, since they contain crucial information about the geometry driven hydrodynamic flow (cf. section §D). More specifically, the comprehensive analysis in [6] revealed both for \( \Psi_{2PP} \) and \( \Psi_{2EP} \) a strong correlation with the reaction plane, consistent with expectations for \( \varepsilon_2 \) being controlled by the almond-shape overlap geometry in non central collisions (Fig. (7)), on one hand, and for \( v_2 \) mostly reflecting a linear response to this geometric deformation, on the other hand. In contrast, \( \Psi_{3PP,35} \) seems completely uncorrelated to the reaction plane, indicating that the third order eccentricity is a result of mere fluctuations. The basic response of \( v_3 \) is predominately driven by its initial counterpart \( \varepsilon_3 \), for \( \Psi_{3PP} \) and \( \Psi_{3EP} \) being strongly correlated, with an approximately linear relation between \( v_3 \) and \( \varepsilon_3 \).

The behavior of \( \Psi_{4PP} \) is more subtle, though \( \Psi_{4EP} \) on average points into the reaction plane. In principle, \( \Psi_{4PP} \) was found anti-correlated with the reaction plane, which was assigned to the positive correlation between the ellipticity \( \varepsilon_2 \) and quadrangularity \( \varepsilon_4 \), intuitively meaning that the initial quadrangular component is rather oriented like a diamond. In other words, the correlation of \( \Psi_{4EP} \) with the reaction plane is weaker than the anti-correlation of \( \Psi_{4PP} \) with that.
plane. This suggests that quadrangular flow does not develop predominantly in the direction of the steepest density gradient associated to $\varepsilon_4$, but in the direction of the $\varepsilon_2$-related density. However, such a statement seems at first sight in conflict with the result depicted in the furthest right panel of Fig. (48), where a correlation peak at zero relative angle between $\Psi_{4E}^P$ and $\Psi_{4P}^P$ was found.

Once more, it should be noted that the calculations above referred to mixed-centrality event classes, thus the resolution of this apparent paradox is achieved by inspecting the centrality dependence of the mode mixing. In deed, it was shown that the planes are correlated to each other in central collisions, whereas they become essentially uncorrelated in mid-central and anti-correlated in peripheral collisions. Conclusively, $v_4$ seems on average to exhibit poor correlation with $\varepsilon_4$, except for very central collisions, where all eccentricity harmonics are fluctuation driven. For larger impact parameters, it seems that the strong $\cos(2\phi)$ collective flow component gives rise to $v_4$, without the need for non zero $\varepsilon_4$, namely the $\varepsilon_2$-induced quadrangular flow receives strong contribution from a purely elliptic deformation of the overlap region. The same mode coupling holds additionally for $v_5$. Further studies quantified the extent to which $v_4$ and $v_5$ arise from nonlinearities of the medium response (e.g. [8]).

B.2 MCG and MC-fKLN initialization

Although qualitative similar, different predictions for the magnitude of eccentricities are estimated by the two primary employed models, owing to the model dependent nature of fluctuations. Without delving into minute details, $\rho_s(x_\perp)$ in Eq. (B.3) is a function characteristic of the initial entropy or energy density production, whose distributions control the input for pressure gradients that subsequently drive the evolution of the transverse flow and its asymmetries. In its simplest form, $\rho_s(x_\perp)$ equals to the number of nucleons that partake in the collision $N_{\text{part}}(x_\perp)$,

$$N_{\text{part}}(x_\perp) = T\left(x + \frac{b}{2}, y\right) \left\{ 1 - \left[ 1 - \sigma_{\text{NN}} T\left(x - \frac{b}{2}, y\right) \right]^A \right\}$$

$$+ T\left(x - \frac{b}{2}, y\right) \left\{ 1 - \left[ 1 - \sigma_{\text{NN}} T\left(x + \frac{b}{2}, y\right) \right]^A \right\} , \quad A \equiv 208 \text{ Pb}, \quad \text{(B.4)}$$

with $N_{\text{part}}$ being calculated in the realm of the MCG model [9]. The notion of the overlap function $T = \int dz \rho(r)$, as estimated based on an azimuthally isotropic and smoothly varying with the nucleus radius $R_A$ matter distribution $\rho(r) \propto (1 + e^{-R_A r})^{-1}$, is used, whereas the additional model parameter $\sigma_{\text{NN}}$ regulates the nucleon-nucleon cross section (e.g. [10]). In case of referring either to energy or entropy density weight, parameterizations are easily obtained within the same MCG approach, based on combination of the spatial coordinates of participation nucleons and binary nucleon-nucleon collisions $N_{\text{coll}}$,

$$\rho_s(x_\perp) \propto \frac{1}{\tau_0} \left[ f N_{\text{part}}(x_\perp) + (1 - f) N_{\text{coll}}(x_\perp) \right] , \quad \text{(B.5)}$$

where $f$ is constrained by multiplicity measurements, and $N_{\text{coll}}$, associated with the minor hard component $(1 - f)$, writes

$$N_{\text{coll}} = \sigma_{\text{NN}} T\left(x + \frac{b}{2}, y\right) T\left(x - \frac{b}{2}, y\right) \quad \text{(B.6)}$$

