Talking about Mathematics

Prompting Discussion among Community College Students in Algebra Tutoring

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For all three of my parents, who taught me to never stop learning, to keep my options open, and that life is full of opportunities for transformation. I love and miss you all so much.
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Abstract

In the United States, community colleges provide educational opportunities to students who have not met the requirements to attend 4-year universities. The pass rates for basic skills mathematics courses such as elementary algebra at these institutions are low. In the interest of supporting students, most community colleges have tutoring centers, which employ professional tutors and faculty-trained student tutors. This research explores the potentials of discussion as a tool in tutoring elementary algebra at a community college in California. Although it does not assess the effect of discussion on student performance, it was meant as a first step in exploring how discussion can be prompted among these students.

Five types of prompts were designed to encourage collaboration and discussion among students as they worked on homework assigned by their algebra instructors during 6 weeks of bi-weekly tutoring sessions (students chose from three different session times which they would attend each week). Qualitative approaches, including coding of dialogue and student interviews, were used with individual and groups of students to explore what types of talk can be distinguished among the students, what types of prompts encourage which types of talk, the practicability of the prompts in the tutoring situation, and the appearance of issues of affect, including confidence and personal views of mathematics.

The prompts were found to be practicable during many of the tutoring sessions, and nine of 20 sessions will be reported on. Five types of talk, defined before the intervention, were identified as occurring in student discussion: exploratory, explanatory, reflexive, challenging, and parallel. Exploratory talk was the most common, occurring in nearly all discussions. Students maintained their view that mathematics is primarily a set of rules, however, they also responded positively to working collaboratively with other students, which was quite different from their experience in the classroom and in the tutoring center. Students found that they benefited from explaining to others and encountering alternate methods of solution. Additional result regarding the types of prompts and talk, as well as indications for tutoring preparation and future research will also be discussed.
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1 Introduction

This research project was inspired by my own experience working as a tutor with adult students struggling with algebra. It is still incredible to me how many times I have heard a phrase similar to “I have A’s in all of my other classes, but this will be the second (or third, or fourth…) time that I have tried to pass algebra.” Some students whom I have worked with have contemplated changing their majors and career paths in order to avoid algebra, although they were excelling in all other parts of their academic programs.

Difficulty with algebra and other courses in basic skills mathematics is in fact a widespread problem for students in post-secondary education. In this research project, I will address some particular aspects of this issue, contributing to a growing body of research on the teaching and learning of basic skills mathematics in community colleges (Perin, 2004).

1.1 Problem

Herriott (2001) wrote that the college algebra course, part of remedial or developmental mathematics at the tertiary level of education in the United States, has a “reputation nationally for failing an unusually high percentage of students.” (p. 2). Herriott reported the pass rates at that time to be 60% nationally, but that some colleges reported pass rates as low as 10%. Biswas (2007) reported on behalf of Achieving the Dream, an initiative to help more community college students succeed, that at least for students at the 27 institutions participating in that initiative as of 2002, the number requiring developmental mathematics education was far greater than those needing English remediation: 70% compared to 34%, respectively.

There are multiple factors influencing both the massive need for remedial mathematics education and the disappointing success rates associated with developmental mathematics courses. Herriott points out that many students have not done well in prior mathematics courses, for example in high school, and have developed fear or anxiety associated with math. Many of the traditionally taught mathematics courses in colleges follow a structure similar to that used in high school, and it is difficult to expect students to achieve greater success the second time they are taught the same concepts in the same way. Remedial courses also often take on the structure of the typical college/university course which involves lecture, notes, and completing exercises for homework. However, since these students are developing basic skills in mathematics, perhaps it is necessary to help them by giving them support and mathematical experiences which are not found in these lecture and drill approaches.
Some schools in the U.S. community college system are currently redesigning their mathematics remediation programs. Many of those that are successful include more personal engagement with the students and discussion of attitudes toward mathematics (Biswa, 2007; Patton, 2009). A common theme in these college-level reforms is the learning community. In this case, the term learning community is used to describe a group of students who attend both a mathematics course and a study skills course simultaneously. The study skills course deals with math and test anxiety, study strategies, and motivational issues, among other things. According to instructors, relationships which are formed within these communities help students succeed with the mathematics material (Marklein, 2009).

Learning assistance and tutoring services are also in place at most community colleges to support students to increase their ‘academic preparedness.’ In Perin’s (2004) qualitative study of tutoring and learning assistance facilities at community colleges across the United States, she hypothesized, and suggested further research regarding the idea, that the ‘generic’ instruction used in developmental mathematics courses is not as effective as the more contextualized learning received through subject-tutoring and learning centers. On the other hand, there was also a worry that tutors supplied too much help to students, so that students were not doing enough of their work on their own. Also, students seeking help in basic skills, as well as college-level courses, may have learning disabilities. For these two reasons, Perin also suggests professional development for tutors which addresses both issues.

Perin begins her paper by mentioning one of the main challenges faced by community colleges. As institutions that are committed to providing ‘open access’ education, they must educate an extremely heterogeneous student body. My intent was to keep in mind the diversity of community college students, with respect to age, ethnicity, learning style, etc., throughout this research.

1.2 Personal motivation
I would like to know: how can we assist community college students in a way which allows them to develop a more positive attitude both toward mathematics as well as their own ability to do mathematics? What type of activities and assistance can benefit them in the future with courses, problem solving, and basic mathematical literacy? Although these questions do not directly address the issues of student performance which I have described above, perhaps such an exploration can reveal the potential of such changes in attitudes towards and view of mathematics to benefit achievement, which can be explored further in future research.
At the very least, mathematics should not pose the kind of obstacle I mentioned above, becoming the one barrier between the student and a desired career or path of study. There is no reason why math should not be taught in a way that allows everyone to develop some mathematical literacy and confidence. One issue that contributes to this apparent ‘impossibility’ is the maintenance of ways of teaching which are not truthful about the nature of mathematics. In many educational contexts, mathematics is approached as a subject of certainty in which both strategies and answers alike are deemed simply right or wrong. From this perspective, problem solving takes on a formulaic and memorized aspect. Alternatively, mathematics and problem solving can be approached as a process of trial and error, the error being just as useful in the process as the success. This view of mathematics can also embrace a more social view where problem solving can be a shared activity. Not only do I think that approaching mathematics in this way can make it apparent that it is a subject in which it is possible to succeed, but in addition, simply by altering the view to one that is less dependent on certainty and right and wrong, there is an opportunity in teaching to take advantage of the benefits that students receive from talking about mathematics and reflecting on their own thinking processes. The reported success of the learning communities which are part of current remedial mathematics reforms could be due to the creation of safe environments where it becomes possible for students to take part in such beneficial discussion.

Adult education theory also indicates the importance of considering the heterogeneity of adult learners, and is influenced both by the assumptions of andragogy (namely, that learners become more self-directed as they mature) and the goals of self-directed learning (Merriam, 2001). A social view of mathematics put into practice in the classroom could give students more control over their learning, which may be even more important in an adult learning environment like a community college, which serves an especially heterogeneous population. Less leading by the teacher and more encouragement to communicate with others about understanding and thinking processes can both encourage self-directed learning by empowering students to value their own understanding, and take advantage of the different learning styles, previous knowledge and background of the adult population.

In this research I will explore some of these issues in the context of tutoring for students in an introductory algebra course at a California community college. Specifically, I will consider how discussion and ‘conversations about math’ can play a role in addressing the issues raised above, including the honest portrayal of the nature of mathematics, creating community among students, and the benefits of students working collaboratively.
2. Theoretical Framework

In this chapter I will define a theoretical basis for this research. First I will define talk and discussion as types of classroom discourse. I will also give an overview of previous research regarding discussion in mathematics classrooms and the apparent effects and benefits to learning and affect.

2.1 Defining ‘talk’ and ‘discussion’

Throughout the literature, social aspects of the classroom and student-centered philosophies are connected to mathematics by the concept of mathematical discourse. Lampert (1990) states that, “Mathematical discourse is about figuring out what is true, once the members of the discourse community agree on their definitions and assumptions.” (p. 42). Moschkovich (2003), on the other hand, defines mathematical discourse by elaborating on Gee’s more general definition of discourse:

A Discourse is a socially accepted association among ways of using language, other symbolic expressions, and ‘artifacts’, of thinking, feeling, believing, valuing and acting that can be used to identify oneself as a member of a socially meaningful group or ‘social network’, or to signal that one is playing) a socially meaningful role.” (Gee, 1996, p. 131)

The mathematical version of this, according to Moschkovich, includes not only those things mentioned by Gee (talking, acting, thinking, writing, etc.) but also “mathematical values, beliefs, and points of view.” (p. 326). Therefore, participating in mathematical discourse (forming or being part of a ‘discourse community’, as suggested by Lampert) involves much more than talking about mathematics and using mathematical language. In fact, becoming competent in a particular discourse (e.g. mathematical) is as multi-faceted as fitting into a new culture. However, talk, and subsequently different types of talk (conjecturing, justifying claims by presenting evidence, explaining, etc.), are specific examples of discourse practices that can be associated with mathematics, and mathematics learning and teaching (Esmonde, 2001).

If we define talk as any purely verbal discourse practice (Pimm, 1987), then the term discussion refers to a specific kind of talk. Discussion, according to Pimm, should include both negotiation of meanings and sharing points of view with others. These specifications are supported if we look at discourse and talk from the point of view of Sfard and Kieran (2001), who consider thinking itself as a form of communication (with oneself). In their view, discourse is “any specific instance of communicating, […] whether predominantly verbal or with the help of any other symbolic system.” (p. 47). So discussion must
be specified as both talk (‘predominantly verbal’) and as something shared with others (not just two people talking near each other), as indicated by Pimm.

### 2.2 The use of talk and discussion in mathematics

Talk (a discourse practice), and subsequently discussion, become part of mathematics itself when we begin to see mathematics as a social activity. From this perspective, Hoyles (1985) defines mathematics as the ability to act in the following ways:

1. Form a view of a mathematical idea
2. Step back and reflect upon it
3. Use it appropriately and flexibly
4. Communicate it effectively to another
5. Reflect on another’s perspective of the idea
6. Incorporate the other's idea into one’s own understanding or challenging (logically) alternative views (p. 112).

In this definition of mathematics, communicating mathematical ideas (to another or to oneself by reflecting upon it) is central. Hoyles also points out the common notion agreed upon by researchers such as Piaget, Bruner, and Vygotsky that the effort of putting ideas into language in order to describe a situation causes the learner to modify how he/she thinks of that situation. Therefore language, even if its primary purpose is communication, leads to reflection and “internal regulation of concepts” (p. 206).

In addition to giving an opportunity for reflection, talk or discussion is the externalization of knowledge. There is a difference, according to Pimm (1987), between tacit and externalized knowledge which highlights the importance of language and words in the process of understanding (in addition to one’s internal process of understanding). Reminiscent of what Hoyles points out regarding the effort of putting thoughts into language, Pimm says, “It may be only when you discover a difficulty in expressing what you want to say, that you realize that things are not quite as you thought” (p. 25).

Hoyles distinguishes between two types of action in conversation (namely: listening and talking) and two types of talk: articulating one’s own ideas by saying them out loud, which serves a cognitive function; and explaining one’s ideas to others, which serves a communicative function (p. 206). For purposes here, we can consider conversation and discussion to be approximately the same concept. Pimm (1987) also distinguished between “talking for oneself” (cognitive) and communicative talk (p. 23). According to Pimm, also in agreement with Hoyles, both types of talking allow reflection on one’s own ideas and thinking and help the speaker to “clarify thoughts and meanings” (p. 24). For example, Moschkovich
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(1996) also reports that students refined their description during peer discussions in order to make sure that they understood each other (communicative talk). In addition, externalizing thoughts makes them more accessible to the speaker. In fact, as remarked by Hoyles, there is a connection between the two types of talking, in that communication assists cognition and vice versa.

Pirie and Schwarzenberger (1988) take this idea of discussion further and develop a definition of mathematical discussion. In their words, mathematical discussion is “purposeful talk on a mathematical subject in which there are genuine pupil contributions and interaction” (pp. 460-461). However, they view mathematical discussion as evidence of understanding rather than part of it, contrary to Hoyles’ view that these social and communicative abilities are part of mathematics itself. In addition, it is important to note that Pirie and Schwarzenberger’s descriptive definition of mathematical discussion does not specify that students must use a specific, formal language or set of words in order to be ‘talking mathematically’. In fact, Moschkovich (1996) points out that there is an important difference between mathematical ways of talking and formal ways of talking. For example, Moschkovich observed that in peer discussion students used everyday language and metaphors to build “shared descriptions of mathematical objects” (p. 266). In Moschkovich (2003) she reiterates that everyday language and approaches to discussion “should not be seen only as obstacles to learning mathematics” (p. 325), but instead we should think carefully regarding what might be part of mathematical discussion in the classroom.

Contributing to the discussion of peer interaction in the mathematics classroom, Dekker & Elshout-Mohr (1998) developed a process model to describe the activities that both facilitate and define mathematical understanding – partially as a response to Hoyles’ assessment of mathematics as a series of socially-based abilities, as discussed above. The model is based on four key activities in which students should take part:

1. Show one’s work
2. Explain one’s work
3. Justify one’s work
4. Reconstruct one’s work

Although students can take part in these activities by themselves, interaction between students stimulates the four activities by giving the opportunity to respond to the regulating activities of other students, which include asking others to show or explain their work, and criticizing the work of others. Pijls, Dekker and Elshout-Mohr (2007) further explains the four key activities by specifying that justification is a ‘reaction to criticism’, and reconstruction, while close to the activity of showing, is indicated by evidence that students’ work has evolved from a previous answer or response to a regulating activity. Dekker and Elshout-Mohr view the process model as a basis for Hoyles’ activities. In other words, learning situations
where there is promotion of the key activities help students to acquire the abilities that Hoyles’ sets out as ‘mathematical understanding.’

Dekker and Elshout-Mohr’s process model uses differences between student answers as a jumping-off point for student discussion (Pijls et al, 2007; Dekker & Elshout-Mohr, 1998). Also investigating students’ social interaction in the classroom, Horn (2005) looked at other mathematical activities (representation, justification, generalization) and what social practices support them. Horn describes the Share, Compare & Analyze (SC&A) routine and how it was implemented in an algebra class. This routine draws students’ attention to the variety among the answers within their group and Horn found that using the SC&A routine to structure student discussion did support their engagement in the three mathematical practices.

2.3 Types of talk in discussion about mathematics

From the literature, we can determine and define types of talk which are likely to arise in mathematics classroom discussion. The types of talk which I have identified within the literature are exploratory, explanatory, reflexive, challenging, and parallel talk.

*Exploratory talk* is defined in the literature as improvisational talk, and rearrangement of thoughts by the student (Barnes, 1976). Cazden (1988) also highlights Barnes description of exploratory talk as speaking “without answers fully intact” and as a “rehearsal of knowledge” (p. 133). This type of talk occurs when the speaker is still searching for a path to a solution and the way is not yet clear. Also, in exploratory talk, students take responsibility for their own reasoning, and the talk is not focused on what Barnes refers to as ‘external criteria’ (e.g. pleasing a teacher; the answer being ‘right’ or ‘wrong’). According to Barnes, this type of talk occurs most when students perceive their audience as more intimate, for example in a small group and without an extreme authoritative presence. Although Barnes and Cazden were not talking specifically about mathematical discussion, we might expect this type of talk to occur in the context of problem solving among mathematics students and in response to questions from the tutor.

*Explanatory talk* is more formally descriptive. Explanations, as defined by Webb (1991), include how to solve a problem and (possibly) elaboration of the process. This type of talk, in contrast to exploratory talk, occurs when the speaker has a clearer idea of the path ahead, and perhaps knows the way quite well. Explanation may include the speaker’s reasoning for deciding the next step in solving a mathematical problem, using specific examples to illustrate a general concept, translating language (formal, unfamiliar)
so that it is understood by other students, or responding to concerns and questions, especially ‘why’ questions, with elaborate descriptions rather than brief or incomplete answers.

In his research on student talk about modeling activities, Barbosa (2008) defines *reflexive talk* as talk which “refers to the nature of the mathematical model, the criteria used in its construction and the consequences of these criteria.” ‘Modeling’ can be paralleled to a more general, problem solving situation, by allowing ‘criteria’ to refer to the method or approach to solving a mathematical problem. In this (more general) case, in reflexive discussion students refer to the nature of their solution or progress on the problem, the method they used to solve the problem and what they did, and the results of this method. Reflexive talk, then, also includes their opinions of the method and their relation to it, such as how well they felt that they understood it, or how they viewed their role in solving the problem within a group. In this way, reflexive talk can be metacognitive, revealing the speaker’s monitoring of their learning process.

*Challenging talk* is a response to confrontation with alternative points of view. Dekker and Elshout-Mohr (1998) refer to this type of talk as ‘justification’, a key activity, which is a result of criticism, a regulating activity. Challenging talk can also occur as a kind of criticism itself, an argument against a particular view or line of reasoning (Weber, Maher, Powell & Stohl Lee, 2008).

Finally, *parallel talk* is identified and defined by Barbosa (2007), again in the context of student conversations about modeling. He defines parallel talk as any discussion or talk which do not fit into his other defined categories of ‘technical,’ ‘mathematical,’ or ‘reflexive.’ Namely, these are bits of discussion that do not contribute to the solution of a task or problem, but instead refer to students’ non-mathematical perceptions of the theme of the problem, or to their ‘social reality’ outside of school. Parallel talk may not be about mathematics, but it occurs during the mathematical discussion. Although it doesn’t directly contribute to the solution, it does not necessarily detract from the mathematical process, and Barbosa indicates that drawing attention to and asking students to expand on their parallel talk can result in talk that contributes to the mathematical discussion.