MC-fKLN initial conditions [11] rely on the physical argument that gluon production saturates at high energies, and prior to QGP, a new form of matter is present, dubbed Color Glass
Condensate (CGC); color, because it is composed of coloured particles, while it evolves on time scales long enough compared to microphysics time scales and therefore has properties similar to glasses, and a condensate, since the phase-space density of gluons is very high. More specifically, the factorized KLN model, a modified form of the original KLN approach, handles more naturally the edge regions of a nucleus, where the density is small and it is not actually expected to find a nucleon at that position. In that sense, the low-density surfaces of the overlap zone may not thermalize and should not be included in the initial condition for hydrodynamics. Pertaining to the saturation scale \( Q_s^2(x, N_{\text{part}}) \), it was shown that the \( Q_s^2(x, N_{\text{part}}) \sim N_{\text{part}} \) definition is in fact a good approximation for computing the multiplicity of gluons, namely

\[
Q_s^2(x, N_{\text{part}}) \propto N_{\text{part}} \left( \frac{0.01}{x_{1,2}} \right)^\lambda \gg \Lambda_{\text{QCD}} ,
\]  

where \( x_{1,2} = \frac{p_T}{\sqrt{s}} e^{\pm y} \) are the (light-cone) momentum fractions of the colliding gluons and the form \( Q_s^2(x, N_{\text{part}}) \sim x^{-\lambda} \) is motivated from deep inelastic scattering experiments, with \( \lambda \approx 0.2-0.3 \) (e.g. [12]). Correspondingly, \( N_{\text{part}} \) can be evaluated in the realm of the MCG model. The \( \rho_s(x, \perp) \) weighting factor gives in the KLN approach

\[
\rho_s(x, \perp) \propto \frac{1}{\tau_0} \frac{d^3N_g}{dxdy} \bigg|_{y=0} ,
\]

and the distribution of gluons at each transverse coordinate \( x, \perp \) produced with rapidity \( y \) is calculated by the \( k_T \)-factorization formula [13],

\[
\frac{d^3N_g}{dxdy} \propto \frac{\alpha_s}{C_F p_T^2} \int d^2k_T \times \phi(x_1, k_T) \times \phi'(x_2, p_T - k_T)
\]

where \( k_T \) amounts to the transverse momentum of the incoming gluons. As for the rest of the notations, \( \alpha_s = g^2/(4\pi) \) corresponds to the strong coupling constant and \( C_F = (N^2 - 1)/(2N_C) \), \( N_C = 3 \), is a SU(3) invariant, while \( \phi, \phi' \) are the employed unintegrated gluon distribution functions.

C  Relativistic Hydrodynamics; an overview

Formally, it is convenient to perform a tensor decomposition both for \( T^{\mu\nu} \) and \( j_i^{\mu} \) (Eq. [2] and [3]) in section §2.2.2 with respect to an arbitrary 4-vector \( u^{\mu} \), the latter usually specified in the fluid rest frame \( F^* \) and physically conceived as the flow of energy (Landau frame) [14]. In that case, the flow velocity is the normalized \( u^{\mu}u_{\nu} = 1 \) time-like eigenvector of \( T^{\mu\nu} \), \( T^{\mu\nu}u_\nu = e\!u\!^{\mu} \), whose eigenvalue equals to the energy density \( e \). Then, the tensor decomposition reads

\[
T^{\mu\nu} = T^{\mu\nu} + \delta T^{\mu\nu} = (e\!u\!^{\mu}u_{\nu} - p_0\Delta^{\mu\nu}) + (-\Pi\Delta^{\mu\nu} + \pi^{\mu\nu})
\]

\[
j_i^{\mu} = j_i^{\mu,0} + \delta j_i^{\mu} = n_i\!u\!^{\mu} + V_i^{\mu} ,
\]

where small deviations from local equilibrium could be approximated by the viscous corrections, \( \delta T^{\mu\nu} \) and \( \delta j_i^{\mu} \) respectively. For the notation of Eq. (C.1), the local 3-metric \( \Delta^{\mu\nu} = g^{\mu\nu} - u^{\mu}u_{\nu} = \text{diag}(0, -1, -1, -1) \) is used. The isotropic pressure \( p = -\frac{1}{3}\Delta_{\mu\nu}T^{\mu\nu} \), namely the space-like component of \( T^{\mu\nu} \), does not necessarily coincide with the equilibrium pressure \( p_0 \). Such a difference is parameterized by the bulk viscous pressure \( \Pi = -\frac{1}{3}\Delta_{\mu\nu}T^{\mu\nu} - p_0 \), whereas the shear stress tensor \( \pi^{\mu\nu} \) is also introduced in the non-equilibrium part of Eq. (C.1). The net flow of charge \( V_i^{\mu} = \Delta^{\mu\nu}j_i^{\mu} \) completes the decomposition of the current \( j_i^{\mu} \).
In the case of inviscid hydrodynamics, the energy-momentum tensor $T^{\mu\nu}$ could be finest understood in the fluid local rest frame, a frame $F^*$ in which only the time-like component in the 4-velocity $u^\mu = dx^\mu/d\tau$ of the system constituents, intuitively viewed as fluid elements, is not zero. For the $T^{00}$ component, according to the $T^{\mu\nu}$ definition,

$$T^{00} = \frac{dp}{dx^1 dx^2 dx^3} = \frac{dE}{dx^1 dx^2 dx^3} = e ,$$

meaning $T^{00}$ describes the energy $E$ (momentum in the 0th dimension) per unit volume ($dx dy dy$ surface area perpendicular to the 0th direction), namely the energy density $e$. Analogously, in the very same frame,

$$T^{11} = \frac{dp}{dx^0 dx^2 dx^3} = p ,$$

where the force $dp/dx^0, x^0 = \tau$ on the 1st direction per unit area in the $\{2,3\}$ direction simply equals to the pressure $p$ in the 1st direction acting on a $dy dy$ surface area. Therefore, in the $F^*$ frame $T^{\mu\nu}$ collapses to

$$T^0_{\mu\nu} = \begin{cases} e & \mu, \nu = 0, 0 \\ p\delta^{\mu\nu} & \mu, \nu = 1, 2, 3 \end{cases} ,$$

which indicates that the energy density and pressure are the same in every space-time point as the system evolves. In any other frame $F$ with an arbitrary fluid velocity $u^\mu$, $T^{\mu\nu}$ is obtained by a transformation of $T^0_{\mu\nu}$, reading (15)

$$T^{\mu\nu} = \Lambda^\mu_\alpha \Lambda^\nu_\beta T^{\alpha\beta}_0 ,$$

or

$$T^{\mu\nu} = (e + p) u^\mu u^\nu - g^{\mu\nu} p ,$$

with $g^{\mu\nu}$ the Minkowski metric tensor.