These types of talk have been defined in order to better understand student discussion. The expectation is that when students talk about mathematics, they will use one or more of these types of talk to communicate mathematical and other ideas.
2.4 Affect, mathematics, and discussion

The affective domain consists of emotions, attitudes, and beliefs. According to Norman (1981), motivation is derived from the concepts of emotion and belief. In the area of mathematics education, beliefs can be about mathematics (‘mathematics is just a set of rules’) or about self (‘I can solve this problem’) (McLeod, 1992). Common beliefs about mathematics – that it is important, difficult, and ‘based on rules’ can contribute to extreme, intense emotions related to mathematics anxiety. McLeod also suggests that working on changing students’ beliefs about mathematics might benefit self-concept and confidence – concepts related to beliefs about the self which contribute to intrinsic motivation. According to Lampert (1990), participation in mathematical discourse can give students having more control over their own learning and a more authentic experience of mathematics. Both Lampert and Yackel (2001) see discussion as a way to emphasize the actions of mathematical reasoning and argumentation, actions which define the process of mathematics (rather than the product, which focuses on right and wrong). Lampert tried in her own teaching to help students learn a different way of thinking about what it means to do mathematics by initiating and supporting interactions in her classroom, which allowed for explanation, argument, and justification from students. The result was that students did begin to act differently toward mathematics, including in future courses.

In their account of qualitative research with previously failing students who begin to succeed in mathematics in higher education, Povey and Angier (2004) note that student responses indicated that the style of learning at the university in question (co-constructed learning and discussion-oriented classroom practice) was instrumental in changing both their attitudes and degree of success in mathematics. Povey and Angier distinguish four types of student responses to questions about their changing performance and views of mathematics. Two of these types are important in this context: 1) mathematics is negotiable and a subject to explore and 2) learning is social, supportive, and collaborative. In the case of responses of type 1), students noted that their new understanding of mathematics included the idea that a different way of ‘doing it’ (solving a problem, for example) does not necessarily mean that one person is wrong. Students also talked about the benefit of discussing with classmates and going away and doing their own work, as well as the benefit of reflection as a way of “untangling what I am thinking about,” as one student noted, “and saying, ‘well I think this about this because…’” (Povey & Angier, 2004).

According to McLeod (1992), beliefs about the self, as opposed to the subject of mathematics, are related to self-regulation. Pape and Smith (2002) connect mathematical discussion and reflection to self-regulated learning. According to them, a self-regulatory student is an active learner who is able to analyze tasks and set goals to accomplish them, making judgments of their own progress along the way. In the context of
mathematics (e.g. problem solving), this means the ability to analyze relationships between parts of the problem and choose an appropriate procedure or “strategy.” Although this particular view of self-regulation seems to take a bit of a static view of mathematics, it is true that problem solving requires knowing where to start (whether that be the right or wrong way is not necessarily the point), and an ability to set goals and monitor progress. Also according to Pape and Smith, the development of self-regulation is facilitated when students are asked to explain and justify their work. This idea is credited to Fennema, Sowder and Carpenter (1999), who suggested that students must reflect on a problem in order to articulate it. Pape and Smith report on a 1998 strategy-embedded developmental mathematics course with underprepared students which, along with learning to take lecture notes, read texts, and be aware of resources, students also learned to take responsibility for their progress in mathematics by reflecting on experiences and connecting outcomes and results to actions rather than abilities. The effect of the growing sense of control exhibited by these students was an increased perception of their own ability to learn mathematics. Becoming more self-regulated learners, they began to believe that they could be successful in studying and learning mathematics.

2.5 Practicability

One criterion for assessing a teaching or tutoring method is its practicability. A method is *practicable* if it is “able to be done or put into practice successfully; feasible; able to be used; useful, practical, effective” (Practicable). I will focus on the last three terms in this definition, as they clearly describe a multi-faceted view of practicability: For a method to be practicable, it must be practical, useful, and effective. The issue of effectiveness is an important one, since I have already made clear that this research will not be about effectiveness in terms of improving student performance. So then in terms of practicability, what are we looking for when we judge for effectiveness? Since I want to explore the use of discussion, there must be some discussion to explore, so one measure of effectiveness is whether or not the method, or type of intervention being used by the tutor produces talk and discussion among students.

Dekker and Elshout-Mohr define two types of teacher interventions: *process help* and *product help*. Their research was done in the context of their process model (Dekker & Elshout-Mohr, 1998), and process help includes teacher interventions which focus on encouraging students’ to engage in key and regulating activities (2.2). This type of help is made clearer when contrasted with product help, which is more focused on the mathematical content of the classroom activities, and instead of simply encouraging student to take part in interaction and collaboration, the teacher performs regulating and scaffolding activities themselves to support students’ problem solving processes. If the desire is that students collaborate and participate in the types of talk defined here, an effective method will include the type of
instruction described as process help, and minimize the amount of product help, which includes more direct explanations from the instructor and less incentive to interact with peers.

2.6 Relation to this research

This theoretical framework is meant to support the aims and help guide the design of the research that will be presented in the following pages. The main goal of this research is to determine the potential for using discussion as a tool in tutoring algebra. In answering this question, it is important to explore the discussion among the students themselves. This framework clearly defines what is meant by talk and discussion, and so what it is that will be explored. Previous research on discussion in mathematics classrooms also reveals the potential benefits of student discussion in mathematics in terms of both learning and affect, which motivates using discussion in a new context, and provides a list of things to look for in students’ talk to reveal the practicability of discussion as a tutoring tool. Finally, the literature contains ideas for encouraging discussion and developing relationships among students which allow for discussion in the classroom, which will help guide the design of this research.
3 Research Design

In this chapter I will describe the setting of the research, and present the research questions. I will also describe the teaching methods, data collection, and analysis methods which were used, and give some pertinent information about the students who participated in the research.

3.1 Research setting

The community college system may not be familiar to many readers of this research, so I will provide some information on the system in the state of California, and describe the particular college in which the research took place, in Santa Cruz County, California.

3.1.1 California Community Colleges

According to the California Community Colleges Chancellor’s Office website¹, the state’s community college system is the largest system of higher education in the United States, made up of 112 colleges throughout California and 2.9 million enrolled students. Community colleges in California, and similarly elsewhere in the U.S., provide basic skills education and training for skilled jobs. They also provide courses which prepare students to transfer to four-year colleges and universities with advanced standing and prepared to take upper-division courses. Community colleges also offer opportunities for lifelong learning, including non-credit courses for the surrounding communities. These institutions are public, affordable compared to 4-year public universities, and provide opportunities for financial aid, as well as academic and financial advising.

3.1.2 Cabrillo College

Cabrillo College is a member of California Community College system. It is located in Aptos, part of Santa Cruz County, on the central coast of California. In the fall of 2009, approximately 16,000 students were enrolled at Cabrillo². The college has an ethnically and otherwise diverse student body which reflects the surrounding communities. In 2009, the student body was 55% white, 28% Latino, and 12% other ethnicities. In the Spring of that year, 55% of students were 25 years of age or less (traditional college age), while 22% were 40 years of age or older. Although it is not required for enrollment, about ¾ of Cabrillo students have a high school diploma or higher level of education.

¹ http://www.cccco.edu/
² Demographic and other data can be found at www.cabrillo.edu
Cabrillo serves primarily as a provider of lower division general education leading to associates degrees (Associates in Arts (A.A.); Associates in Science (A.S.)), as well as transfer to four-year colleges and universities. The college also provides occupational education and training, which can lead to completion of associate degrees or certification. Basic Skills Education is provided to prepare students to succeed in college-level courses. This includes any courses deemed below the level of first-year university courses, including Elementary Algebra (Math 154).

A high school diploma is not necessary for enrollment in Cabrillo or most community colleges. Anyone who is 18 or older can enroll, and pursue any of the programs discussed above, including transfer to a four-year university.

Other issues, besides previous education, which may concern community college students more than other university or college students include responsibilities related to jobs and families, as well as the need to commute long distances to attend classes.

### 3.1.3 Mathematics and Elementary Algebra at Cabrillo

In spring, 2009, the year before this research took place, nearly 60% of students taking the mathematics placement test placed into Elementary Algebra or below (‘Essential Math’). During the 2008/2009 school year, the success rate for level 100 classes (algebra, including elementary and intermediate; geometry; tutor preparation; etc.) was 53%. During the spring semester of 2010, when this research took place, there were 15 Elementary Algebra classes offered across the three Cabrillo campuses, including one online course. The enrollment limit for each of the 15 classes was 39 students, and all classes and most waiting lists of students trying to add the course were full.

The Math Learning Center (MLC) at Cabrillo is a tutoring resource open to all math students. Full-time tutors as well as student tutors trained by faculty are available. Students sit at numbered tables and sign in and request a tutor through a computerized system. Tutors stay with each student up to ten minutes per request for help, to encourage independence on the part of the student, as well as allow tutors to get to all students seeking help. Most of the students involved in the research also spent time in the MLC each week.
3.2 Research questions

A method has been designed to answer the following research questions:

Over-arching question: What are the potentials of discussion as a vehicle for basic skills algebra tutoring?

Sub-questions with relation to tutoring sessions:

1. What categories of talk can be discerned within small groups of community college students?
2. Which categories of talk can be related to which types of prompts?
3. What types of (or which) discussion prompts are practicable for use by a tutor?
4. What affective constructs, if any, can be observed within the discussion?

The term *practicability* (sub-question c) was used because it encompasses several aspects which would be required of a successful tutoring or teaching tool: usefulness, effectiveness, and practicality, as described in Chapter 2 (section 2.5). It is important to note once again that effectiveness here refers to how well the prompts encourage student collaboration and talk, and not the improvement with regard to student achievement.

3.3 Teaching Method

To explore the practicability of discussion as a tool in tutoring community college algebra students, five types of discussion prompts were designed to be implemented in a tutoring setting (Table 3.1). A more complete explanation of prompts and list of planned examples can be seen in Appendix A. The role of the prompts was to promote discussion among students. Specifically the prompts are a set of tools for the teacher to use with groups or individual students to induce student engagement in the types of talk defined in section 2.3, the key and regulating activities described by Dekker & Elshout-Mohr (1998), and the mathematical activities described by Horn (2005). The prompts can be thought of as a method of providing process help and avoiding product help (2.6).

Volunteer students from five of 15 algebra classes met twice a week for 1.5 to 2 hours to work on homework problems. The sessions began the first week of March, 2010, and ended the first week of April, after a week-long spring holiday. Meetings took place in one of two reserved rooms in the Student Activities Center (SAC) in the main building of the college. Students sat around a single, large table (Figure 3.1), and were free to talk to one another and ask the tutor questions as needed as they worked on homework assigned by their teachers. All problems assigned by teachers and discussed in the tutoring sessions were from the book *Introductory Algebra for College Students* (5th Ed.) by Blitzer (2009).
The tutor’s role was to 1.) Encourage students to work together by recognizing when students had similar questions and concerns, were working on the same section or problem, or when one student had the knowledge to help a struggling student; and 2.) Administer discussion prompts informally at appropriate points during the session, either to individuals or groups of students. Prompts were implemented in the context of particular problems, as well as more general discussion of mathematical concepts. Prompts were intended to be the primary tool of the tutor-researcher during the sessions, and activities defined as product help, such as scaffolding and hints directed toward the mathematical concepts were minimized.

Researcher preparation for sessions involved solving the problems assigned to the students, predicting potential areas of difficulty, and deciding which prompts would be appropriate to use with which problems. Despite this preparation before each session, there was a large element of spontaneity in the

<table>
<thead>
<tr>
<th>Discussion Prompt Categories</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Critique:</strong> Interpret the meaning and purpose of the problem (Barbosa, 2007; Patrick, 1999)</td>
<td>What might make the problem more: ‘doable,’ interesting, approachable?</td>
</tr>
<tr>
<td><strong>Representation:</strong> Draw or otherwise represent an idea visually, transition from one representation to another (e.g. graphical to algebraic), identify types of representation (Janvier, 1987)</td>
<td>Represent the problem visually (draw it, or…) and describe the representation to your group.</td>
</tr>
<tr>
<td><strong>Explanation:</strong> Describe their solutions explaining ‘how’ and/or ‘why’; look into and compare different methods of solution; Respond specifically (Moschkovich, 2003)</td>
<td>Can you explain what you did?</td>
</tr>
<tr>
<td><strong>Justification:</strong> Defend one’s view of a mathematical situation (Weber, Maher, Powell &amp; Stohl Lee, 2008; Yackel, 2001)</td>
<td>Defend the method you have used to solve the problem.</td>
</tr>
<tr>
<td><strong>Contextualization:</strong> Connect to previous knowledge; make a concept more meaningful. (Carraher &amp; Schliemann, 2002; Van Oers, 1998)</td>
<td>How do you understand this term/concept outside of mathematics?</td>
</tr>
</tbody>
</table>

Table 3.1: Discussion prompt types and examples

### 3.3.1 The tutor-researcher’s role

The tutor’s role was to 1.) Encourage students to work together by recognizing when students had similar questions and concerns, were working on the same section or problem, or when one student had the knowledge to help a struggling student; and 2.) Administer discussion prompts informally at appropriate points during the session, either to individuals or groups of students. Prompts were implemented in the context of particular problems, as well as more general discussion of mathematical concepts. Prompts were intended to be the primary tool of the tutor-researcher during the sessions, and activities defined as product help, such as scaffolding and hints directed toward the mathematical concepts were minimized.

Researcher preparation for sessions involved solving the problems assigned to the students, predicting potential areas of difficulty, and deciding which prompts would be appropriate to use with which problems. Despite this preparation before each session, there was a large element of spontaneity in the
implementation of the prompts. The role of the tutor in this informal situation included judging when and what prompts to implement in the context of the session itself.

![Figure 3.1: Room where most tutoring sessions took place](image)

### 3.3.2 Participants

Eight Elementary Algebra students attended the tutoring sessions regularly. Five of these students were interviewed after the last tutoring session (see section 3.4). Six of the students were female, and two male. There was a wide variety of ages, educational and professional backgrounds, and mathematical backgrounds within the group. An overview of the participating students can be seen in Table 3.2.

<table>
<thead>
<tr>
<th>Name*</th>
<th>Age</th>
<th>M/F</th>
<th>Interviewed?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ainsley</td>
<td>20’s</td>
<td>F</td>
<td>No</td>
</tr>
<tr>
<td>Ariana</td>
<td>30’s</td>
<td>F</td>
<td>No</td>
</tr>
<tr>
<td>Danielle</td>
<td>20’s</td>
<td>F</td>
<td>Yes</td>
</tr>
<tr>
<td>José</td>
<td>20’s</td>
<td>M</td>
<td>Yes</td>
</tr>
<tr>
<td>Kelly</td>
<td>40’s</td>
<td>F</td>
<td>Yes</td>
</tr>
<tr>
<td>Maryanne</td>
<td>40’s</td>
<td>F</td>
<td>Yes</td>
</tr>
<tr>
<td>Nick</td>
<td>20’s</td>
<td>M</td>
<td>Yes</td>
</tr>
<tr>
<td>Sidney</td>
<td>20’s</td>
<td>F</td>
<td>No</td>
</tr>
</tbody>
</table>

*Table 3.2: Overview of student participants. *All student names are pseudonyms.*
The majority of the participants were in their 20’s and had completed most of or graduated from high school, where they had had some experience with basic algebra concepts. Two of the five students interviewed were over 40, and had not been required to complete algebra in high school. Most students also had a full- or part-time job in addition to attending school as a full-time student, and some were single parents with multiple children at home.

### 3.4 Data Collection

This project can be classified as qualitative, exploratory research (Bell, 2009). As is appropriate for this type of research, data collected was in the form of audio recordings of tutoring sessions and semi-structured interviews.

All tutoring sessions were audio recorded. When more than three students were present, two recorders were used to capture individual conversations between students or between a student and the tutor. The data from the tutoring session recordings was used to help answer all four of the research sub-questions and the over-arching question.

A semi-structured interview was also designed to be administered after the last tutoring session to individual students. The instrument includes questions designed to elicit responses which could be used to help answer all four of the sub-questions as well as the over-arching research question. The interview schedule, including the correspondence to the research questions, can be seen in Appendix B. Questions relating to affect (sub-question d) were adapted from Schoenfeld’s questionnaire exploring student perceptions of mathematics and school practice in the context of a problem-solving-related research (Schoenfeld, 1989). The instrument also contains questions not related directly to the research, including about students’ ideas of how they may need math or algebra in the future, their previous experience with tutoring, and their ideas for improvement of this project for future implementations. These were added, and the questions were arranged in an order which allowed the interviewer (here, the researcher) and each student to have a conversation about the student’s experience with mathematics and algebra. Questions specifically about discussion were left for later in the interview in an effort to avoid leading the students towards particular types of answers when asking more generally about the sessions and their opinions about and attitudes toward mathematics.

This project was a form of action research (Bell, 2009). After each session or at the end of each week the data was reviewed and it was noted what actions had been successful and what needed to be changed, with regard to the amount and how much the tutor interacted with the students, as well as which prompts
had been used, which seemed ‘successful,’ and which might be appropriate to try in the next session. With this information, along with weekly algebra assignments that I obtained from the college instructors, it was possible to plan and alter the method slightly for upcoming sessions.

3.5 Analysis
The methods used to analyze all collected data will be described in this chapter, including analysis of the recordings of tutoring sessions as well as student interviews.

3.5.1 Analysis of tutoring sessions
Most tutoring sessions were transcribed, at least in part, and excerpts of student-student and student-tutor discussion from nine sessions were chosen to be transcribed completely. These nine sessions were chosen in order to exemplify the use of the discussion prompts described earlier in this chapter and in Appendix A, the variety of discussion following these prompts, and the various groupings of students which occurred during the sessions. The process of choosing which sessions were analyzed is discussed in more detail in Chapter 4.

The methods used to analyze the transcriptions of discussions were inspired by grounded theory (Strauss & Corbin, 1990; Barbosa, 2007). Student talk, and occasionally tutor talk, arising after the use of the discussion prompts was categorized. The process of categorization, or coding, was similar to ‘focused coding’ described by Walter and Hart (2009). However, while they determined categories from the data itself, the categories of talk were defined from the literature before beginning the data collection. As discussed in section 2.3, the categories of talk which we might expect to see in classroom discussion include exploratory, explanatory, reflexive, challenging, and parallel. These are briefly described again briefly in Table 3.3 and Appendix C.

Each student statement in the excerpts transcribed for analysis was identified as fitting into one or more categories of talk. In addition, since prompts were often unplanned, the prompts which began the discussion, or additional prompts within the discussion were also categorized as one of the types discussed in section 3.4 and Appendix A.

Less structured analysis was also applied to the transcripts of the tutoring sessions. The transcripts were read with the intent of observing connections between the types of talk, and other aspects of the situation and discussions which may have impacted the types of talk, as well as the apparent success or lack thereof.
of the prompts. They were also read for signs of affective constructs, including motivation, views of mathematics, and confidence.

<table>
<thead>
<tr>
<th>Category of talk</th>
<th>Defining aspects of student talk</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exploratory</td>
<td>o Improvisational</td>
<td>“Where is this 25 coming from?”</td>
</tr>
<tr>
<td></td>
<td>o Rehearsal of knowledge</td>
<td></td>
</tr>
<tr>
<td></td>
<td>o Not focused on ‘external criteria’</td>
<td></td>
</tr>
<tr>
<td>Explanatory</td>
<td>o More formally descriptive than exploratory talk</td>
<td>“Yeah because, in the first instance [...] the large one has less area than the two small ones. But they’re the same price.”</td>
</tr>
<tr>
<td></td>
<td>o Elaborate descriptions</td>
<td></td>
</tr>
<tr>
<td></td>
<td>o Specific examples for general concepts</td>
<td></td>
</tr>
<tr>
<td>Reflexive</td>
<td>o Referring to the nature of the solution method or progress toward solution</td>
<td>“I’m just confused because I don’t know how I’m going to get this.”</td>
</tr>
<tr>
<td></td>
<td>o Opinions of solution method and how well they understood it</td>
<td></td>
</tr>
<tr>
<td></td>
<td>o View of their own role in the solution</td>
<td></td>
</tr>
<tr>
<td>Challenging</td>
<td>o Preceded by criticism</td>
<td>“It just doesn’t make sense to call it an undefined slope.”</td>
</tr>
<tr>
<td></td>
<td>o Argument against a mathematical viewpoint or line of reasoning</td>
<td></td>
</tr>
<tr>
<td>Parallel</td>
<td>o Does not contribute to the solution of a problem</td>
<td>“Yeah, but I let the contractor do that!”</td>
</tr>
<tr>
<td></td>
<td>o Non-mathematical talk which is not reflexive</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.3: Categories of talk used in grounded-theory-inspired discussion analysis

3.5.2 Analysis of student interviews

All interviews were transcribed in their entirety. Responses which contributed to answering the research questions were grouped first according to research question, then according to interview question. Analysis was structured in the following way: Once transcripts of student interview responses to particular questions were grouped, similarities and trends among the responses in a given group were identified, as well as outliers, or students who had much different opinions or responses than the others. The responses were also connected and compared to the tutoring transcripts in order to confer or deny the
analysis of the tutoring sessions, as described above, in terms of the practicability of prompts as well as issues of affect.
4. Results

In this chapter I will present the data from this research. The data presented were gathered during tutoring sessions and student interviews.

I recruited students to participate in tutoring sessions through class visits where I made a brief announcement and had interested students fill out a form indicating their availability. In the week before the announcements, I met instructors and sat in on classes. I used the forms which students submitted to create a schedule and invited students via email to attend time slots which seemed to fit their schedule.

Tutoring sessions were bi-weekly, from 1.5 to 2 hours in length. Students from five classes (five different instructors) participated on a volunteer basis, and work done in the sessions was determined by what students needed to complete for homework. Homework problems were assigned by all instructors out of the same textbook: *Introductory Algebra for College Students* (5th Ed.) by Robert Blitzer. Unless otherwise stated, students sat around a single, large table, sometimes in pairs, as will be discussed in the following section.

All tutoring sessions were recorded. Selected sessions were transcribed, and the transcriptions analyzed. Excerpts from these transcripts are presented here in section 4.2.

After the final tutoring session, I interviewed five of the participants. Completing the interview schedule, which can be seen in Appendix B, required between 30 and 45 minutes depending on the student. I transcribed the interviews in full and analyzed the transcripts. Select responses and analysis are presented in section 4.3.

4.1 Attendance and group work issues

Students working together often had different instructors, which meant that they might be a little ahead or behind one another in the text. This, combined with poor attendance, contributed to a lack of opportunities for students to work together on the same problem. Regarding attendance, since there was quite a bit of interest in my project after my visits to and announcements in classes, in an attempt to accommodate as many students as possible, I organized three bi-weekly sessions. My intent, which I made clear to students, was that they join one of the three and remain committed to it for the entire time (about six weeks). However, although many students expressed interest in the tutoring sessions, very few showed up at the sessions, even when their attendance was confirmed by phone and/or email. Therefore I
allowed the few committed students to come to as many sessions as they liked in an effort to have a maximum amount of students present at one time. This meant that students did not have a chance to get to know each other and form a community, as I had hoped. Also, despite this concession, attendance was rarely above three people, and often, especially during some of the first sessions, only one student attended. A total of 25 sessions were held (one time slot began a day late due to location issues). Five of these sessions were not attended by any students, five sessions were attended by only one student, six sessions were attended by just two students, and the remaining nine sessions had an attendance of three to four students.

4.2 Tutoring sessions

The following discussions were recorded during the tutoring sessions which took place throughout the month of March, 2010. The recordings were transcribed and analyzed using the discussion prompts and categories of talk described in Chapter 3 (also see Appendices B and C). Reported sessions are listed chronologically. Due to the attendance issue described above, quite a few of the discussions which take place during these sessions are student-tutor discussions rather than between or among students. The discussion excerpts are introduced with an explanation of what was going on in the session at the point when the discussion took place, and the entire session surrounding the excerpt is briefly described, including less successful attempts to prompt discussion.

In 11 of the 20 sessions which were attended by one or more students (see section 4.1), few discussion prompts were used. Although discussion prompts were planned for every session, during some sessions there were fewer opportunities to apply them and the session took on a more traditional style, where tutoring primarily took the form of product help (as opposed to process help; see section 2.3). In the remaining nine sessions, I was able to apply the prompts. These will be reported below. The nine reported sessions were chosen in the interest of giving an overview of sessions with varying numbers of students, as well as a variety of examples of the five different types of prompts. In the transcripts below, ‘C’ refers to the tutor-researcher.

4.2.1 Week 1: March 3, 2010

Only one student, Kelly, attended this session. She was working on simplifying variable expressions using the order of operations. Until about 30 minutes into the session, Kelly worked through problems, sometimes on her own, sometimes talking through them out loud and asking me for reassurance. I tried prompting discussion by asking her to explain her method (Explanation prompt), but she didn’t seem to have the confidence to describe her own method, and instead in these cases responded with questions of
her own and did not give complete or elaborate explanations.

After simplifying one particular expression, Kelly wanted to take one more step and combine two unlike terms: $3x + 3$. I tried explaining why she ‘couldn’t’ do this in a few different ways, including comparing this situation to adding apples and oranges, or $3x + 3y$, which she agreed couldn’t be done, as well as reminding her that $3x$ is just $x + x + x$. She claimed that her instructor had explained this last point in class, but Kelly said that she didn’t “see it,” I asked her to explain what she needed clarified, but she was not able to be more specific. After giving various examples of the relationship between multiplication and addition on the board, keeping Kelly involved by questioning her about each example, she indicated that she thought of $x$ and 1 as the same thing, so I introduced a new variable, $a$, via her suggestion. The new variable made combining like terms clearer to Kelly, but she still said that she “doesn’t see the logic” of not combining unlike terms. Given that she volunteered that there was a lack of logic, I wanted to prompt her to explain her own logic:

1. C: (Discussing the expression ‘$5a + 5$’) If you want to explain to somebody ok ‘I want to put these together,’ you know just from (your instructor) telling you that you can’t combine these together, right?
2. Kelly: right.
3. C: But you still kind of want to, so how would you justify combining them together. Could you…think of a reason why? 00:36:36-4
4. Kelly: No, I've been told... no justification! (laughs) It doesn't work! I get laughed at! So I have like a whole number, no variable, I have a 3 (referring to the original problem) times an $x$. So technically, going by that idea that I can't combine unlike terms, then I can't do this (multiply 3 times $x = 3x$).
5. C: You can do that, because you're multiplying.
6. Kelly: Ok - then I'm going to argue... if I can do that, why can't I...? (referring to combining $3 + x = 3x$).

Since Kelly was consistently making an error by combining ‘unlike’ terms, such as $3 + x = 3x$, my intention was to prompt her to justify this action and in turn challenge her justification. Using a justification prompt (line 3), I pushed Kelly to challenge the rules that she was being asked to take for granted, and encouraged her to have the confidence to explain her own understanding. Kelly’s resulting talk was initially reflexive, insisting that she knew that her method was wrong, and in fact it would be “laughed at.” This talk transitioned into exploratory talk (lines 4, 6) as she began to reason through a description of her understanding. Although this was only a discussion between teacher and student, it enabled me, the instructor, to gain a better understanding of what sense this alternate method was made to Kelly, and a better idea of what to use as a counterargument to increase Kelly’s understanding.
As this discussion progressed, I asked Kelly to think about the meaning of $5a$ vs. $5 + a$, by which her concern (If multiplication means addition, why doesn’t addition mean multiplication?) became extremely clear. I responded that this was a good point, and gave her my own argument, by translating the above expressions into language: “five a’s” and “five more than a.” I also plugged in some values for $a$ as counterexamples in order to make the concept concrete. Kelly indicated that this helped, that she was “really thinking about it,” and that she didn’t remember ever going “this far” with it. After this she worked silently on similar problems, and the session returned to a format similar to that from before the excerpt, where most of our discussion was about understanding and remembering the order of operations, as well as operations with signed numbers. Since Kelly’s prior profession had been as a bookkeeper, at times we referred to positive and negative numbers as ‘black’ and ‘red’ instead, which put what she saw as very different math into a familiar context. This is not, however, reported as a use of a contextualization prompt because Kelly herself introduced this context without a prompt from me, the tutor, to induce this discussion. The issue of unlike terms came up later when she was afraid to ‘combine’ $3$ and $x$ by multiplying, and I reminded her of the language behind $3x$ and $3 + x$ as I described above.

4.2.2 Week 2: March 8, 2010

The students attending this session were Maryanne, José and Kelly, who arrived late. Before the following excerpt, Maryanne and José, who have the same instructor, discussed their class and their frustration with math and the material, their desire to be done with math courses altogether, how age effects the ease of learning (Maryanne was much older than José), and their course loads. Maryanne was concerned about finishing the homework. She had skipped a lot of problems because she had no one to help her, so she had a list of problems that she was prepared to ask me about. I suggested that, when possible, the two students work together. After beginning their homework, José helped Maryanne with a problem he had already done. I attempted an Explanation prompt, asking if there was another way to do the problem, which elicited a simple response of “I don’t know;” so I showed another method on the board, comparing it to José’s method. José and Maryanne worked together off and on throughout the session, usually with José helping Maryanne with something he had already done. However, José wanted to make progress on his own work, so I spent a lot of time helping them individually to set up and solve problems. They were each aware of the other’s discussions with me, interjecting and giving helpful feedback at times, even though they were working on different problems. Maryanne solved most homework problems by finding something similar in her lecture notes and setting up the book problems exactly as the instructor had done. While working in this way, Maryanne read the following problem from her book and immediately asked for help before trying any method of her own:
The American football field is a rectangle with a perimeter of 1040 feet. The length is 200 feet more than the width. Find the width and the length of the rectangular field.
(Blitzer, 2009)

She felt that she should have known how to solve this and that it had something to do with the width and length. She also noticed that a problem about a basketball court from her lecture notes was similar, but she decided not to use this example as a guide for solving the football field problem for fear that she would confuse the two contexts.

1. Maryanne: [...] I look at it, and I don't even know where to begin. But I know its length... isn't it like length times width...?
2. C: Well, have you ever had to do anything like this before? Like Prompt (Co)
   anything where you had to figure out how much of something... have you ever had to buy a fence, or...
3. Maryanne: Yeah but I let the contractor do that! Para
4. C: So you... ok.
5. Maryanne: That's his job. I did get a fence like last year! I put a whole fence Para
   around my whole yard... that's his job.
6. C: So you didn't measure anything... just said “you measure, you buy...”?
7. Maryanne: Yeah - here's a check! (laughs) Para
8. C: You just trust him?
9. Maryanne: Well, I mean I have a pretty good idea. I can go out there with a Explore
   measuring tape and go 'ok from here to here it's this many feet... from here to here it's this many feet... Explore
   from here to here it's this many feet' (pantomiming measuring three sides of a rectangle). And then I take those
   three different feets that I got, and I add 'em up.
10. C: Do you see how that might be similar to this? Prompt (Co)
11. Maryanne: Yeah like if my... fence was in this shape (points to rectangular Explore
    "football field" which she has drawn on her homework paper) around my back yard. And say this was 8ft, 8ft (the two 'widths') and this was 20 feet across the back, well I know that there's 16 and 20 and you put those together and you get 36.

As with the previous excerpt (4.2.1), this is a teacher-student discussion. A contextualization prompt (line 2) was used with the intent of prompting the student to consider an alternate ‘real-life’ context with which she may have had experience (unlike the ‘real’ contexts of the basketball court and football field). As a result of the prompt, parallel talk regarding a personal experience of getting a new fence for her yard progressed into exploratory talk about finding approximately how much fencing she would need and would pay the contractor to buy. A second (re)contextualization prompt was used to return the discussion to the context of the football field (line 10), and the student was able to use exploratory talk to connect the two contexts by relating her previous exploratory talk to her diagram of the football field (line 11).
After this discussion, Maryanne required step-by-step guidance to write out the equation for the perimeter of the field and solve, the difficulty being primarily with considering the variable expressions as numbers themselves. To guide Maryanne to complete the problem, I helped her to put her verbalized statements, similar to the descriptions she gave in lines 9 and 11, above, into written expressions. During this episode, José worked on his own, across the table from Maryanne, and was not involved in the above discussion.

4.2.3 Week 2: March 8, 2010

Soon after the discussion described in 4.2.2, Kelly arrived after completing her first math exam (she had a different instructor than Maryanne and José). Maryanne was working independently on the following problem:

*If the quotient of three times a number and five is increased by four, the result is 34.*

*(Blitzer, p. 156)*

After setting up an equation, which she did by talking through it with me, and solving it up to the point \(\frac{3}{5}x = 30\), Maryanne got stuck. I wrote the equation on the board, asking Kelly and José to come up to the board (and closer to where Maryanne was working) to give their input. José solved the equation on the board by multiplying both sides by the reciprocal of \(\frac{3}{5}\). He made a multiplication error, but the discussion proceeded without anyone noticing. José and Kelly remained at the board and Maryanne in her seat nearby, and the following discussion took place.

1. **C:** Is there any point in there that... you don’t understand how he got to the next (step)?
2. Maryanne: (with Kelly) No. […]
3. Maryanne: Well, it's just... really what you're doing is multiplying both sides by 5. Well you could have just done it by 5!
4. **C:** Could you... (to Kelly, who seems to agree) Do you agree... she could have just done it by 5?
5. Kelly: That was kind of where I was going, but his... it works out the same way. […] You want to try it? (picking up a pen), see, it still does it. (Writes out problem on board and starts to solve). Multiply by 5... It's going to be a little longer on this side, I think…

Kelly worked through the problem on the board with interjections and help from José and Maryanne. Meanwhile, Maryanne was also doing the problem on her own paper.

6. Kelly: Yeah... I guess it doesn't come out right. You still have to... 'cause if you're going to do a fraction, it's gotta be a fraction, right? […]
Talking about Mathematics

7. José: But there's still... you still have the 3x.

8. Maryanne: Oh, I see, and I didn't cancel the 3... 3x... oh you're right!

At this point in the discussion, Kelly and Maryanne finished solving to find that the answer was 50, but they were concerned that it didn’t agree with José’s solution and assumed that their solution was incorrect. At this point I decided to interject to suggest that José may have made a careless error and he began to go back through his work on the board.