Again, for inviscid hydrodynamics, the number of $10 + 4N$ independent variables reduces to 6; one each for the energy density and pressure, three independent components in the flow velocity, while the last unknown variable accounts for the net baryon density $n_i u^0$. The baryon flux $n_i \vec{u}$, $\vec{u} = (u^1 i, u^2 j, u^3 k)$ vanishes in ideal hydrodynamics, since local equilibrium implies isotropic properties (Eq. C.5); if a current were to be developed, a direction in space would be defined, hence having spoiled isotropy. In relativistic viscous hydrodynamics though, in which the strict assumption of local equilibrium is relaxed to near local equilibrium, the baryon flux might not be zero; such a transport phenomenon is called diffusion.

For a near equilibrium state being described, the space of thermodynamic variables has to be extended relative to the case of ideal hydrodynamics, meaning that $1 + 3 + 5 = 9$ additional quantities are introduced; $\Pi, V^\mu$ (net baryon flow) and $\pi^{\mu\nu}$ respectively. Although the theory of dissipative relativistic fluid dynamics has been formulated quite early (16), first order in $\delta T^{\mu\nu}$ and $\delta j^\mu_i$ deviations theory suffers from unphysical superluminal signals, leading to numerical instabilities. The problem of causality is removed in the Israel-Stewart second order theory (17) at the expense of rendering the dissipative fluxes dynamical, whose kinetic equations of motion need to be solved simultaneously with the hydrodynamic evolution of Eq. (2) and (3) in section 2.2.2 (e.g. (18)). The dynamical equations that control the three dissipative flows are called the relaxation equations, for introducing non zero relaxation times, the

\[ \text{Like } T^{\mu\nu}, \text{ the shear tensor } \pi^{\mu\nu} \text{ has } 10 \text{ components, though only } 6 \text{ are independent on the grounds of the } \pi^{\mu\nu} u_\nu = 0 \text{ orthogonality; if the additional traceless condition } \pi^{\mu\mu} = 0 \text{ is imposed, the number of independent components is reduced to } 5. \]
later related to the respective dissipative coefficient; for instance the shear stress relaxation time \( \tau_\pi \propto \eta \) with \( \eta \) the shear viscosity\(^4\).

The temperature dependence of \( \eta/s \), as expected both by pQCD and lattice QCD \(^{[10]}\) in section §5, is usually not considered during calculations (e.g. \(^{[25]}\) in section §5), thus \( \eta/s \) is taken as an average over the temperature history of the expanding fireball. However, \( \eta/s(T) \) is essentially manifested in its centrality and collision energy dependence, though for the later meticulous investigation should be performed, e.g. the influence of pre-equilibrium dynamics, before a tight constraint would be drawn (e.g. \(^{[19]}\)). Bulk viscosity \( \zeta \), though is calculated one order of magnitude less than \( \eta \), it is expected to be dominant near the critical (transition) point though (e.g \(^{[20]}\)). Last, conductivity \( \lambda \) remains the least studied coefficient, albeit its effect is considered negligible in the baryon-free environment of relativistic collisions.

### D Pressure gradient-driven flow

Although the initial transverse velocity of the fluid could be received as zero, the acceleration might be in general different than zero. For sake of simplicity, Eq. (2) (cf. section §2.2.2) is solved for the case of inviscid fluidity. Taking under consideration the general formula for \( T^{\mu\nu} \) (Eq. (C.7)), the energy-momentum conservation is written as the sum of three terms

\[
(\varepsilon + p) u^\mu \partial_\mu u^\nu + \partial_\mu ((\varepsilon + p) u^\mu) u^\nu - \partial^\nu p .
\]

(D.1)

Further contraction with the projector \( \Delta_{\alpha\nu} \) implies,

\[
\Delta_{\alpha\nu} \partial T^{\mu\nu} = (\varepsilon + p) (u^\mu \partial_\mu u_\alpha + u_\alpha u^\mu \partial_\mu u^\nu) + \Delta_{\alpha\nu} [\partial_\mu ((\varepsilon + p) u^\mu)] u^\nu - \Delta_{\alpha\nu} \partial^\nu p ,
\]

which in \( F^* \) simplifies to

\[
(\varepsilon + p) u^\mu \partial_\mu u_\alpha = \Delta_{\alpha\nu} \partial^\nu p \Rightarrow \frac{\partial}{\partial x^\nu} ((\varepsilon + p) \hat{u}) = -\nabla^\nu p .
\]

(D.3)

Note that \( \Delta^{\mu\nu} \) and \( u^\mu \) are orthogonal, since \( \Delta^{\mu\nu} u_\nu = (g^{\mu\nu} - u^{\mu} u^{\nu}) u_\nu = u^{\mu} - u^{\mu} (u^\nu u_\nu) = 0 \), and the normalization of the flow velocity demands for \( u_\mu u^\mu = 1 \Rightarrow u_\mu \partial_\mu u^\mu = 0 \). For a fluid at rest, \( u^\mu \partial_\mu \equiv u^0 \partial_0 \) and \( \vec{u}, \vec{v} \) represent ordinary spatial vectors, while \( \Delta_{\alpha\nu} \) projects space-time onto regular space; \( \Delta_{\alpha\nu} \partial^\nu p \) could be then viewed as the pressure gradient. In the non-relativistic limit, Eq. (D.3) is Newton second law applied to a fluid element (Euler equation), since \( \varepsilon + p \) reduces to the mass density. In the relativistic limit, the system pressure contributes to the inertia of a relativistic fluid, whereas in both cases the force \( -\nabla p \) pushes the fluid towards lower pressure.