9. José: 10 times 5 is 50... x equals 50. It IS 50! (To K) It would have been wrong if you didn't do your math.

10. Kelly: ...talking about it really helps us to connect... missing dots.

11. Maryanne: Well... and even making a mistake helps... because that's something that will help me remember it. Whatever works in this case, let me tell you.

Initially (line 1), I used an Explanation prompt to ask Kelly and Maryanne to look critically at José’s mathematics and make sure that they understand each step. Although Kelly and Maryanne’s initial verbal response was a simple “no,” they also both took a close look at José’s solution and suggested the existence of an alternative method. Sensing that this discussion would end with this mere suggestion, I reinforced the idea of a possible alternative method, giving my own suggestion that they continue in that direction (a second Explanation prompt, line 4). The ensuing discussion was exploratory. The students used their own, incomplete and informal language to describe what they thought might work (line 3), to describe what they were doing to accompany their actions on the board (line 5), and to help each other (line 8). Finally, in lines 10-12, all three students used reflexive talk to discuss how the process of working together on this problem helped them, both to understand and to come to a final solution, with José acknowledging that they had used multiple methods, referring to Kelly’s method as “your math” (line 10).

During the remaining hour, Kelly and Maryanne had some short discussions about their past experiences with algebra, which were caused by general frustration with their work. The two of them also discussed the math requirements for the degrees they were pursuing, arguing that math is not necessary for them and expressing frustration that they were required to take it at all. Maryanne even suggested that she had considered quitting school altogether because of this particular introductory algebra course.

These parallel discussions mentioned above were short and interspersed throughout the tutoring session. For the remainder of the session, all three students worked primarily independently, although they
communicated about practical things such as assignments and due dates. I moved around the table, helping them individually by checking in and responding to their questions.

4.2.4 Week 2: March 9, 2010

Two students, Nick and Danielle, attended this session. Later on, Nick and Danielle would often attend the same sessions, and the following account, including the discussion excerpt, is somewhat typical of these future sessions. Throughout the session there was little communication between the two students. Nick and Danielle were members of the same algebra class with the same instructor, and were both working on the geometry section of the book, where the algebra concepts they had learned were applied to problems in geometry (area of polygons and circles, volume, properties of triangles). During this session, I tried prompting discussion between the two students in a number of different ways, none of which resulted in significant student-student communication. For example, prompting them to compare answers was barely acknowledged. Danielle at one point communicated on her own, asking Nick if he had figured out how to do a particular problem. He gave her a short, procedural answer. I prompted Danielle to question him more and asked him myself to elaborate, but neither one was productive. Later I asked Danielle about that particular problem and it was clear that she had not fully understood Nick’s directions and was still puzzling over it, although she had not asked any clarifying questions. At one point I prompted them to work on a problem together. Danielle’s reaction was to say “it’s so easy! You already know the answer just by reading the question.” This comment seemed to cause Nick to become disinterested. At this point, I tried a Justification prompt, encouraging Danielle to defend her method. However, when asked to do this her confidence in her method waned and she admitted that she had gotten “weird answers.” Nick began working independently on the problem while Danielle and I talked through it on the other side of the table. After Nick felt he had solved the problem sufficiently, he asked Danielle what she had found the answer to be. A similar exchange occurred while we all worked on the following problem.

Which one of the following is a better buy: a large pizza with a 16-inch diameter for $12.00 or two small pizzas, each with a 10-inch diameter, for $12.00? (Blitzer, p. 213)

We had different ideas of what the answer should have been, and I decided to prompt both students as well as myself with an Explanation prompt, proposing that we all compare our methods. Nick did not respond and worked silently until he had gotten the same answer that I had, leaving Danielle to discuss her method with me.

1. C: **If you guys didn't get the same thing then we should compare.** Prompt(E)
2. Danielle: Yeah because, in the first instance [...] the large one has less area than the two small ones. But they're the same price. Explain
3. C: [...]... But I found that the 16 inch pizza had more area than the two little ones combined.


5. C: What did you do?

6. Danielle: I did, um, I combined the two radiuses, so it's 10...

7. C: Oh! That's interesting, ok, so... you did... I didn't do that. I said... one of the small pizzas has a radius of 5, so that means that the area is 25π inches squared [...]  

8. Danielle: Where is this 25 coming from?

9. C: π r squared.

10. Nick: Oh! It's a much better deal to go with the large pizza.

11. C: Why do you think that?

12. Nick: Because I did the math (laughs). 

Asking Nick and Danielle to compare their methods with each others’ and mine was an Explanation prompt (line 1). Danielle responded (line 2) to the prompt by explaining why she thought her answer made sense, which was different from mine, and I in turn explained my answer to Danielle. She used exploratory talk to discuss why my answer was different from hers and to understand my method (lines 4, 6). My talk was similar, although I explained my method more specifically and asked Danielle questions about her method (another Explanation prompt, line 5). Nick, on the other hand, did not take part in comparing methods and instead went back through his own work and interrupted excitedly with his answer. I attempted to prompt him to explain his method, but he answered reflexively, though not very revealingly, that he had “done the math.”

I attempted to use a Contextualization prompt to discuss the possibility of a restaurant ‘tricking’ customers by using a pricing scheme similar to the one in the problem, but the idea was not taken seriously by the students and there was no significant talk following the prompt. For the remainder of the session, Nick and Danielle worked independently and I helped them both individually to interpret and solve word problems in the geometry section of the book, including showing brief, similar examples on the board. They did not work on the same problem simultaneously again for the rest of this session.

4.2.5 Week 2: March 11, 2010

Two students, Nick and Ainsley, attended this session. Before the following excerpt, Nick was studying for his first exam by working on a practice test, specifically problems regarding definitions of the subsets of the set of real numbers. Ainsley was working on corrections for her most recent exam. What led to the discussion below were the final steps of simplifying an expression, which was marked incorrect on Ainsley’s exam. Ainsley knew from class discussion that the expression \( \frac{5n}{5n} \) should simplify to 1, but she
was confused because in sharing with her classmates after getting their papers back, she saw that a fellow student received full credit for the expression $\frac{n}{n}$. I asked her first to simplify $\frac{5}{5}$, which she quickly said was 1. I then asked her if/how the expression $\frac{5n}{5n}$ was different. Since she was not sure, I decided to bring Nick into the discussion:

1. C: Let's ask our partner over here... (to Nick) So, do you agree that this is 1 (showing $\frac{5}{5}$)? ...**How is this different** $\frac{5n}{5n}$? **Prompt (E)**
2. Nick: Because you don't know what $n$ is. $n$ could be... 7... $n$ could be 5. So. 5 times 5 is 25... Well, yeah, it is 1, because it's always going to break down to 1. Yeah, it is. **Explore**
3. Ainsley: Ok. But the answer here is $n$ over $n$... I guess, because I looked at someone else's paper that got it right. So how does that work out? 'Cause my answer was $1n$. **Ref**
4. Nick: No that's not right, because say $n$ equals 3. So 3 over 3 is... **Explain**
5. Ainsley: Is 1. **Explain**
6. Nick: Is 1. And it does not equal 3...1... 3 times 1. That's 3. **Ref**
7. Ainsley: Ok. Alright. I'm just confused because I don't know... how I'm going to get this. **Ref**
8. Nick: What are you lost on? **Ref**
9. Ainsley: I mean I understand what you're saying, but the $n$ over $n$ part throws me off. I don't think I would ever arrive at $n$ over $n$. **Ref**
10. Nick: Don't think of it as being abstract. Like it's not abstract. It's just being replaced by something else. It's just a number being represented by a letter. That's all it is. It could be anything in the world. **Explore**

A prompt (explanatory, line 1) is used to bring another student into the discussion. Nick’s response is both explanatory and exploratory. Ainsley is primarily the listener, but she responds with reflexive talk expressing worry over her understanding of what is being explained (lines 3, 7, 9). Ainsley’s talk seemed to promote Nick’s use of exploratory talk in order to make the concept clear to both himself and Ainsley. Nick also uses exploratory talk to describe to Ainsley his understanding of the concept of variable (line 10).

Ainsley worked independently for the remainder of the session, asking me short, procedural questions about the exam problems she was correcting. Ainsley left as soon as she finished correcting her exam, and Nick remained. There was some student-teacher discussion, notably the use of Critique prompts applied to applications used in word problems. For example, I asked him, “**who has to use this kind of stuff?**” in the context of a question regarding an auto parts bill and the price of labor, to which he replied “**no one**” since in his opinion the mechanic would provide the information which he had been required to solve for in the problem. Nick’s talk in response to this prompt could be classified as reflexive and parallel.
4.2.6 Week 3: March 15, 2010

One student, José, was present at this session. He was working on the first sections of the chapter introducing linear equations, which involves evaluating equations for different values of $x$ and completing and graphing from tables.

Early attempts at prompting discussion were not completely successful. For example, after guiding José through graphing the point $\left(\frac{5}{2}, \frac{7}{2}\right)$ by ‘converting’ the $x$ and $y$ values into decimal numbers, and estimating halfway between the units on the coordinate plane, I asked him if he thought that there might be another way to do this (Explain prompt). To this he replied “I’m sure there is. What is it, though? Show me.” This type of talk was typical of José, however, he often needed less showing than he indicated, and just a small suggestion or question on my part would produce an elaborate explanation or exploration in reply.

After completing a set of graphing exercises where he created tables of values for equations which were given in the form $y = mx + b$, I prompted José to consider the relationship between the equation and the graph:

1. C: **Do you think you could have predicted which line would be steeper if you just looked at the equations? What do you think the equations have to do with how the lines look?**  
   Prompt(R)
2. José: Well, the higher the number is the more slanted it goes?  
   Explore
3. C: Which number?  
   Explore
4. José: The higher the $y$ is. Well if you have a short, like, 2, and a 10 $y$, you know, it's going to be really, like, slanted. But if you have a long $x$ it's going to be more like horizontal.  
   Explore
5. C: Oh, so you mean if you plug something small in for $x$ and you get a big $y$, then it's going to be steep.  
   Explore
6. José: Yes.  
7. C: So in number 61, you plugged in a small $x$ and you got kind of a bigger $y$, so...  
   Explore
8. José: The only positive number that I plugged in, which was two, what I got was a 5, which is pretty high. I had to multiply by two, so that's why I knew it was gonna be higher so that's why I didn't want to put a higher number right here (as one of the choices of $x$ in the table) ’cause then I was going to have to make the graph bigger.  
   Explain

The prompt in this case is representational. I asked José to consider a connection between the graphical and symbolic representations of a linear equation. José responded initially with exploratory talk (lines 2,
4), and as I expressed understanding of his thinking, he proceeded to explain the precise method which allowed him to predict which line would be more “slanted” (line 8), connecting the two representations via the third, numerical, representation. Although he hadn’t yet begun to use the mathematical terms (slope and y-intercept), José was able to explain why the line would be steeper and successfully transition among three different representations.

José needed little assistance and worked on his own for the remainder of the session, although he did make remarks about the equations he was graphing and we had brief discussion about which numbers would be ‘smart’ to pick to plug in for \( x \), by considering how to make the math easier (fewer fractions!) and what to expect for the \( y \) value (keep it small enough to fit on the paper).

Later in the session I used a representational prompt to begin a discussion about where the lines cross the axes in which José determined that a line will cross through the origin if the \( x \) is “being multiplied by a number,” but not “subtracting or adding another number.” In fact, there was an extended conversation throughout this session during which José became apparently more comfortable and more willing to describe his observations and to hypothesize and explore.

We also talked briefly again about slope (but using the term ‘steepness’), and José seemed to have retained the idea that a larger number multiplying the \( x \) (whether negative or positive) indicated a steeper line. We ended the session by talking about the prospect of working with other students from his class, since he indicated that working with me helped him feel less stressed about the homework, but José indicated that it was difficult to make plans with people from his class because he didn’t know any of them very well, and that everyone had their “own life” and were too busy.

4.2.7 Week 3: March 16, 2010

Four students attended this session: José, Nick, Maryanne, and a student who attended very rarely, Sidney. For most of the session, Nick and Maryanne worked near each other and often together, and José and Sidney sat near each other and helped each other sporadically, especially when José was working on problems similar to those Sidney had already done. I went back and forth between the two pairs, and recorded them separately. About 30 minutes into the session, Nick and Maryanne had completed the following problem:

\[
A \text{ lab technician needs to mix a 5% fungicide solution with a 10% fungicide solution to obtain a 50-liter mixture consisting of 8% fungicide. How many liters of each of the}
\]
fungicide solutions must be used? (A table with missing entries is provided). (Blitzer, p. 192)

I prompted discussion by asking them the question, “Are problems that directly relate to your life easier to solve?” I explained that I was curious because I assumed that an application relating to fungicide had little to do with their lives, and I wondered whether other applications seemed easier, giving the example of a pizza problem we had worked on in a previous session as an application they might consider to be more familiar. A variety of talk resulted from this Critique prompt, including reflexive, exploratory, and parallel:

1. Nick: No. Math is math is math, isn't it? Is it relevant that you actually do go out and buy pizza and you might be saving money...? 00:40:20-8

2. C: Maybe the actual problem isn't relevant, but does the fact that it's about something that is familiar make it easier to work on?

3. Maryanne: I think it would for me.

4. Nick: I think it makes it tangible. Because math is an abstract subject. […]

5. Maryanne: See for me it would be better if they used diapers... and... you know, because I'm a mom, I've got 4 kids. Or if they used, 'You go to the grocery store and you have to buy...' you know, things that would be more relevant to my life, then I can think about them... 'cause I'm very visual. It doesn't work that way for me, like you just said (to N), 'math is math is math'.

6. Nick: Well it is an abstract subject. And if you can make it relate to your life... like, um, 4 is 4 dollars to me (referring to a recent problem), minus .5 which is half a dollar...

7. Maryanne: Right. See and I had to really work on that to see that in my head.

8. Nick: (asks C how math makes sense to her)

Maryanne’s parallel and reflexive talk centers around analysis of her own life and learning style (lines 3, 5), and she reflects on how difficult it is for her to understand numbers as money, something that is easy for Nick (line 7). Nick’s talk is more conceptual and primarily exploratory. He contemplates with reflexive talk the abstractness of math (lines 1, 4), and in response to Maryanne’s analysis of which applications would be relevant for her, Nick uses explanatory talk to describe a relevant situation for him (money, line 6).

This was also Maryanne’s last session. She dropped the class later this week, and during this session she expressed numerous times that she was anxious about how long problems like the one quoted above were...
taking. After the above conversation, Nick and Maryanne started working on another problem about an alcohol mixture:

*How many ounces of a 15% alcohol solution must be mixed with 4 ounces of a 20% solution to make a 17% alcohol solution? (Blitzer, p.193)*

The following discussion took place as Nick and Maryanne worked together toward a solution to the problem above. They worked together, using a table that Maryanne had set up with my help, determining which expressions they needed to fill it in. As they progressed through the problem, I gave them a little bit of guidance, similar to line 3, below, but only when they got stuck and motioned me over to their end of the table. José and Sidney sat near the other end and I moved between the two pairs of students as they worked. The following is an example of an unprompted student-student discussion.

1. Nick: This is a tricky problem. So, with the other problems, where do you go after you get to this step? 01:07:45-5
2. Maryanne: Well we did it differently, because the chart was set up. So we copied the chart. It's easy just to fill in the blanks. That's basically what we're doing - filling in the blanks of the chart. We're supposed to get an answer here (pointing to one of the blanks in the table). (To C) This is what makes me want to just drop (the class). [...] Maybe the math lab would have been better... because they're trained to just sit there and show you how to do it. Because I would never spend this much time and it's making me anxious to spend way too much time on it.
3. C: [...] **How would you use this to set up an equation?**
4. Maryanne: Like we did over here. (Nick and Maryanne work together to set up an equation using the table and then solve it) [...] And now we're going to divide the 2 and that's how we get the 6 (ounces). [...] But see that was just ridiculous. It shouldn't take me like half an hour... I need to have a different strategy.
5. Sidney: As long as you get it, it will go quicker. You need to like solidly get first.
6. Maryanne: (talking over) Yeah but see I spent too much time on it to get it. Now I don't even know where we're starting from. [...] I need to be taught how to set it up to the problem. And then, once I get to here (solving the equation), I could figure that out.

The parallel and reflexive talk above (lines 2, 5-7) was unprompted, in that it was not induced by a prompt used by me, the tutor. The Explanation prompt used here (line 3) was not the cause of any significant talk. Regarding the remainder of this session, from my point of view, there had not been too much time spent on this problem, and the two of them made quite a bit of progress on their own. I made this opinion clear to Maryanne and encouraged her to start on another similar problem, which she did. She
made significant progress on her own, more quickly and with greater ease. However, she decided to leave before completing it to get help from the college’s Math Learning Center (MLC) (“math lab,” line 2).

4.2.8 Week 3: March 18, 2010

Only one student, Nick, attended this session. Nick was working on graphing linear equations using intercepts. At times he predicted what the intercepts would be without calculating them, so I prompted discussion by asking him to explain his predictions. This resulted in short answers such as “I don’t know” or “the answers usually look like this.” A second Explanation prompt was unsuccessful in getting him to elaborate. This session was similar to the session with José which was described in 4.2.5, where we had an ongoing conversation about the material he was working on, although he solved many problems independently.

About one hour into the session we began discussing horizontal and vertical lines. The problems gave graphs of horizontal and vertical lines and asked the student to write equations for them. This was the first point in the book that students had been directed to go from the graphical representation of the line to the symbolic, so I used a Critique prompt, asking Nick why he thought they were asking him to do this task. He answered with exploratory talk: “So we understand... x and y intercepts?” He then immediately began successfully writing equations for each line, beginning with the horizontal line $y = 3$. I then used an Explanation prompt, asking, “How would you describe the relationship between x and y?” With some further explanation of the question, he decided that the y value was constant and the x value was changing, “because it’s just going along that same line... there’s no slope to it. Does it have a negative slope? No I guess it doesn’t. It’s an even slope.” Nick then suggested the term ‘slopeless,’ which we laughed about. This set the tone for being critical of the mathematical terminology. The next line was vertical, and he wrote an equation indicating a constant value for x. I asked, “If this has no slope, what about (the graph he had just written the equation for)?” and how he would describe it in terms of slope (Explanation prompt), to which he replied that it was a “perfect slope... because it doesn’t deviate off of the axis”:

1. C: Well if you saw that... do you ever think of slope as something in real life? 01:01:04-0
2. Nick: Like a building. Like the angle of a building... is going straight up... the slope would be...
3. C: You mean like a wall?
4. Nick: A wall. Like this wall. It’s going straight up. So it’s going at a perfect slope (laughs). [...]
Here we discussed whether you could climb up a ‘perfect slope’, and Nick wrote down equations for the remaining horizontal and vertical lines. Later in the session, Nick began working on a group of problems with the following directions:

*Find the slope of the line passing through each pair of points or state that the slope is undefined. Then indicate whether the line through the points rises, falls, is horizontal, or is vertical.* (Blitzer, p. 254)

At this point, we came back to this term ‘perfect slope’ and it helped Nick with some frustration regarding the concept of ‘undefined slope.’ Nick was working with a pair of points which lie on a vertical line, which forced him to consider the book’s option: “...or state that the slope is undefined.” (Blitzer). At first he thought that “you can’t do it,” (can’t find the slope connecting them) but after comparing to a horizontal line, he realized that the two points would be on a vertical line. He jokingly asked if he could write, “*perfect slope*” as the answer, and when I answered that he could if he explained that this meant ‘undefined,’ the following discussion took place:

5. Nick: What does it mean 'undefined'? An 'undefined slope'?  
6. C: (Explains briefly on the board: If $x = \frac{5}{0}$, then $0 \times x = 5$. So there is no definition for $x$.)  
7. Nick: Ok. Is this still referring to slope? It just doesn't make sense to call it an undefined slope. Because it is defined. It's going straight up. You know exactly what it is. I'm defining it. It's going up.  
8. C: So what would you rather call it?  
10. C: (pause). Ok. Well... why is that 'perfect' and not this one... horizontal?  
11. Nick: That's the 'anti-slope.'

Prompting Nick to consider a slope in a ‘real-life’ context (Contextualization prompt, line 1) led to an exploratory teacher-student discussion. Although this didn’t directly prompt the challenging talk which Nick used to argue for his view on vertical lines and ‘undefined slope’ (line 7), in the directly-resulting discussion (lines 2, 4) Nick connected his term ‘perfect slope’ to the contextualized idea of slope, or more specifically, to the contextualized idea of a vertical line. This discussion is quite different from the recontextualization discussion in 4.2.2, in that Nick has already been successful with the mathematics, but putting it into a context resulted in requesting more insight into the terms and concepts surrounding undefined slope and vertical lines. I later used an Explanation prompt, asking Nick to explain/justify this view to another student (see 4.2.11).

Between the two excerpts above, we also discussed an application problem about an eagle’s path where the student is asked to interpret what the eagle is doing during the time period displayed on the graph.
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(Blitzer, p. 245). We compared our interpretations of what the eagle was doing, which turned out to be a little bit different. Using a Critique prompt, I asked him what might have made this problem better, since he seemed annoyed by it. His main problem with it was that it was too easy, implying that it was condescending – that “a second-grader could easily do it.” This type of talk was reflexive. He also used exploratory talk to describe his reasons for this opinion, in terms of how he understood the problem.

4.2.9 Week 4: March 22, 2010

The students who attended this session were José, Kelly, and Nick, who arrived late. In the beginning of the session, José and Kelly worked independently. I worked a lot with José in the beginning of the session, helping him talk through finding x- and y-intercepts. I asked Kelly briefly about a recent exam, which she was very disappointed with. She was behind and frustrated and seemed to not quite know where to begin. When she did get started on her assignment, she began by preparing to graph a line given a point and the slope. As she prepared, Nick arrived and immediately showed me his recent exam grade, which he told me was the highest in the class. He initially sat at the opposite end of the table from Kelly, but later I asked him to move closer since they were working on similar problems. After greeting Nick, I asked Kelly to explain to me how she used the information given to graph a line. She had plotted the point, but was having trouble connecting the slope value, 5, to the idea of slope, or ‘rise over run.’ She also seemed to be having additional trouble with vocabulary, confusing slope and y-intercept, so I prompted Kelly to contextualize the concept of slope:

1. C: **If you just heard the word slope out of the context of math... what would you...** Prompt(Co)
2. Kelly: (interrupting) Slope. Falling down!
4. Kelly: Yeah... not rising up!
5. C: Not rising up!
6. Kelly: So it throws me off when it goes 'rise over run'!
7. C: Oh! Well, what if you're going... ok, so slope... you think of falling down... like a ski slope you mean... like that?
8. Kelly: yeah, probably...
9. C: You can go up a ski slope, too, right? But what makes it a 'slope'...? Explore
10. Kelly: It's at an angle... it's not vertical straight up and down and it's not horizontal across... so it falls at an angle - there's an angle like a 45 degree angle.
11. C: Ok... so it's not flat. Explore

Kelly’s response to this prompt was primarily exploratory talk, which allowed me, the instructor, to ask
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further exploratory questions in order to clarify and modifying her idea of slope. Her everyday understanding of the term slope was restricting her mathematical understanding, so I helped her to modify and extend this everyday understanding through a discussion directing her to explore her own contextualized understanding of the term ‘slope.’ The result of this discussion was that Kelly verbalized a more complete description of the concept of slope (line 10).

4.2.10 Week 4: March 22, 2010

Immediately following the discussion in 4.2.9, Kelly began working independently on the following problem, with which she began the session:

\[ m=5, (-2,6). \text{ Write an equation in point-slope form and simplify to slope-intercept form.} \]

(Blitzer, p.270)

Kelly decided that graphing and “thinking about things visually” was helpful to her, so she began graphing the line as the first step in writing the equation. When she seemed to be having trouble, I asked her to explain to Nick what she was doing.

1. C: (To Kelly) Why don’t you explain how you’re doing this problem...? Prompt(E)
2. Kelly: (to N) See then it’s a... but it’s a positive rise. (Kelly has trouble explaining what she is doing, she’s not sure of how to graph a slope of 5).

Nick asked her what problem she was working on, and said that it should be easy to graph. He asked her about her graph, and looked for himself to see that she had already graphed the point (2,6). Nick then explained how to graph the slope \( \frac{5}{1} \). Kelly wrote notes as he talked, and then asked “how do you determine whether it’s positive or negative?” and how she should graph the line depending on the sign of the slope. Nick looked in the book to find examples of graphs of linear equations with positive and negative slopes and used them to help him show as he explained to Kelly:

3. Nick: We're given lines... and these are perfect examples right here... this is positive...this is the positive side over here and this is negative. Ok. And this line over here, the blue line, which direction is it going? Explain
4. Kelly: Down. Explore
5. Nick: no no no.
6. Kelly: Oh.
7. Nick: Is it going positive or negative? Explore
8. Kelly: Oh. Negative. Explain
9. Nick: No. it's going... this way. So the line could start down here and it's going this way. Explore
11. Nick: Which direction is it going? Explore
12. Kelly: So then it's going to the right... it's going up... up to the right
13. Nick: Which is... which is positive.
15. Nick: This line right here, the red line, is it positive or negative?
17. Nick: Why?
18. Kelly: Because it's... well... if you're going straight up it goes into the negative.

In response to Kelly’s incomplete explanation (responding to the Explanation prompt, line 1), Nick began using exploratory talk to guide her through recognizing the sign of the slope of a linear graph. Kelly responded mainly with exploratory talk and one word answers. However, Kelly’s exploratory talk apparently adapted to the type of language and amount of information which Nick was looking for. By the end of this excerpt, she described positive slope as going “up and to the right” rather than simply ‘up’ and describes a line with negative slope as “up... into the negative” instead of the less explanatory “down.” This progression, seemingly towards explanatory rather than exploratory talk, seems to be due to how Nick responded to Kelly’s exploratory talk. At times he indicated that she was simply wrong, but often his questions indicated that she had not given enough information, and she seemed to adapt to the language Nick used during the discussion in order to communicate with him. It may be important to note that the Explanation prompt itself does not seem to have directly brought about this discussion, which was instead the result of using a (any) prompt which indicated that both students should be involved (‘student a, explain your method to student b’), as well as asking them to sit together early on in the session.

After this discussion, Kelly was able to complete her graph and write an equation in slope-intercept form ($y = mx + b$). Kelly and Nick began working on writing equations in point-slope form, and I prompted them to discuss the different forms of the linear equation by asking why they think this is called ‘point-slope form’ and if there is a form which they prefer, both of which resulted in some talk but no discussion between the students. José worked independently for most of this session, and I split my time between encouraging Kelly and Nick to communicate about their work and helping José with graphing exercises. Kelly had a hard time understanding what she was being asked to do and remembering the question being asked as she moved through each problem, and I asked Nick to explain to her how he had solved the problem above, which resulted in an interaction between Kelly and Nick in which Kelly was less involved than the discussion described above, possibly due to the solution in question, which involved simply plugging in numbers to the general point-slope form of the linear equation. I attempted a Critique prompt, asking the two students to critique this process: “Why do you think they’re asking you to do this?” This prompt was not met with any significant response, but Nick and Kelly continued to work together off and
on, especially when I was working with José, with Nick helping Kelly through the problems and correcting her when she made a wrong step. Nick gradually worked more and more on his own, and by the end of session, I was moving among the three students who were primarily working alone, instead of back and forth from Nick and Kelly to José.

### 4.2.11 Week 3: March 23, 2010

Two students, Nick and José, attended this session. José was late, so Nick was the only student present for the first hour of the session. Curious about the mathematics in future chapters, instead of beginning his homework Nick opened his book to the chapter on rational expressions and started working through some of the first problems, which required the student to state at what values of the variable various rational expressions were undefined. Since Nick was the only student there and had been demonstrably successful in his class, I worked with him for quite a while on these problems, introducing him to a few things, such as factoring quadratic trinomials, which he would encounter in the future. When José arrived, Nick was working on his assigned homework, and the two students mainly worked independently and did not communicate with each other. When José was not sure whether a slope of \( \frac{0}{1} \) was “undefined... or... horizontal,” I attempt to prompt a student-student discussion by taking advantage of the previous discussion about undefined slope with Nick (4.2.8):

1. **C:** So what is... undefined? What is the difference between undefined and... (zero slope)? **01:09:47-7**
2. José: I don't know. What is the difference? What would be an undefined line in a graph? [...] 'Cause you can... you can graph that, right? The horizontal line? You can graph it obviously; I mean it's going to be horizontal. What about an undefined one?
3. **C:** N knows about ‘undefined lines’. (To N) You know a lot about these (referring to previous discussion of 'perfect slope')
4. Nick: (adamantly) There's no such thing as an undefined line. Doesn't exist.
5. José: (to C) Why do you got to torture us? Why can't you just...
6. **C:** No... He has some really intense opinions about undefined lines and I wanted him to tell you!
7. Nick: There's no such thing as an undefined line. Well an undefined line is a straight, vertical line... but they call it undefined. It's undefined because you can't…
8. José: It's undefined so it's not vertical, then?
9. **Nick:** It is vertical. An undefined line is a vertical line. But... I don't see why they call it undefined when it's straight up-and-down line.
10. José: It's vertical, right?
11. **Nick:** It's a vertical line is an undefined line.
12. C: **Well is there a difference between saying that the line is undefined and the slope is undefined? Because it's the slope that's undefined...**

13. Nick: Well the slope creates the line. (Pause). Because from the slope you can create a line.... if you have a given point. [...] So it's one and the same.

Nick’s frustration, which he makes apparent through challenging talk, seems to be mainly with the terminology, and taking the time to discuss this frustration helped him to remember and explain to another student that vertical lines have undefined slope. This discussion also apparently allowed him to begin to form a more complete understanding of the relationship between a line and its equation through exploratory talk. He still seems to have a gap in his understanding with regard to the meaning of undefined, but talking about it in his own words and arguing for his own language gave way to beginning to further explore why the term undefined is used.

At the end of this discussion, both students turned the conversation to parallel, personal discussion, and then immediately returned to working silently and independently for most of the remainder of the session in a more traditional tutoring style similar to previous sessions I’ve described, where I allowed them to work alone and helped them as they needed and requested it. For the most part, my role during the remainder of this session was reassurance. Because I was working this way, Nick spent a lot more time talking and discussing during the remaining 30 minutes, but only with me, simply because he asked for help at a much higher frequency, and in these cases I asked him to explain what he was doing and at times compared methods with him.

**4.2.12 Summary of Session Data**

Table 4.1 summarizes the data reported in section 4.2. All accounts of prompts and discussions in the sessions reported are listed. Listings for which the *Categories of talk* has been left blank indicates that no significant talk resulted from the associated prompt. I define ‘significant talk’ as any talk falling into the categories of talk defined in Chapter 2. Where there is no prompt listed, the talk reported was unprompted.

The data shows that all types of prompts designed were attempted, although Explanation prompts were used most often, and other types of prompts, especially Justification and Representation, prompts were used very rarely. We can also see from the data that all types of talk occurred, and that any type of talk can lead to exploratory talk. Also, nearly all implementations of prompts in these nine sessions led to some form of ‘significant’ talk.
Table 4.1: Overview of reported session data.

4.3 Student interviews

Five students were interviewed individually in the week following the last tutoring session, using the instrument in Appendix C. Responses from all five interviews are discussed below. Not all of these students were in tutoring sessions together regularly, although they did overlap to some extent, as seen in the excerpts above. Because of this, they had very different experiences within the tutoring sessions. For example, some students came to a majority of sessions where there were no other students or only one, while others came to fewer sessions but with better attendance. I will begin by presenting responses to
interview questions 9-12, which were designed to find out how students perceived discussion within the tutoring sessions. Questions 2-5, which relate to motivation will be discussed at the end of this section. Questions not designed to contribute to answering a particular research question will not be discussed.

4.3.1 Types of discussion perceived by students

Students had a difficult time recalling discussions, their own and others’ contributions, or how the discussions began. Below are some examples and analysis of student responses to the portions of the interview designed to evaluate the different types of prompts and talk.

4.3.1.1 Interview question 9: What kinds of discussion did you take part in during our sessions?
Interview question 10: What started this discussion?
Interview question 11: What was your contribution to the discussion?

Two students, José and Nick, mentioned specific discussions in response to question 9. Both students recounted discussions where they had begun interacting by helping another student who was having trouble:

Nick: I did help that one lady… (4.2.10 with Kelly). She was frustrated and she didn't know what to do. You didn't want to give her the answer so she got even more frustrated. I tried to sit down with her and talk with her and work it through. […]

Although José also recounted a discussion which began with him helping another student, Maryanne, he put more emphasis on the resulting teamwork/discussion aspect of working with both Maryanne and Kelly, instead of focusing on his success in teaching or helping Maryanne to complete the problem:

José: First we talked about it a little and I thought it would be easier if I just wrote it on the board because we could work from that. And it kind of did because it made that lady (Maryanne) see ‘oh, ok, so that's what it is...’ But then the other girl still caught my mistake... Yeah, it helped everybody out.

C: Did you just write it? Or did you kind of explain...  
José: Yeah I kind of did explain the steps... that's when that other girl caught my mistake - when I was supposedly explaining how it was done...

Other students described general types of discussions, although in these cases they didn’t have a clear idea of what prompted the discussions:

Maryanne: We talked about learning styles. We talked about things like what worked for us and what didn't work for us. We talked about maybe helpful ways of getting through the class. Our frustrations. That's about what I remember.

C: What started those discussions (about learning styles and frustrations, etc.)?
Maryanne: Probably me. I don't remember…

Kelly: Well there were only two times that there were 3 or 4 of us, and, yeah, almost all at once we would start talking about the same problems... and even that one day
that one kid was in a different class (Nick), he knew what we were doing because he'd already done it, so he kind of joined in and gave his ideas... and that was nice.

It seems that most students remember and value the discussions arising after Explanation prompts. They also primarily mention discussion based on explanatory and exploratory talk. However, Danielle noted specifically the existence of Critique prompts. She gave general examples of these when asked what she remembered discussing during the tutoring sessions:

**Danielle:** There's a lot of stuff that we talked about...you asked us a lot of open-ended questions about 'what's the purpose of doing this,' or the definition of something, like 'what does it mean to find a solution?'

**C:** Do you remember how you responded or took part in these discussions?

**Danielle:** When you had a question I tried to answer your question.

### 4.3.1.2 Interview question 12: (Referring to a specific discussion or problem not mentioned as a result of questions 9 and 10). Describe any discussion about this problem.

Before the interviews, I reviewed recordings of tutoring sessions which I had made a note of wanting to discuss further with students, and chose a number of discussions to remind them about in the interviews if they did not bring them up themselves. In Nick’s case, I was curious about his perception of our discussion about undefined slope and his preferred term, ‘perfect slope’ (4.2.8):

**Nick:** It's actually really upsetting. [...] I had no idea what they were asking. I couldn't tell you if it was defined or undefined, I didn't know what it meant. ... And before that I had no idea - there was no way I could have known, because I wasn't able to put it in my own words...