Therefore, the system thermal pressure acts against the surrounding vacuum and transforms the spatial deformations to momentum anisotropies. For inviscid fluids, expansion is an isentropic process and only the volume augments. Variations in energy, entropy and number density are simply related to \( ds/(\varepsilon + p) = ds/s = dn/n \). Making use of Eq. (D.3), which is valid to first order with respect to \( u^{1(2)} \) components,

\[
\frac{\partial u^{1(2)}}{\partial x^0} = -\frac{1}{\varepsilon + p} \frac{\partial p}{\partial x^{1(2)}} = -c_s^2 \frac{\partial n_s}{\partial x^{1(2)}} ,
\]

(D.4)

where the velocity of sound \( c_s^2 = (\partial p/\partial \varepsilon)^{1/2} \) controls the propagation of disturbances; a soft equation of state corresponds to a small \( c_s^2 \). Integration of Eq. (D.4) with a Gaussian entropy density

\(^4\)For easiness, viscous pressures and their associated relaxation times are respectively scaled with the entropy density \( s \) and the temperature \( T \), resulting in the specific viscosities and scaled relaxation times.
\[ s \propto \exp \left( -\frac{x^2}{\sigma_x^2} - \frac{y^2}{\sigma_y^2} \right), \] yet approximate assumption about the initial profile, and constant \( c_s^2 \), yields \( u_{x(y)} = \frac{c_s^2 x(y)}{\sigma_{x(y)}} t \), (D.5)

having substituted 0, 1 and 2 indices by \( t, x \) and \( y \) respectively, while \( \sigma_{x(y)} \) represents the widths of the Gaussian entropy distribution. The almond shape of the overlap region in non-central collisions, as illustrated in Fig. (7) and (8), is characterized by \( \sigma_x < \sigma_y \), thus conversely implying \( u_x > u_y \), particles are directed with greater velocity parallel to the \( x \)-axis as compared to particles in the proximity of \( \phi = \pm \pi/2 \). Since the Gaussian entropy profile implies lower densities at the furthest regions of the fireball, more particles are expected to be emitted along the less deformed \( x \)-direction.

In fact, the effect of mass dependent anisotropic emission can be analytically derived assuming that collective motion dominates over thermal motion [21]. Without introducing a specific fluid-velocity profile, like in the blast-wave parameterization (e.g. [22] and references therein), the fluid transverse velocity can be alternatively decomposed in harmonic terms. In other words, the azimuthal expansion of \( u \) reads [23]

\[ u(\phi) = u_r + u_a = u_r + 2 \sum_{n=1}^{\infty} u_n \cos \left[ n \left( \phi - \Phi_n \right) \right], \]

for which each coefficient could be related to initial density profiles; \( \phi \) is taken as the particle azimuthal angle, while \( \Phi_n \) are simply the deformation angles (Eq. (B.3)). For instance in [9] in section §2, it was shown that a radially symmetric Gaussian distribution results in one and only component being present in Eq. (D.6), the radial flow \( u_r \); the distinct fluid velocity component that is separated from the anisotropic velocity modulations \( u_a \) (Fig. (49)). In contrast, an initial elliptic deformation gave rise to the second order harmonic \( u_2 \) in the fluid velocity profile. In general, the balance between the evolution of radial and anisotropic flow should exhibit a centrality dependency, with \( u_a \) stronger competing with \( u_r \) in non-central collisions.

![Figure 49: Radial flow modulation by the anisotropic component. The solid ellipse shows the overlap region and the arrows show the direction and magnitude of the expansion velocity (7) in section §4.](image)

For our purpose, it is sufficient to consider symmetric collision systems, thus neglect odd harmonics, and to drop the \( \Phi_n \) dependence, like in [21], meaning

\[ u(\phi) = u_r + 2u_2 \cos (2\phi) + 2u_4 \cos (4\phi) \].

Interestingly then, making use of the constant \( T_f \) freeze-out process (Eq. (E.2)) and recalling
the leading effect \(dN/d\phi \propto 1 + 2v_2\cos(2\phi)\), it is revealed that up to first order in \(u_2 \ll 1\) \[15\]

\[v_2(p_T) \propto \frac{u_2}{T_f} \left( p_T - m_T \frac{u_r}{u^0} \right), \tag{D.8}\]

where \(u^0\) stands for the time-like component of the fluid velocity \((u^\mu u_\mu = 1) \Rightarrow u^0 = \sqrt{1 + u(\phi)^2}\). Eq. (D.8) features the dependence of the \(p_T\)-differential \(v_2\) on the particle mass. For light particles \(m_T \approx p_T\) and \(v_2\) increases linearly with transverse momentum, whereas for heavier particles \(m_T\) is larger at the same value of \(p_T\), hence resulting in smaller \(v_2\). Parenthetically, Eq. (D.8) manifests that the mass ordering effect is profound enough only if \(u_r\) is a significant fraction of the velocity of light, hence experimental data on anisotropic flow could be considered strong evidence for relativistic fluid dynamics.