Most students had very brief responses to this question. Maryanne didn’t have any comments on the discussions and problems I brought up, and Kelly said that she ‘barely’ remembered the justification discussion (4.2.1).

Danielle and José were able to remember some tutor-student discussions, but had little to say about the content and even less to say about their own role in the discussions. I reminded Danielle of the pizza discussion (4.2.4):

**Danielle:** Yeah, that was a good discussion. We sort of figured it out after a while. We talked about expectations, like we expect bigger pizzas to cost less... I can't really remember what the outcome was of that...

I asked José to comment on the representation discussion (4.2.6):

**José:** Oh, yeah. Well just by looking at it before graphing it...yeah it was kind of hard to imagine well how would they be, but after doing a problem or two then I started figuring it out... before you graph it can give you a hint about how the lines are going to look.
There is a lot of information in this data from interview questions 9 through 12 regarding both the types of talk and types of prompts, and there is also indication of what types of interaction worked best for certain students, which students had trouble with peer collaboration in general, and the general atmosphere of the sessions. Although one student, Danielle, took note specifically of the Critique prompts used in her sessions, most students interviewed remembered the discussions following prompts, and remembered taking part mainly in exploratory and explanatory talk.

4.3.2 Student opinions about the tutoring sessions and discussion

4.3.2.1 Interview question 2: Do you plan to take further algebra/mathematics courses?

-Will this (tutoring/discussion) experience help you in these future courses?

Please explain.

Students did not express a clear idea of how the experience would benefit them in the future, regardless of their plans for further study in math.

Nick planned to study science and work toward attending medical school, and so had definite plans to take more math courses in the future. His response was not at all related to discussion or what occurred during our tutoring sessions, but instead focused on his success in his algebra class and ability to help other students in the class:

**Nick:** (relates a situation where he helped other students in class because he finished first). As soon as he (the instructor) wrote down the problem, I was writing it down and it took me 2 or 3 minutes to complete a problem. And then after I was done I went around to the rest of the class and helped everyone else do it. I was the only person to be able to complete it.

In fact, very few students gave a specific answer about how this experience would help them in future, although they all answered affirmatively. Maryanne, who adamantly protested taking any math beyond introductory algebra (which she planned on retaking the next semester), responded by explaining what she found generally helpful about the sessions:

**Maryanne:** …it allowed for working with a much smaller group and other students who had similar issues... it also allowed for us to work together, so it wasn't just you as the tutor helping us but helping each other.

Danielle and José both indicated that they would probably take more math courses in the future, depending on which academic path they chose. Neither student indicated clearly how the experience would help them in the future, besides the notion, similar to Maryanne’s, that being able to talk about the problems with me and other students was helpful.
Kelly planned on taking intermediate algebra and statistics which are both required for her planned Associates degree in social work. Kelly was the only student to respond not only that the experience was positive and useful, but to also indicate a specific way in which the experience might help her in mathematics courses in the future:

**Kelly:** The way you did it like with a group and let us feed off of each other, I really like that, so it’s kind of like a private study group. So in the future, if I look for a tutor I would like to have a tutor that does it that way... with other students because we got a lot of feedback.

**4.3.2.2 Interview question 13:** Which types/which discussions mentioned were the most beneficial?

There were some similarities among student responses to this question. Three students stated that more collaborative discussions that involved as many other students as possible were the most beneficial. José and Kelly, specifically, mentioned the ‘argument’ or conversation aspect which occurred when more students were involved:

**José:** Well obviously when there was more people... when we were all right there working on the board... you get to argue... well not argue, but there's just more people to talk about the problem with, meaning it's just going to be more in your head about it, because you had more conversations than just one.

**Kelly:** I think they all were. Being able to just communicate depending on the situation and who was in the room and who was willing to cooperate and work together like a team.... everybody does see things differently.

José also commented in response to a later question that,

**José:** If we could have gotten packed in there it could have been very cool in there, like, a lot of talking like, “no, no this is how you do it.”

Maryanne, on the other hand appreciated being able to commiserate with other students and connect with respect to having similarly negative math experiences, including about frustrations with specific problems or the course in general:

**Maryanne:** …it allowed a place to vent out the frustration in a place where the others understood and possibly felt the same way …what was really helpful to me was that one particular day when that lady who might have been a little bit older than myself was there (Kelly)... and she told me it was her 5th time taking it! That actually gave me relief because then I know that it's ok if I can't make it this time - I can do it again and try again and just keep going. So to have those kinds of discussions and find out those things about other people are helpful.

Maryanne also mentioned that any problems or discussion where I or students wrote on the board were beneficial to her, because she considers herself a visual learner.
Nick and Danielle both mentioned tutor-student discussions as being the most beneficial. Nick talked specifically about the undefined slope discussion (4.2.8) as well as generally, regarding the idea that putting concepts into his own words in discussion with someone who knew more than he did helped him to learn the ‘language’ of math:

**Nick:** The type of discussion that helped me the most was where I could take a problem and be able to put it in my own words. …when you're able to sit down with someone who knows that language and be able to put it into your own words, then you can develop an understanding yourself.

Danielle specifically mentioned the pizza discussion as being the most beneficial, primarily because the resulting answer at the end was “surprising.”

**4.3.2.3 Interview question 14:** *Did you learn from this experience?*

Only one student, Maryanne, gave a response which was related to the discussion aspect of the sessions:

**Maryanne:** I learned that it was ok to come to the conclusion that I couldn't cram the course in and take it the way I was taking it. That just because I'm a good student in all my other areas doesn't mean I'm gonna ace this. And... I was able to come to the acceptance of having to do it a different way (she dropped the class and planned on taking a two-semester option beginning the following Fall)... In some ways it was almost like a mini counseling session. A math counseling session.

Danielle’s, José’s and Kelly’s responses mainly centered on the sessions helping them be more productive and efficient and learning certain math skills, and Danielle reiterated that my lack of explanation, meaning when I didn’t just give the answer, helped her to be more perseverant. Nick, on the other hand, indicated that he learned in the sessions primarily by asking questions, and that the ability to ask as many questions as he wanted was what created the learning experience:

**Nick:** Whether it's a silly question or not, if I can look at a problem that I'm facing and... even if I do understand it, but just to be able to ask questions to solidify my understanding. It helps me out a lot. And through this tutoring I was able to sit down, and even if I understood how to solve the problem - maybe not completely and thoroughly - I'd be able to ask you and more concretely set that foundation...

Nick was specific about the idea that it was asking me, the tutor, questions which helped. He was unsure about the idea of working with fellow students to find solutions or learn new things. He preferred working with other students only when he already knew the answer and could teach them.

**4.3.2.4 Interview question 15:** *Which discussion(s) did you like/enjoy the most?*

Again, the answers to this question can be classified by interest in student-student interaction. Kelly and Maryanne again mentioned an interest in being able to communicate with other students:

**Maryanne:** Probably the ones where I felt like we were all on the same page and the
others were having as much of a struggle as I was because it made me feel not so alone and not so dumb.

**Kelly:** To be personal... discussing my problems. It was very important to me. Because by expressing the problems that I had I was able to get answers.

Nick thought the same tutor-student discussion that had been beneficial was also the most enjoyable (undefined slope 4.2.8):

**Nick:** It was (enjoyable). It was also... it wasn't just a challenge - it was also creative as well. Because I could take a problem that I'm facing and be able to put my own spin on it.

Although students did not refer to particular prompts in their responses to these questions, which were meant to determine which prompts were practicable, their responses do contribute data which can help determine, in combination with the tutoring session data (4.2) which, or which types of, discussion prompts are practicable in a tutoring setting. Student opinions regarding the tutoring sessions and the discussions that took place were positive. Although they didn’t seem to have a clear idea of how the experience might benefit them in the future, they liked being able to help classmates outside of the tutoring sessions because of the extra work we did there, as well as the opportunity afforded by the structure of the sessions and my presence to put things into their own words and to take time to talk about concerns and questions. A final positive aspect according to most responses was the opportunity to work and interact with other students.

### 4.3.3 Affective constructs and student opinions about algebra

#### 4.3.3.1 Interview question 3: *How likely do you think it is that you will succeed in this (Introductory Algebra) and future courses?*

Most students were generally positive about their future success in the introductory algebra course and future math courses, stating emphatically that they had no doubt that they would succeed, or that if they didn’t succeed the first time, they would the second, or “one way or another.”

Maryanne was the only student who expressed significant doubt regarding her ability to succeed in mathematics. She planned to retake Introductory Algebra as a two-semester course. When I asked her how likely she thought it was that she would succeed in this, the only math course she plans to take, she answered

**Maryanne:** Well, being that it's going to be stretched out and they're going to be taking much more time on it… I'm hoping that that will allow me... first of all a lot less stress to get through it so fast. And that I'll be able to have more time to focus on
the steps and get a grasp on it.

**4.3.3.2 Interview question 4: How accurate are the following statements, in your opinion?**

Below are examples and brief discussion of student responses to question 4, organized by statement. I will focus on the student responses with regard to the final statement, as this is the most useful with regard to evidence of issues of affect. Responses to the first four statements will be briefly summarized.

- **“In algebra, there is always one right answer.”**
  Some students felt that they didn’t know enough about mathematics to answer this question, and one student stated that it was accurate because there are “different ways of writing things in mathematics.” Only two students, Maryanne and José, thought that this statement was absolutely accurate.

- **“In algebra, there are usually multiple correct answers or ways to solve the problem.”**
  All students answered that this was accurate, most emphatically, and Nick indicated that he had seen this during the tutoring sessions, but not necessarily that he didn’t know it prior to the sessions.

- **“Algebra is mostly facts and procedures that you have to memorize.”**
  All five students thought that this was accurate at least to some extent, and Kelly and Maryanne specifically explained that this was what they found the most challenging about algebra:

  - **Maryanne:** Accurate. For me! That's how it seemed like it was. And, that was my downfall was having to remember them - the procedures.

  - **Kelly:** Yes. That's my problem. *(elaborate?)* Eight years of trying to get through algebra has taught me that! You’ve got to follow the order or sequence.

  Nick, Danielle, and José stressed the importance of understanding, but all indicated that there are aspects of algebra, formulas especially, which require memorization.

- **“You either know the right way to solve a problem or you can’t solve it.”**
  This statement was interpreted in many different ways, in contrast to the previous statements which were more straight-forward. One of the main ideas which came from two student responses (Nick and Danielle) was that there is no ‘right way’ to solve a problem, only the most expedient or efficient way. Maryanne and Kelly, on the other hand, felt that the statement wasn’t accurate, Maryanne because she equated it to the idea of there being multiple ways to solve most problems, and Kelly because she had experienced solving a problem, or thinking she had, but not having the right answer. José was the only student who
thought that the statement was accurate because of how easy it is to get the wrong answer when solving math problems (“just by having the negative where it’s not supposed to be”).

-“In algebra it doesn’t help to collaborate with students who know less than you do.”

All five students responded that this statement was inaccurate, although, again, some were more emphatic than others. Kelly and Maryanne specifically referred to learning from the other person, and Maryanne decided that it isn’t really a matter of one person knowing and one not knowing, but different people understanding different things:

**Maryanne:** Well, I don't even know if it's a matter of them knowing more or less. They might just remember how to do a certain step that you don't know how to do, and they may not know how to do the rest and you do. I mean, it just could be any little piece.

Nick, José, and Danielle referred the act of explaining as having a positive influence on the explainer’s learning experience:

**Nick:** A person might not know anything about algebra, but simply talking to them and explaining to them what the problem is, you would have a much better chance of coming to a solution.

**José:** That's not true, that actually helps me. Because I end up teaching them which in return stays in my mind forever, once I've taught someone.

**Danielle:** I would say inaccurate... I think you can benefit from learning from people who know less about a certain math problem...it gives you a better understanding of the challenges of the problem and how it works…

These three students address the benefits of externalizing knowledge, as discussed in Chapter 2 (refs). It is implied that these tutoring sessions reinforced this view regarding explaining and teaching others, since all three students had a chance to do this in their tutoring sessions, and are, presumably, drawing on these experiences as in their responses. However, there is not enough information here to know for sure what impact the tutoring/discussion experience had on their opinion about this particular statement.

**4.3.3.3 Interview question 5:** Have any of these opinions (Answers to question 4) been influenced by your experience in these tutoring/discussion sessions?

Maryanne, Danielle and José answered question 5 generally, not specifying a particular statement. Their answers referred to diverse aspects of the tutoring sessions as influencing their views on these statements, I suppose depending on the statement they had in mind. The lack of clarity here is a flaw in my interview method, since I did not always ask them to elaborate on which statement they were considering.

**Maryanne:** Yeah. Because that's where I got to do the interactive work. We didn't get to work interactively in the classroom.
Danielle: Yeah, a little bit, I would say so. It was very accepting. Like you didn’t mind if we had different ways of doing the problem…

José: Yeah. Definitely it did. I was learning the whole time I was there, so yeah, of course.

I did ask José to elaborate on this point, and he said, apparently in reference to the last statement about working with people who understand less, that because of the tutoring sessions he was well prepared and had been able to help other people in his class and at the MLC.

Kim did not think that her views had been influenced, but only reinforced in a positive way by the tutoring sessions.

4.3.3.4 Interview question 6: Has your attitude toward being ‘stuck’ on a problem changed?
   -Have these sessions given you any strategies for getting ‘unstuck’?

Most students answered that their attitude toward being stuck had not changed, but some indicated that the sessions had given them some strategies for getting ‘unstuck.’ Nick and Danielle, however, indicated that their attitude had actually been changed by the sessions:

   Nick: It's made me rethink that there is only one way to do a problem. To be able to effortlessly go to the beginning and start over, as opposed to being stuck half way through a problem and trying to finish it… I don't know if I did that before, but now I do.

   Danielle: Yeah, sort of, you just... it has made me more perseverant.

Kelly, although she said that she still got just as stuck and just as frustrated, also said that she had learned from the sessions to go back to the beginning of problems and start over:

Maryanne and José answered negatively to both parts of the question, José indicating that he still got mad when he couldn’t figure it out and that the only strategy he had learned was to ask me (the tutor). Maryanne said, “When I was stuck, I was stuck!”

   Maryanne: I didn't stumble across a certain thing that would help generate my memory on how to do a problem. I'm still trying to find that.

Overall, students had a positive opinion regarding their potential success in mathematics, although notably the weaker students were less sure of their success in the introductory algebra course. Students had some strong opinions about algebra, including that there are multiple ways to solve problems, and all students felt that to some extent, algebra was about memorizing rules and procedures. All students felt strongly that collaborating with students who ‘know less’ is beneficial, and for various reasons, including
the educational value of teaching and explaining to others, as well as taking advantage of the different types of knowledge held by the collaborators when solving a problem. Although most students did not feel that they had gained any tools for working on their own and getting ‘unstuck’, three students did state that the tutoring experience helped them to become more perseverant when working on math problems.

4.3.4 Summary of Interview Data

Five of the students who participated in the tutoring sessions reported in 4.2 were interviewed in the week following the last session. Students expressed that they liked the sessions, partially because they received extra help from me, but they also mentioned the opportunity to work together with other students. Students did not often recall prompts or characterize the discussion, but they recalled specific discussions and overall found discussions involving Explanation prompts and exploratory/explanatory talk more beneficial or enjoyable than others. Students expressed an appreciation for the opportunity to talk about their problems (both with homework problems as well as general frustrations) and ask a lot of questions, and they found that a benefit of working in these sessions was being able to help classmates outside of the sessions, in algebra class or in the tutoring center. However, students did not have a clear idea of how the experience might benefit them in the future, although some students indicated that they had become more ‘perseverant’ when solving problems.

Students were for the most part sure of their future success in algebra and more advanced mathematics, and although they were certain that problems could be solved in multiple ways, they tended to view memorization as having a central place in the learning of algebra. However, they also felt that collaborating with other students was beneficial to learners of all levels.

In the following chapter, I will primarily use the data described in 4.2 to make conclusions about the first two research sub-questions, and I will draw on both session and interview data to answer the questions of practicability and affect, as well as the overarching question.
5. Conclusions

The over-arching question in this research is, “What are the potentials of discussion as a vehicle for basic skills algebra tutoring?” Four sub-questions were developed for the purpose of finding an answer to this question. Below I will discuss the conclusions of each sub-question, considering the findings presented in Chapter 4.

5.1 Sub-question a

What types of talk can be discerned within small groups of community college students?

This question must be expanded to include discussion between tutor and student, due to the structure of the sessions. Once this is done, it is possible to discern all of the types of talk which were defined in Chapter 3.

The most common type of talk, not only associated with all types of prompts, but also accounting for the bulk of student talk overall, was exploratory talk. This category is defined as talk which indicates that the student is not entirely sure of the mathematics, but they are practicing using their knowledge through speech. Since these learners were new to algebra and in a fast-paced course, the majority of the time when students talked about a problem, for example when describing how they might approach it, or even how they had already solved it, their talk fit into this category.

Exploratory talk was used for a variety of purposes. For example, exploratory talk was used to describe a context in response to a Contextualization prompt (4.2.2, 4.2.9), to work through an idea verbally and come to an answer (4.2.5), and to describe mathematical relationships in nonmathematical terms (4.2.6). Situations where students used their own, original terms (4.2.8), as well as when they gave incomplete answers to questions from peers (4.2.10), were also classified as exploratory talk. For exploratory talk to be a useful category for future research in talk among students, it may be necessary to either modify this definition, or divide the category up into more specified types of talk.

Parallel talk was also common during the sessions, although not as a result of tutor-implemented prompts. However, to some students this talk was quite important, as it often took the form of discussion of frustration with mathematics, and students reported that this type of talk helped
them feel more confident and less worried or anxious, with one student even equating the sessions to “math counseling sessions” (4.3.2.3). In the reported discussion, parallel talk played an important, though rare role as a precursor to both exploratory (4.2.2) and reflexive talk (4.2.7).

Reflexive talk was also common, and found frequently in the reported discussions. Reflexive talk, like parallel talk, was used to express concern about students’ understanding and success in mathematics, although in reference to particular concepts and problems (see 4.2.5, 4.2.7) rather than mathematics or algebra in general. Reflexive talk, like exploratory talk, took many forms. For example, it was used to express a fear of being made fun of when attempting to respond to a Justification prompt (4.2.1), and to comment on the benefit of teamwork, giving value to a classmate’s solution (4.2.3).

Explanatory talk, the definition of which implies knowledge of process and content, was less varied as well as less prolific in student discussion. However, it did not only occur when one student already had the answer, as in student-student teaching situations such as 4.2.5. In fact, explanatory talk primarily occurred in tutor-student discussions in which students responded to being asked explicitly to explain their own methods (4.2.4, 4.2.6).

One student was reported using challenging talk. This type of talk showed up in the transcript as one or two lines accompanied by discussion that was primarily exploratory (4.2.11) or reflexive (4.2.8). The primary purpose of this challenging talk was to critique mathematical terminology, and the student later named this particular discussion (4.2.8) as beneficial because it allowed him to put concepts into his own words.

5.2 Sub-question b

Which types of talk can be related to which types of prompts?

From the data presented in Chapter 4, it is not possible to determine a pattern with regard to what types of prompts produce what types of talk, or which types of prompts are more successful at producing certain types of talk. However, it is possible to discuss the types of talk and in which situations they were observed. Table 5.1 gives an overview of the relation between types of talk and types of prompts.
5.2.1 Exploratory talk

Exploratory talk occurred as a result of every type of prompt, as can be seen in Table 5.2.1. Nearly all prompts which led to significant talk or discussion resulted in exploratory talk (Table 4.2.1). It can also be seen in Table 5.1 that only exploratory and reflexive talk were linked to Justification prompts. In one of these two discussions, the only one where exploratory talk occurred, exploratory talk is the more confident, mathematical talk which follows the reflexive talk regarding concern at being laughed at (4.2.1). Similarly, in 4.2.2, the student begins to use exploratory talk after using parallel talk to set up a context in response to a contextualization prompt. Two Explanation prompts led to responses consisting of only exploratory talk (4.2.1 and 4.2.10), but these were both short exchanges between student and tutor. Contextualization prompts also led to discussions which were primarily exploratory (4.2.8, 4.2.9), and exploratory talk was common in unprompted discussion, including in the student-student teaching episode in 4.2.10.

5.2.2 Reflexive talk

Reflexive talk can also be related to every type of prompt although it occurred with less frequency than exploratory talk. Both implementations of Justification prompts led to reflexive talk. In the first case, as mentioned before (5.2.1), the student expressed concern about being laughed at because they knew that their understanding was incorrect (4.2.1). In the second case, the student was prompted to defend their solution method, and the result was that they faltered in their confidence of the correctness of their answer, expressing this with reflexive talk (4.2.4). Reflexive talk is also commonly found in discussions following Explanation prompts. In 4.2.3 the three students involved in the conversation reflected on their own explanations, methods, and the differences among them. However, in 4.2.4, reflexive talk was used in a flippant way, and
actually signaled an end to the discussion. In a third discussion (4.2.5), reflexive talk was used to express concern and frustration in response to another student’s explanation and explanatory help. Reflexive talk was one of the many types following the use of Critique prompts – in fact all incidents of Critique prompts which led to significant talk led to reflexive talk. In two of these situations, reflexive talk was used to describe the student’s negative criticism of a particular problem (4.2.5, 4.2.8), while in the third, the student used reflexive talk to describe their own methods of understanding and learning (4.2.7). This, of course, relates to the nature of the Critique prompts themselves, which asked for critique of particular problems in the first two cases, and critique of applications in general in the second. Reflexive talk was also used in response to the single implementation of a Representation prompt in order to describe the reasoning behind using a certain method (4.2.6). Following a Contextualization prompt, reflexive talk was used to make clear what part of a particular concept was confusing (4.2.9).

5.2.3 Explanatory talk
Explanatory talk can be related to three types of prompts: Critique, Explanation, and Representation. Most cases of explanatory talk are linked to Explanation prompts, but we cannot know whether or not this is simply because Explanation prompts were used at a higher frequency than any other type. In one case of explanatory talk linked to an Explanation prompt, the student’s explanation is directly related to the prompt. In this case, when asked to compare methods (4.2.4), one of the students explains her method to me, the tutor. However, in a second case, the explanatory talk is not the direct result of the prompts, but comes after the reflexive talk of the second student which better indicates their question and concern, and in this way better prompts the first student to explain (4.2.5). Again, in the case of the Representation prompt, the explanatory talk directly results from the prompt, although it is preceded by exploratory talk, which seems to serve the purpose of solidifying the student’s understanding of their own method well enough to finally use explanatory talk. In the case of the Critique prompt, which was followed by many different types of talk, a student used explanatory talk to clearly describe, for the benefit of a fellow student, the mathematical details of a particular context used in mathematics problems which made sense to him (4.2.7: using money in solving decimal value problems).

5.2.4 Parallel talk
Parallel talk occurred often in unprompted discussions, as mentioned by Maryanne in her interview response (4.3.1.1), and reported in the session data (4.2.3). However, it can be related to
only two types of prompts: Contextualization and Critique. In the case of both types of prompts, there were specific situations where parallel talk served the important purpose of setting the stage for the student to relate mathematics to their own experience (4.2.2, 4.2.5). This is especially true in the case of contextualization, where connecting to student experience and therefore pre-knowledge is the focus of the prompts, and there parallel talk is serving as a precursor to other (exploratory) talk.

5.2.5 Challenging talk
Challenging talk can only be related to an Explanation prompt, although it was also reported in an unprompted discussion between tutor and student. In the prompted discussion, challenging talk was used to by one student to express criticism and frustration regarding mathematical terminology, in direct response to the prompt (5.2.5). This was followed by exploratory talk attempting to reconcile the mathematical terminology with the students’ ‘everyday’ understanding of the terms.

5.3 Sub-question c
What types of discussion prompts are practicable?
All five types of prompts used in the sessions were practicable for the group of students being studied here. However, they were used in different situations, and in many cases led to different kinds of talk, so the nature of their practicability is different for each type of prompt, and varies within each category. To be practicable for use in tutoring, the prompt must be useful, practical and effective in the tutoring situation. It also should be restated here that the data indicating the prompts’ practicability is found in only nine of 20 sessions. In the remaining 11, the opportunities to use prompts, which is one indication of practicability, were very rare.

5.3.1 Explanation
Explanation prompts were successful in getting students to follow through with exploring an alternate method (4.2.3). One student also connected Explanation prompts with the opportunity to put concepts into his own words and clarify his understanding (4.3.2.2). It was also often effective to use Explanation prompts to induce student-student discussion. In these cases, the Explanation prompts led to a variety of types of talk between students (4.2.5, 4.2.10, 4.2.11). However, using Explanation prompts with the intent of getting students to work together was not always effective (4.2.4) and this seemed to depend as much on the students themselves as on other aspects of the situation in which the prompt was being implemented, such as in the context
of what type of problem, and surrounded by what types of talk and discussion. This is further discussed in sections 6.2 and 6.3.

5.3.2 Justification
As a tutor, I found the talk resulting from this type of prompt extremely useful in one of two cases. In this particular case (4.2.1), the student’s exploratory reply to the request to justify her method revealed aspects of her reasoning which I had no way of knowing otherwise. I was able to use what she said to add to the discussion and the tutoring process. By means of contrast with the second implementation of a Justification prompt, which was not as useful from a tutoring perspective, this first implementation was in the middle of a longer discussion, and between only tutor and student. The second case, which produced only reflexive talk, was in the presence of a second student, which may have impacted the ensuing discussion, or lack thereof. Also the actual prompts were quite different. In the first case, I asked Kelly to defend her method, and there was an understanding that although we both knew that it was the ‘wrong’ method, I wanted to hear her interpretation of the concept. However, in the second case, I simply asked Danielle to defend her method, and there was no indication whether it would be right or wrong, or what kind of a reaction she would get from me or the other student.

5.3.3 Representation
The one implementation of a Representation prompt reported was very effective in producing a variety of types of talk. Using the Representation prompt started a dialogue between student and tutor about the relationship between different representations, and succeeded in getting the student to look ahead, use exploratory talk to consider new concepts such as slope and y-intercept, and use explanatory talk to describe his own mathematical methods. In this way the prompt was both practical and useful. The student himself referred to the activity of predicting what the graph would look like as “hard,” but also said that after doing the same kind of predictions with a couple of problems he began to “figure it out.”

5.3.4 Critique
One particular implementation of a Critique prompt produced the widest variety of talk reported in a single discussion. This prompt was used not in the context of a particular problem, as most prompts were, but to ask about ‘real-world’ applications in general (4.2.7). The types of talk varied widely because the two students involved in the discussion used different types of talk to reply to the prompt, and the two students responded to one-another, while still sticking to their
own types of talk. This Critique prompt led to talk about learning styles, which helped students verbalize how curriculum might change in order to be more effective for them (4.2.7, 4.2.5), as well as discussion of one student’s generalized view of the abstractness of mathematics.

Critique prompts made an impact on students, but perhaps at times simply as something notably different from the norm. Danielle referred to them specifically when asked about the type of discussion that took place during the sessions in general (4.3.1.1). However, Danielle was not involved in any significant discussion following from Critique prompts.

5.3.5 Contextualization

The nature of the Contextualization prompts themselves were quite similar across implementations, but the three situations were different (different students, different problems), as were the ensuing discussions.

All uses of Contextualization prompts can be seen as cases of reminding students of pre-knowledge and connecting it with the mathematics (4.2.1, 4.2.2, 4.2.8). In the case of 4.2.8, the talk following the discussion following the prompt allowed the student to put a concept into his own words and helped him, in his own estimation, to clarify the language and terminology, which to him was the key difficulty in this case (Nick, 4.3.1.2).

In 4.2.1, the discussion following the prompt served as a tool for the tutor to understand student confusion linked with everyday meanings. This understanding was applied to further questioning of the student which resulted in a more complete verbalization from the student of the concept of slope. This discussion also acknowledged and placed importance upon the student’s ideas and methods of understanding. Acknowledgment of student methods was also apparent in 4.2.2, where the student talked more extensively about the non-mathematical aspects of her pre-knowledge, using parallel talk, but with additional prompting from the tutor was able to connect it with mathematics. In this case, a second Contextualization prompt led immediately to the student relating the mathematics of the familiar context with the book problem.

5.4 Sub-question d

*What affective constructs, if any, can be observed?*

Students reported that they viewed working with other students as beneficial for various reasons. Some students found that explaining or just talking to other students, or specifically teaching a concept to other students helped them to better understand and retain understanding of
mathematical concepts (4.3.3.2). Talking reflexively during a tutoring session, students referred to the usefulness of sharing their different methods (4.2.7), and a student from this discussion, Maryanne, indicated later on that when students interact and help each other, it is not necessarily an issue of one person knowing more and the others knowing less, but instead that different students know and understand different aspects of a problem (4.3.3.2). Maryanne also indicated that the “interactive work” in the tutoring sessions, as opposed to the more independent work in the classroom, had an impact on her view of the benefits of collaborating with others.

The majority of the students involved continued to view algebra, at least to some extent, as a set of rules that must be memorized. This was especially true for the weaker students, which I define as those that expressed the most concern regarding their past and future success in mathematics. For example Maryanne, who dropped the class as well as the tutoring sessions during the third week of the implementation, did not feel that she had gained any strategies for getting ‘unstuck’ because she hadn’t found the “certain thing that would generate my memory on how to do a problem” (4.3.3.4). Both Maryanne and Kelly primarily pointed to their inability to memorize procedures as the reason for their difficulty in algebra (4.3.3.2).

Other students mentioned becoming more perseverant because of these tutoring sessions, and being willing to go back to the beginning of a problem and start again without getting discouraged. However, increased perseverance was not a trait of all students. The unprompted discussion in 4.2.7 revealed not only a desire, on the part of Maryanne, for tricks and rules for solving problems rather than less directed guidance, but also frustration about the amount of time spent working on the problem and insecurity in her ability to learn the mathematics.

5.5 The over-arching question

What are the potentials of discussion as a vehicle for basic skills algebra tutoring?

The findings and conclusions above show that there is a place for discussion in basic skills algebra tutoring. Many of these prompts produced rich discussions, which began building bonds between students, revealed the method of understanding behind students’ frustrations, and brought students’ attention to alternate solution methods. All students interviewed recognized that teaching and explaining to others benefited their learning experience, and the types of prompts used exhibited the potential to activate student participation in “negotiation of meanings” and “sharing points of view with others,” two requirements, according to Pimm (1987), of mathematical discussion.
Different types of discussion prompts have different potential for use in tutoring situations, as can be seen by the wide variety of talk in response to the variety of prompts. Although most students implied that Explanation prompts were the most beneficial, all prompts had the potential to lead to significant discussion which impacted the learning process. Student discussions and tutor-student discussions were also helpful from the point of view of the tutor, as a tool for determining the educational needs of the student.

In addition, for many students the discussion structure of the sessions allowed them to vent personal frustrations regarding mathematics and to feel more comfortable asking questions and discussing difficulties, which may have helped them to feel more comfortable and more motivated to take part in the mathematics.

Although this is a small research and in no way capable of being generalized or indicative of results that would come from a similar project in a different situation, the results herein suggest that tools for engaging students in discussion, specifically the five types of discussion prompts which are exemplified here, can be a useful part of the basic skills tutor repertoire, both as tools for engaging students in the externalization of knowledge with a tutor, as well as for encouraging student-student interaction.
6. Discussion

6.1 The appearance of ‘key activities’

There is evidence that students took part in some of the key activities defined by Dekker and Elshout-Mohr (1998, 2004) (2.3). Showing and explaining work occurred, and the evolution of student answers due to interaction both with me and each other pointed to the activity of reconstruction. Justification, an activity similar to that of challenging talk in this research, was the rarest type but did occur nonetheless. This type of activity is a ‘reaction to criticism,’ and students seldom expressed confidence to defend or justify their methods in the face of criticism. The connection of challenging talk and affect will be discussed further below (section 6.4).

A difference from Dekker and Elshout-Mohr’s research is that the tutor (teacher) plays a much larger role in inducing discussion by taking part in key and regulating activities with the students, instead of depending primarily on, and encouraging, students to use the activities in their peer discussions. According to Dekker and Elshout-Mohr (1998):

In principle a student who works alone can perform all the key activities, but it takes a lot of self regulation. In communication with other students the key activities will take place in a more natural way. (p. 306)

However, both the tutoring (rather than classroom instruction) aspect of this research, as well as the community college setting made consistent interaction with other students difficult. In addition, as discussed above, there are issues of affect, specifically lack of confidence, which may not be as persistent in younger students. Therefore, this role of the tutor being the performer of regulating activities in order to stimulate key activities in the students may be thought of as an adaptation of Dekker and Elshout-Mohr’s process model specifically for an adult education, tutoring context.

6.2 Comments on practicability

A challenge with the use of any discussion prompt was that the appropriateness (and hence the usefulness, practicality, and effectiveness) of the prompts depended both on the students and the problem at hand. As stated in Chapter 3, I planned ahead the types of prompts I intended to try that day, based on the types of problems I expected the students to be working on. Very often I deemed these planned prompts inappropriate during the actual session. Sometimes this was due to student preparation or pre-knowledge being different than what I predicted, or perhaps due to my observation that the students present were having trouble or resisting communicating with one another (evidence of this is discussed in 6.3). Also,
students often had their own plan for what they would accomplish during the session, and this was not something that I wanted to discourage. These factors clearly played a bigger role in the 11 unreported sessions (out of 20), in which very few prompts were used. However, I still wanted to utilize the types of prompts and encourage talk during the tutoring sessions, so, in adapting to the structure of the actual session, many of the prompts which were implemented and resulted in discussion were not planned ahead of time. They were still based on my definitions and examples, but implemented in the moment, based on the immediate situation of the student (activity, social situation, etc.). In this way, having a repertoire of prompts was useful, and all types of prompts at times resulted in some rich discussion, especially when they were implemented thoughtfully and with the concept, problem, and student in mind. As can be seen from the design of the prompts (Chapter 3 and Appendix A), choosing which to implement also depends on the mathematical topic, and therefore requires both pedagogical and content knowledge with regard to the mathematical concepts being studied. Recognizing ‘openings’ for prompts, and opportunities to further discussions with and between students also necessitates both of these types of knowledge on the part of the tutor/teacher. Because of the potentials of discussion in this tutoring context, I suggest on the basis of this research that tutor pedagogical content knowledge (PCK) (Shulman, 1987), and therefore potential professional development (Perin, 2004), may include a repertoire of prompts and how to use them.