Similar predictions hold for higher order terms and in leading order in \(u_2 \ll 1\) and \(u_4 \ll 1\)

\[v_4(p_T) \propto \left( \frac{u_2}{T_f} \right)^2 \left( p_T - m_T \frac{u_r}{u^0} \right)^2 + \frac{u_4}{T_f} \left( p_T - m_T \frac{u_r}{u^0} \right), \tag{D.9}\]

Now, there exists both a linearly generated and a quadratic dependent term, the later identified as the cross-talk between anisotropic flow of different harmonic order. The analytic form of Eq. (D.8) and (D.9) could be interpreted as a signature of hydrodynamic evolution, yet could serve as a consistency check with numerical calculations.

**E  Thermal decoupling**

The fluid system created in heavy-ion collisions will eventually decouple and experimental observables originate from the surface of the kinetic freeze out. Kinetic freeze out is generally regarded as the end of the hydrodynamic description, and incorporated by the Cooper-Frye formula. A space-time hypersurface \(\Sigma\) - a collection of elements \(d\sigma_\mu\) in the 4-dimensional Minkowski space- is defined, on which free-streaming particles are emitted according to a thermal distribution in the fluid rest frame \(F^*\). In inviscid hydrodynamics, such an assumption translates to an integration over \(\Sigma\), meaning that

\[\frac{dN}{p_T dp_T dy d\phi} \propto \int_\Sigma d\sigma_\mu g_i f(u^\mu p_\mu), \tag{E.1}\]

where \(g_i = 2s + 1\) is the spin \(s\) degeneracy factor of particle species \(i\). The Cooper-Frye prescription transforms the space-time dependence of the hydrodynamic fields into the momentum dependence of the particle spectra, as long as the phase-space distribution function \(f(u^\mu p_\mu)\) is known. Though Eq. (E.1) is valid for any \(f\), the distribution function is usually assumed to the boosted thermal distribution, namely

\[f(u^\mu p_\mu) \propto \frac{1}{\exp \left( \frac{<(u^\mu p_\mu)}/T_f \pm 1 \right)}, \tag{E.2}\]

with \(T_f\) the freeze-out temperature; particles of different species or with different \(p_T\) might not decouple in one common \(T_f\). For the sake of simplicity, chemical potentials are considered zero (e.g. [26]). Alternatively, a distribution close to the boosted thermal distribution \(f \rightarrow f_0 + \delta f\) could be considered (viscous fluid), where \(\delta f \ll 1\) is the dissipative correction.\(^5\) It is

\(^5\)Viscosity modifies the thermal distribution function \(f \rightarrow f_0 + \delta f\) with the viscous corrections typically taken as \(\delta f \sim p^\nu p^\mu \pi_{\mu\nu}(x^\nu)\) (e.g. [24]). Because the exact form of \(\delta f\) depends on the details of the unknown underlying microscopic theory, systematic analyses investigate the uncertainty introduced in final observables by the uncertainty in \(\delta f\) (e.g. [23]).
worth noting that according to the kinetic theory, if the fluid velocity $u^\mu$ and temperature $T_f$ were to depend on the space-time position $x^\mu$, Eq. (E.2) would define a state of local thermal equilibrium. Moreover, the covariant form of Eq. (E.2) safeguards the inclusion of the total number of particles emitted from all (fluid) elements that move independently - as dictated by the ideal gas assumption.

As a Lorentz scalar then, the 4-vector $u^\mu p_\mu$ in the exponent is invariant and can be viewed as the energy $E^*$ of the particle in the fluid rest frame. In deed, in $F^*$, where $u^\mu = (1, 0, 0, 0)$, $u^\mu p_\mu$ reduces to $p_0$, which equals to the particle mass in case the later is at rest. Focusing on the mid-rapidity region $y \approx 0 \Rightarrow p^3 \approx 0$,

$$E^* = u^\mu p_\mu = m_T u_0 - \vec{p}_T \vec{u}, \quad (E.3)$$

having introduced the particle transverse mass $m_T = (p_T^2 + m^2)^{\frac{1}{2}}$, namely the time-like component (energy) for a particle with $p^3 = 0$. On one hand, Eq. (E.3) delineates that the suppression factor $\exp(-E^*/T)$ is minimized in case that the particle transverse momentum $\vec{p}_T$ is parallel to the fluid velocity; fluid and particle have the same azimuthal angle $\phi$. On the other hand, the suppression factor is found minimum in directions of maximum fluid velocity, while for a given $|\vec{p}_T|$, the larger the mass $m$ the greater the suppression. Combining with the result of Eq. (D.5), it is expected then the preferential emission of lighter particles along the $x$- relative to the $y$-direction ($u_x > u_y$) to be more pronounced, if compared to heavier species; for example, pion emission should be found more anisotropic than protons in a hydrodynamics scenario, as previously highlighted (cf. section §D).
The ALICE experiment at LHC

F.1 ALICE detector prototype

Figure 50: The ALICE experiment consists of the central-barrel detectors ITS, TPC, TRD, TOF, PHOS, EMCal, and HMPID, which are embedded in a solenoid with magnetic field $B = 0.5 \, T$ and address particle production at mid-rapidity. The cosmic-ray trigger detector ACORDE is positioned on top of the magnet. Forward detectors (PMD, FMD, V0, T0, and ZDC) are used for triggering, event characterization, and multiplicity studies. The MUON spectrometer covers the pseudorapidity $-4.0 < \eta < -2.5$.