6.3 Student compatibility

As mentioned briefly in reference to the practicability of prompts (5.3), an obstacle to the use of prompts in the tutoring sessions was the relationship between students. Some students had trouble feeling comfortable enough in the sessions to participate to the extent that is required for discussion, and this was revealed in the interviews. For example, Danielle identified a degree of competitiveness which I was not aware of at the time of the sessions, but which could have contributed to the difficulty of engaging her and accompanying students in peer discussion. She said the following when I asked her to elaborate on this point:

Danielle: Just a little bit. Like I didn't really feel like it was like, 'let's all get together and help each other' it was more like comparing what chapter we were on and, like, not really communicating that much. I only had one other student working with me most of the time (usually Nick), and you sort of had to probe him to help me or for me to help him or whatever, so it wasn't really that collaborative.
Danielle’s comment brings to attention the idea that some groupings of students were less compatible in terms of engaging in discussion than others. There are a number of changes to the methodology which could address this. One, which was not entirely in my control, is better attendance, which would mean that there would be more students working together and more of a choice of who to work with and how to interact. Also, the short amount of time spent working together was not adequate for building the types of relationships required for a non-competitive learning community, so extending this type of tutoring project through an entire semester or longer could be another solution. Finally, one student suggested that the sessions could begin with planned group activities. My intent with this research design was to directly address the limited amount of time available to most community college students, who may have families and jobs, and to focus on using the completion of their homework to spur discussion. However, other students also agreed that ‘enrichment’ activities for a portion of the session that would allow them to work in groups on the same problem would be a good use of their time. This change to the structure might contribute to the formation of student relationships and interaction, including during the homework completion part of the session.

6.4 Affect in the results and in the literature

Lampert (1990) indicated that supporting interactions (allowing for explanation, argument and justification) between students caused students to think differently about “what it means to do mathematics,” and Povey and Angier (2004) mentioned that students viewing learning as social, supportive, and collaborative and that mathematics is a “subject to explore” changed both their attitude about and degree of success in mathematics (Chapter 2.3). Students in this research refer to knowing mathematics as the result of their actions (explaining, talking, teaching), rather than ability (Pape & Smith, 2002). It appears that the interactive work done in these sessions helped students to begin to form this view, or at the very least reinforced it (4.3.3.3).

Treisman (1992) also found in his ongoing study beginning in the 1980’s, that minority student success in college-level mathematics (beginning with calculus) blossomed when their math activities drove them to work together and interact as a group. Although this project, unlike Treisman’s, did not address achievement, or involve students on a long enough time frame to do so, student responses were clear about the perceived benefits of working in groups, even after such a short time. Not only was this much different from their previous experience with mathematics, but many of these students found that it was something that was missing both from their classroom experience as well as other tutoring experiences.

Challenging talk seemed to be especially related to affect. I assumed that challenging talk would result
from the use of Justification prompts, but this did not occur during this research. However, it is difficult to conclude anything from this since only two discussions prompted by Justification prompts are reported, while Explanation prompts were used much more frequently. However, only one Explanation prompt can be linked to challenging talk. The rarity of challenging talk could be related to this paucity of the appropriate prompts, but I also think that the possible relation to the issue of affect, specifically confidence in voicing mathematical views, is worth discussing.

As mentioned above in reference to both exploratory talk as well as students’ participation in Dekker and Elshout-Mohr’s ‘key activities,’ students did not exhibit certainty or confidence in their own methods, and when questioned regarding their methods they responded usually with acquiescence to mine or others’ views, rather than defending their own. Even when specifically asked to defend or justify, students were hesitant. Lubienski (2000a), describing research about socio-economic status related to student discussion, although with a much larger group of younger students, noticed similarly that some students were “fearful of saying or believing the wrong thing.” She also noticed that students who did feel comfortable and confident voicing their own mathematical views and opinions also tended to be more prone to discussing general principles of mathematics and not just particular contexts. A smaller version of this can be observed in the research reported here, where the only student who used challenging talk (Nick) was also the student most likely to talk generally about mathematics in discussions, as he did following the Critique prompts in 4.2.7 and while helping another student (4.2.5). Lubienski (2000b) also mentions perseverance, and that the same students who felt comfortable and confident also were also perseverant when solving problems and were less likely to get discouraged. However, in this research, even students who were not always confident in discussion cited being more perseverant after participating in the tutoring sessions, specifically Danielle and Kelly (4.3.3.4). Similar methodological solutions apply here as to student compatibility issues (6.3). Better session attendance and a longer period of time with the same group of students could lead to relationships between tutor and student, and among students. In general, the opportunity to form more of a learning community could increase the comfort and confidence of students to voice their mathematical views.

Finally, Kelly and Maryanne both found discussion of their past experience with mathematics to be a beneficial part of the tutoring sessions. In fact, Maryanne felt that for her, this was the most beneficial aspect of the experience, as it increased her confidence to know that others had gone through similar difficulties, and she was surprised to find people with fellow students with very similar experiences to hers. Patrick (1999) observed similar responses from her adult students in an ‘everyday’ mathematics course. Her students identified the ‘sharing of mathematical stories’ as the most useful activities they took
part in during the course. Patrick went further to conclude that these stories were ‘key’ to helping students overcome personal barriers to learning mathematics, and to form bonds with each other.

6.5 Recommendations for future research

There are a number of recommendations which follow from this project. First, as mentioned multiple times in the text above, it could be useful to do a similar study for a longer period of time, for example for an entire semester or a year, with one group of students. This could allow for more exploration of the building of a learning community, changes in attitude toward mathematics and confidence in discussing mathematical views. This type of long term study could also be done with students using a college tutoring center, such as Cabrillo’s MLC (see section 3.1.3). Another suggestion which also comes from the discussion above, and which was suggested by the students themselves, is adding introductory, possibly ‘enrichment’ tasks to the session, separate from work assigned by the instructors and designed to orient students to the norms of peer, mathematical discussion in a more structured way.

Further research in this area might also consider other types of prompts. For example, Daniel, Lafortune, Pallascio & Schleifer (1999) worked with philosophical ‘prompts’ in an elementary mathematics classroom. In that case, the more ‘deeply’ philosophical topics were not pursued by the students, and it would be interesting to try introducing such topic to adult students working with basic mathematics. This type of prompt might be best combined with the idea of including more structured, planned tasks, as discussed above.

Community college students are especially diverse when it comes to previous knowledge, age and life experience. To further explore the use of discussion in tutoring/teaching algebra, it would be useful to work with these prompts with traditional high school algebra students in 8th or 9th grade (14 or 15 years old). This could also be an opportunity to consider how such prompts could be use in a classroom setting, and perhaps as technology becomes more of a learning tool in these ‘regular’ classrooms, discussion and prompting can be explored as possible contributions to the pedagogical methods within learning environments using ICT.

Also regarding the ages of the participants, and of community college students in general, this research has neglected to incorporate issues of adult education theory by looking at design or results from this perspective. The teaching method took into consideration students’ self-direction, and the comments from students (on perseverance for example) may point to an increase in self-direction, but it would be illuminating for the field of adult education to look at similar methods considering previous research in the field, as well as the accepted difference between child and adult learners. Notably, much of the
research on discussion in mathematics, and therefore the bulk of my theoretical framework, has been done with children in more traditional school settings. Considering the similar ideas from an adult education perspective would also contribute to the variety of educational environments investigated by this area of research.

Finally, the problem which motivated the research was introduced as one of achievement and performance of community college students in basic mathematics (Chapter 1). With indications that discussion and methods of encouraging discussion among students has potential in tutoring basic skills mathematics, a future step in research is to look into the effectiveness of such methods in terms of improving achievement. This would go hand in hand with the longer period of research that I have suggested above, and perhaps in consideration of adult education constructs.
7. References


Van Oers, Bert (1998). From context to contextualizing. Learning and Instruction, 8(6), 473-488.


8. Bibliography


# Appendix A: Prompt types and examples

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<tr>
<th>Type of Prompt</th>
<th>Description of Type</th>
<th>Examples</th>
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| Critique       | Critique prompts ask student to interpret the meaning and/or purpose of the problem. They may ask students to assess the problem from a personal, ‘real-world’ and/or mathematical standpoint. | **General**
1. Why are we being asked this?
2. What is the problem/task about?
3. Why is it being asked in this way?
4. Does this question make sense (realistically/mathematically)?
5. If not, why not? What could help it to make more sense? What would you like to change?
6. What might make it more: ‘doable,’ interesting, or approachable?

**Task-specific**
7. What can you say about the relationship(s) between (among) the variables (for example in a conversion task/’solve for _ in terms of _’).
8. Linear equations: What do the parts of the equation mean/refer to (x, y, constants).
9. Line fit problems: Where do you think this data came from? What is the math that you/the book are doing with it good for? |

| Explanation     | Explanation prompts ask students to describe their solutions, and/or to look into different methods of solution. One aim is to get students to be specific, and this includes the students being explained to when they ask questions of the explainer. | **General**
1. What are the different ways your group can think of to solve this problem?
2. Why does a particular method ‘always’ work? Make up a problem for the members of your group to do where you think the method will not work. Work on each others’ problems and come up with a group explanation of when the method will/will not work.
3. Trade solutions with a group member, check/correct, and explain ‘why’ for all corrections.
4. Lead the group through solving a problem step-by-step. Group should ask questions, making sure that they completely understand each step.
5. Explain how to solve the equation

**Task Specific**
6. Why is division by zero undefined? |
| Representation | Representation prompts ask students to draw or otherwise represent problems visually, to comment on or explain different types of representations, or to consider alternate representations from those given in the problem. | **General**  
1. Represent the problem visually (draw it, or…) and explain the representation to your group.  
2. What types of representation did you transition between in order to solve this problem (or set of problems)?  
**Task-specific**  
3. Write an original word problem that can be solved using a linear equation. Solve the problem. (Addition: Edit/rewrite the problem if it needs it once you’ve solved it, and give it to the members of your group to solve).  
4. Predict what the graph of a linear equation will look like. Predict how the graphs of two or more linear equations will be different.  
5. Offer/discuss other explanations for a trend (slope) observed among points on a scatter plot, besides that one caused the other. |
| Justification | Justification prompts challenge students to defend their own mathematical point of view. | **General**  
1. Is there a solution method which you prefer? Explain to your group. Defend your ‘favorite’ method.  
2. If there is variation among the answers in your group and/or your solution method, justify your answer and/or your method with an explanation of why it makes sense. |
| Contextualization | Contextualization prompts lead students to consider a new context for a mathematical context. An everyday context may add new meaning to a situation by connecting to previous knowledge. | **General**  
1. Had you heard of this term before learning it in mathematics class?  
2. How do you understand this (term/concept) outside of mathematics?  
3. What real situation do you think of in connection to this term/concept?  
4. Have you ever done anything in your everyday life similar to what is being done in this problem? |
### Appendix B: Interview Schedule

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<thead>
<tr>
<th>Interview Questions</th>
<th>Answers</th>
<th>Research sub</th>
<th>General</th>
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<tbody>
<tr>
<td><strong>1.</strong> What is your purpose for taking this Introductory Algebra course?</td>
<td></td>
<td>a</td>
<td>X</td>
</tr>
<tr>
<td>- Do you expect to need mathematics/algebra in your life in the future?</td>
<td></td>
<td>b</td>
<td></td>
</tr>
<tr>
<td>- Do you expect to need mathematics/algebra in your work (planned or current profession) in the future?</td>
<td></td>
<td>c</td>
<td></td>
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<td></td>
<td></td>
<td>d</td>
<td></td>
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<tr>
<td><strong>2.</strong> Do you plan to take further algebra/mathematics courses?</td>
<td></td>
<td>a</td>
<td>X</td>
</tr>
<tr>
<td>- Which courses? For what purpose?</td>
<td></td>
<td>b</td>
<td>X</td>
</tr>
<tr>
<td>- How likely do you think it is that you will succeed?</td>
<td></td>
<td>c</td>
<td></td>
</tr>
<tr>
<td>- Will this (tutoring/discussion) experience help you in these future courses?</td>
<td></td>
<td>d</td>
<td></td>
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<tr>
<td>Please explain.</td>
<td></td>
<td></td>
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<tr>
<td><strong>3.</strong> How likely do you think it is that you will succeed in this (Introductory Algebra and future courses)?</td>
<td></td>
<td>a</td>
<td>X</td>
</tr>
<tr>
<td>- Will this (tutoring/discussion) experience help you in these future courses?</td>
<td></td>
<td>b</td>
<td></td>
</tr>
<tr>
<td>Please explain.</td>
<td></td>
<td>c</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>d</td>
<td></td>
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<tr>
<td><strong>4.</strong> How accurate are the following statements, in your opinion? Please elaborate a little on each one.</td>
<td></td>
<td>a</td>
<td>X</td>
</tr>
<tr>
<td>- “In algebra, there is always one right answer.”</td>
<td></td>
<td>b</td>
<td></td>
</tr>
<tr>
<td>- “In algebra, there are usually multiple correct answers or ways to solve the problem.”</td>
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<td>c</td>
<td></td>
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<tr>
<td>- “Algebra is mostly facts and procedures that you have to memorize.”</td>
<td></td>
<td>d</td>
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<tr>
<td>- “You either know the right way to solve a problem or you can’t solve it.”</td>
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<tr>
<td>- “In algebra it doesn’t help to collaborate with students who know less than you do.”</td>
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<tr>
<td><strong>5.</strong> Have any of these opinions (Answers to question 4) been influenced by your experience in these discussion sessions?</td>
<td></td>
<td>a</td>
<td>X</td>
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<tr>
<td></td>
<td></td>
<td>b</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>c</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>d</td>
<td></td>
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<tr>
<td><strong>6.</strong> Has your attitude toward being ‘stuck’ on a problem changed?</td>
<td></td>
<td>a</td>
<td>X</td>
</tr>
<tr>
<td>- Have these sessions given you any strategies for getting ‘unstuck’?</td>
<td></td>
<td>b</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>c</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>d</td>
<td></td>
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<tr>
<td><strong>7.</strong> Do you have any prior experience with tutoring?</td>
<td></td>
<td>a</td>
<td>X</td>
</tr>
<tr>
<td>- Describe your prior experience.</td>
<td></td>
<td>b</td>
<td></td>
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<td></td>
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<td>c</td>
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<tr>
<td></td>
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<td>d</td>
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<tr>
<td><strong>8.</strong> Compare/contrast your previous experience with this discussion group experience.</td>
<td></td>
<td>a</td>
<td>X</td>
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<tr>
<td></td>
<td></td>
<td>b</td>
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<td></td>
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</tbody>
</table>
9. What kinds of discussions did you take part in during our sessions?
   - What was your discussion about? (solving the problem, helping each other, other)
   - How did other students participate? (Explaining to each other, arguing, deliberating, listening, sharing ideas)

10. (Referring to an answer to question 9) What started this (a particular type of) discussion?
    - Was it a question you asked?... I asked? What kind of question?

11. What was your contribution to the discussion?

12. (Referring to a specific prompts/problems not mentioned as a result of questions 9 and 10) Describe any discussion about this problem after being given the prompt. (Elaborate similarly to question 9.)
    - What was the discussion about?
    - If you contributed, how did you contribute?
    - (How did the other members of your group participate?)

13. Which types/which of discussions mentioned were the most beneficial?
    - How were they beneficial?
    - Why do you think they were beneficial?

14. Did you learn from this experience?
    - What did you learn?
    - What type of activity (discussing, explaining, working alone) during the sessions provided the best learning experience?

15. Which discussions did you like/enjoy the best?
    - Which did you find the most interesting? Why?
    - Which did you find the most helpful? Why?

16. Do you have any recommendations for the next time I work with students in a group tutoring setting?
    - Things to change.
    - Things to keep.
### Appendix C: Talk categories

<table>
<thead>
<tr>
<th>Category of talk</th>
<th>Defining aspects of student talk</th>
<th>Examples from data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exploratory</td>
<td>o Improvisational</td>
<td>“The higher the number is the more slanted it goes… the higher the y is.”</td>
</tr>
<tr>
<td></td>
<td>o Rehearsal of knowledge</td>
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<td></td>
<td>o Not focused on ‘external criteria’</td>
<td></td>
</tr>
<tr>
<td>Explanatory</td>
<td>o More formally descriptive than exploratory talk</td>
<td>“The only positive number that I plugged in, which was two, what I got was 5, which is pretty high.”</td>
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<tr>
<td></td>
<td>o Elaborate descriptions</td>
<td></td>
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<tr>
<td></td>
<td>o Specific examples for general concepts</td>
<td></td>
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<tr>
<td>Reflexive</td>
<td>o Referring to the nature of the solution method or progress toward solution</td>
<td>“I’m just confused because I don’t know how I’m going to get this.”</td>
</tr>
<tr>
<td></td>
<td>o Opinions of solution method and how well they understood it</td>
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<td></td>
<td>o View of their own role in the solution</td>
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<tr>
<td>Challenging</td>
<td>o Proceeded by criticism</td>
<td>“It just doesn’t make sense to call it an undefined slope.”</td>
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<td></td>
<td>o Argument against a mathematical viewpoint or line of reasoning</td>
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<tr>
<td>Parallel</td>
<td>o Does not contribute to the solution of a problem</td>
<td>“Yeah, but I let the contractor do that!”</td>
</tr>
<tr>
<td></td>
<td>o Non-mathematical talk which is not reflexive</td>
<td></td>
</tr>
</tbody>
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