F.2 Kalman filtering

Starting from the input digits that each detecting component registers, the ultimate goal of event reconstruction is to create an output file format, which is easily accessible for any further analysis purposes. In principle, adjacent digits, i.e. clusters of energy, that were allegedly deposited by the same particle crossing the sensitive element of a detector, are locally reconstructed (e.g. positions, signal amplitudes and time etc.).

What is commonly referred as tracking reconstruction is the process of identifying clusters of energy depositions within the detector material, on one hand, and ascribing particle trajectories to them, on the other hand. As highlighted in section §3, the first step of cluster finding
is executed separately over each detector, since the signal characteristics depend on the particular detector technology. The subsequent task of assigning clusters to particle tracks and the fitting of the track parameters is done by a Kalman filtering procedure.

In dense environments, as the instance of LHC-related conditions, following the presumed particle trajectory, or stated alternatively, collecting hits compatible with such a trajectory along the way, involves major complications (Fig. (51)). If the hit density were to be small, most of the times a track following algorithm would accept the hit with the smallest deviation from the path. In the presence of high hit density though, using a quality estimator in each propagation step between detector layers has been proven to provide with the optimal solution [27]; a tree of track candidates is constructed, whose branches are followed up concurrently. Kalman filtering consolidates an ideal mathematical machinery supporting a sophisticated implementation, in terms of such a concurrent track evolution strategy.

![Figure 51: Schematic view of the Kalman filter implementation for track following. In a high-density environment, ambiguities intermittently result in fake track candidates. In the current case, a seed having being instantiated from track 1 was propagated through five (fictitious) detector layers. An ambiguous situation was caused by nearby tracks 2 and 3, while the propagation proceeded upstream from the right to the left. Under a concurrent track evolution strategy, fake candidates were discarded and ambiguities were resolved in favor of the correct track selection [28].](image)

**F.3 TPC technical specifications**

To explain the principle of a TPC, it is easier to consider first a simple imaginary TPC box as illustrated in Fig. (52). The box is filled with a typical mixture of noble (e.g. Ne) and quencher (e.g. CO₂) gas. The choice of the relevant composition is crucial, since it regulates the charge transport in the drift volume and the amplification processes. Typically, a drift gas with low charge diffusion, low atomic number and large ion mobility is needed to fulfill the requirements for good momentum resolution. At the bottom of the box a plane of anode wires is found. A uniform electric field is created by bringing the plane opposite to the anode wires to a large negative potential. In the presence of the electric field, electrons and ions created by the medium radiation are accelerated towards the anode and cathode, respectively. As the electrons reach the anode wires, the avalanches induce electric signals on the cathode pads next to them, hence the transverse \( xy\)-position of a charged particle can be promptly obtained by collecting all signals in the volume of the box; the \( z\)-coordinate is given by the drift time.
The principle of a TPC is explained with the help of an imaginary box-shaped TPC detector. The transverse position ($x$- and $y$-coordinate) of a charged particle is obtained from the signals induced in the cathode pads and the drift time determines the $z$-coordinate [29].

In HEP experiments, particles radiate outwards from the collision point, and, owing to the overall geometry of the experiment, the TPC should be a cylinder surrounding the beam pipe. More specifically, the ALICE TPC (Fig. (14) in section §3.2.1) is the largest detector of its type ever built with active volume of nearly $90^3$ m$^3$. The separation into two detection volumes is delineated both by a central -100 kV aluminized electrode and multi-wire proportional chambers (MWPC) in each end-plate, the latter divided into 18 trapezoidal sectors. The TPC measures the charge deposited on up to $63 + 96 = 159$ padrows. The radial segmentation of the (cathode) pad readout system, offered by the MWPC, exhibits varying pad sizes, which is optimized for the radial dependence of the tracking density (Table (53)).

The field cage formed on the basis of such a design, i.e. with a central high-voltage electrode and two opposite axial resistive potential dividers, create a highly uniform electrostatic field in the common gas volume. Because of the drift gas composition, yet optimal in terms of the momentum resolution requirements, of Ne, CO$_2$ and N$_2$ in $85.7 : 9.5 : 4.8$ relative mixture (until the end of 2010), the field cage has to be operated at a rather high gradient voltage of $400 \text{ V} \cdot \text{cm}^{-1}$, for distortions being exclusively limited to the intrinsic TPC space resolution. The intrinsic spatial granularity of the TPC system is limited by the charge diffusion and amounts approximately to 1 mm.

The latter is related to the spatial resolution of the points along the particle path, while the effect of multiple scattering cannot be always disregarded. If there are $N$ equidistant points along the trajectory, it can be shown that the relative momentum resolution $\sigma_{p_T}/p_T$ due to the measurement error $\sigma$ on the space points writes [29]

$$\left[ \frac{\sigma_{p_T}}{p_T} \right]_{\text{spatial}} \propto \frac{1}{\sqrt{N}} \frac{p_T [\text{GeV}/c]}{0.3ZB [T] L^2 [m^2]} \sigma,$$

with $Z$ the charge of the particle, whereas the contribution of the multiple scattering depends on the detector material $X_0$, meaning [29]

$$\left[ \frac{\sigma_{p_T}}{p_T} \right]_{\text{mult.scat.}} \propto \frac{1}{0.3B [T] L [m]} \sqrt{\frac{L}{X_0}}, \quad \beta = \frac{v}{c}.$$

The total error on the particle momentum is simply the quadrature sum of Eq. (F.1) and (F.2). It is then realized that the momentum resolution improves for large values of $N$, while it significantly worsens for higher $p_T$. The multiple scattering contribution becomes dominant at lower-$p_T$.

The velocity dependence of the specific energy loss, i.e. the truncated mean of the clustered charge long Landau-tailed PDF [30], can be compactly parametrized as a function of the $\beta \gamma =$

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\( p/m_0 \) variable \([31]\):

\[
-\left(\frac{dE}{dx}\right)(\beta\gamma) = \frac{P_1}{\beta P_4} \left(P_2 - \beta P_4 \ln \left(\frac{P_3 + \frac{1}{(\beta\gamma)P_5}}{\beta}\right)\right), \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}}.
\] (F.3)

Parameters \( P_1-P_5 \) contain information about the traversed medium(s) according to the Allison-Cobb specification \([32]\), meaning that their values are fixed, given the gas mixture, while their enumeration is simply conventional. Therefore, the energy loss per unit path length in the TPC depends on particle rest mass \( m_0 \) in a fixed momentum window, hence allowing for prompt identification as Fig. (17) (cf. section §3.2.1) illustrates.

| Azimuthal-rapidity coverage | \(-0.9 < \eta < 0.9 \) for full radial track length
| Radial position (active volume) | \( 848 < r < 2466 \) mm
| Radial size of vessel (outer dimensions) | \( 610 < r < 2780 \) mm
| Radial size of vessel (gas volume) | \( 788 < r < 2580 \) mm
| Length (active volume) | \( 2 \times 2497 \) mm
| Segmentation in \( \varphi \) | \( 20^\circ \)
| Segmentation in \( r \) | 2 chambers per sector
| Total number of readout chambers | \( 2 \times 2 \times 18 = 72 \)
| Inner readout chamber geometry | trapezoidal, \( 848 < r < 1321 \) mm active area
| pad size | \( 4 \times 7.5 \) mm\(^2\) \((\varphi \times r)\)
| pad rows | 63
| total pads | 5,504
| Outer readout chamber geometry | trapezoidal, \( 1346 < r < 2466 \) mm active area
| pad size | \( 6 \times 10 \) and \( 6 \times 15 \) mm\(^2\) \((\varphi \times r)\)
| pad rows | \( 64 + 32 = 96 \) (small and large pads)
| total pads | \( 5,952 + 4,032 = 9,984 \) (small and large pads)
| Detector gas | \( \text{Ne-CO}_2-\text{N}_2 \) \([85.7-9.5-4.8]\)
| Gas volume | \( 90 \) m\(^3\)
| Drift voltage | 100 kV
| Anode voltage (nominal) | 1,350 V (IROC)
| Gain (nominal) | \( 7,000 \) to \( 8,000 \)
| Drift field | 400 V/cm
| Drift velocity (NTP) | 2.65 cm/\( \mu \)s
| Drift time (NTP) | 94 \( \mu \)s
| Diffusion (longitudinal and transversal) | 220 \( \mu \)m/\( \sqrt{\text{cm}} \)
| Material budget (including counting gas) | \( X/X_0 = 3.5\% \) near \( \eta = 0 \)

Figure 53: General technical parameters of the ALICE TPC \([36]\).

### F.4 VZERO technical specification

Each detector is segmented in four rings in the radial direction, while each ring is further divided in eight sections over the azimuthal direction (Fig. (20) in section §3.3.1). Different designs are applied to VZERO-A and VZERO-C detectors to comply with their integration constraints, since the former is located away from the nominal vertex, whereas the VZERO-C array is fixed on the front face of the hadronic absorber (Fig. (12) in section §3.1.1). Whenever a charged particle traverses the scintillator material, photons are released from atoms excitation. Subsequently, fiber-optic cables transmit the produced light to photo-multiplier tubes (PMT), after luminescence photons had been shifted down in frequency by wavelength shifting (WLS) fibers.
F.4.1 Centrality determination with VZERO

A typical distribution of the VZERO amplitude is presented in Fig. (54) and fitted based on the assumption that the registered signal is proportional to the primary charged particle multiplicity, the latter considered strongly dependent on the collision geometry. Since nuclei are extended objects, yet spherical in the $^{208}$Pb case, the initial interaction region is parameterized by the impact parameter $|\vec{b}| = b$, i.e. the nuclei transverse distance measured relative to their centers. The particle multiplicity distribution writes then

$$\langle dN_{ch}/dy \rangle = \frac{1}{\sigma_{AA}^{geo}} \int d^2b G (b) P (b) , \quad \text{(F.4)}$$

with $\langle dN_{ch}/dy \rangle$ the average multiplicity density over a rapidity $dy$ window. In other words, phenomenological approaches $G (b) \propto b$, based on the Glauber Monte-Carlo model [9] for instance, are combined with particle production mechanisms $P (b)$, so as to examine the geometry origin of $\langle dN_{ch}/dy \rangle$ in $AA$ collisions. One possible candidate about $P (b)$ assumes a negative binomial distribution (NBD), having been successfully applied in $pp$ and $p\bar{p}$ collisions [33], to be folded with independent particle emission sources, whose yield corresponds to a weighted admixture of soft and hard interactions [34].

Moving a step further, integration in Eq. (F.4) could be limited over a specific range of impact parameters $[b_1, b_2]$ and be normalized to the total geometric cross section $\sigma_{AA}^{geo} \sim \pi R_A^2$. In the field of heavy ions, it is thus customary fractions of $\sigma_{AA}^{geo}$ to be studied, i.e. ranges of impact parameters to be substituted by the notion of centrality classes. The red curve in Fig. (54) constitutes such an attempt of experimentally determining centrality regions, hence resolving fractions of $\sigma_{AA}^{geo}$. More specifically, each simulated Glauber geometry is convoluted with a probability according to the NBD formula in order the VZERO amplitude to be approximated; the optimal model parameters were found by a $\chi^2$ minimization. Because of the poor fitting behavior in the low amplitude region, the normalization could be redefined as $\sigma_{AA}^{geo} \approx 90\% \cdot \sigma_{AA}^{geo}$ in Eq. (F.4), meaning that any given centrality class $c_1$-$c_2$ is obtained as

$$c_1-c_2 = \frac{\int_{V_{c_1-c_2}} dV (dN/dV) \int_{V_{90\%}} dV (dN/dV) }{ \int_{V_{90\%}} dV (dN/dV) } , \quad \text{(F.5)}$$

where variable $V$ represents the VZERO amplitude, $dN/dV$ accounts for the measured distribution, while the Glauber-NBD simulated $\sigma_{AA}^{geo}$ receives 90% of its maximal value at $V_{90}$ (Fig. (54)).
Figure 54: *(left)* Sum of VZERO-A and VZERO-C amplitude distribution recorded in MB\(\text{AND}\) Pb-Pb collisions (coincidence of VZERO and SPD) with \(|z_{\text{vtx}}| < 10\) cm vertex position. Machine-induced background and parasitic, from de-bunched ions, collisions are removed using the timing information from VZERO and ZDC, with the latter being additionally employed in reducing the electromagnetic dissociation background. The amplitude distribution is simulated for various values of the two NBD parameters, mean multiplicity per nucleon-nucleon collision \(\mu\) and its width \(k\), and the relative \(N_{\text{part}}(\text{soft})-N_{\text{coll}}(\text{hard})\) admixture \(f\) used for the Glauber Monte-Carlo geometry. *(right)* A magnified version of the most peripheral region (additionally captured in the inset of the left panel) is accompanied by different trigger signals (markers) and simulations (lines). Percentages of the total (geometric) cross section refer to the NBD-Glauber fit (dashed line) [35].

G The \(V^0\) distinctive decay topology

![V0 decay topology](image)

Figure 55: Possible transverse topologies for decays of neutral particles to pairs of oppositely charged particles in a magnetic field.

H Cross-check for systematic deviations

H.1 \(v_3(p_T)\)

Once more, it was examined whether the harmonic values separately for particle and antiparticle species are statistically compatible. The third order harmonic \(v_3\) as a function of \(p_T\), both for \(\Lambda\) and \(\bar{\Lambda}\), is illustrated in Fig.(28) for the (10-20) % centrality class. Their numerical difference is depicted on the bottom panel, with values having been extracted following the
procedure previously described in section §5.1. Any observed deviations are attributed to statistical fluctuations, as the relevant information in the inset delineate.

**Figure 56:** (top panel) Dependence of the third order harmonic on transverse momentum $p_T$ separately for $\Lambda$ (filled cross) and $\bar{\Lambda}$ (open cross). Symbols are placed at the $p_T$ weighted average. (bottom panel) Numerical difference of the interpolated $v_3^\Lambda (p_T)$ relative to the extracted $v_3^{\bar{\Lambda}} (p_T)$; deviations were found to be compatible with statistical fluctuations.

**Figure 57:** The $p_T$-differential evolution for $v_3^{\Lambda(\bar{\Lambda})}$ over $p_T = 0.6-8$ GeV/c for the (10-20) % most central collisions separately for the nominal selection criteria (filled red, Table 2) and the operated cross-check variations (filled color, Table 3).
Figure 58: The $p_T$-differential evolution for $v_3(\Lambda)$ over $p_T = 0.6-8$ GeV/c for the (40-50) % most central collisions separately for the nominal selection criteria (filled red, Table 2) and the operated cross-check variations (filled color, Table 3). The largest variation was found to be given by altering the position of the transverse radius (Fig. (22)) at $\text{Rad}_Y = 8$ cm.

H.2 $v_4(p_T)$

The $v_4(p_T)$ compatibility separately for particle and anti-particle species has been inspected. The fourth order harmonic distributions as a function of $p_T$, both for $\Lambda$ and $\bar{\Lambda}$, are presented in Fig. (59) within the (10-20) % centrality class. Their numerical difference is depicted on the bottom panel, with values having been extracted following the procedure described in section §5.1. Deviations are well embraced by statistical fluctuations, as was confirmed by the performed statistical test.
Figure 59: (bottom panel) Dependence of the fourth order harmonic on transverse momentum $p_T$ separately for $\Lambda$ (filled cross) and $\bar{\Lambda}$ (open cross). Symbols are placed at the $p_T$ weighted average. (bottom panel) Numerical difference of the interpolated $v_4^\Lambda(p_T)$ relative to the extracted $v_4^\Lambda(p_T)$. The statistical test revealed deviations being compatible with statistical fluctuations.

Figure 60: The $p_T$-differential evolution for $v_4^\Lambda(\bar{\Lambda})$ over $p_T = 0.6-8$ GeV/c for the (10-20) % most central collisions separately for the nominal selection criteria (filled red, Table 2) and the operated cross-check variations (filled color, Table 3).
Figure 61: The $p_T$-differential evolution for $v_4^{L(\bar{\Lambda})}$ over $p_T = 0.6-8 \text{ GeV}/c$ for the (40-50) % most central collisions separately for the nominal selection criteria (filled red, Table 2) and the operated cross-check variations (filled color, Table 3). The largest variation was found to be given by altering the cosine of the pointing angle (Fig. 22) at $\cos \theta_P = 0.999 \text{ cm}$. 
References


