Abstract

There is still a lot of debate concerning the nature of dark matter. One of the explanations is that dark matter is made of MACHOs: Massive compact halo objects. Here, we derive constraints on the possible masses of these MACHO dark matter components. Microlensing measurements have excluded MACHO masses between 0.15 and 20 $M_\odot$ to form all of the dark matter at a 95 % CL. Simulations to the effects of dark matter MACHOs on the evolution of a galaxy have excluded masses from 1 up to a hundreds of solar masses to be the only component of dark matter and MACHOs with masses of a few tens of solar masses are even excluded to form more than 4 % of the total dark matter in the universe. We also show that the constraints from dynamical heating strengthen when a range of MACHO masses is present.
1 Introduction

One of the greatest mysteries of modern day astrophysics is the phenomenon called dark matter; a type of matter that doesn’t interact with light but does have a gravitational impact on its surrounding. Although most scientist agree that there is something called dark matter, there is still a lot of debate concerning the nature of dark matter [1]. The explanation for the fundamental components of dark matter even range from ultralight axions [2] to super massive black holes [3]. For this paper we assume that dark matter is made of MACHOs (Massive Compact Halo Objects). The goal of this thesis will be to find constraints on the mass range of these dark matter components. A lot of research has already been done to these constraints : Brandt (2016) and Kouhiappas (2017) placed constraints by looking at the dynamical impact of these MACHOs on a galaxy [4] [5] and alcock (2001) placed constraints by looking at the microlensing effects of these MACHOs [6]. Here, I will reproduce the results of these studies and strengthen some of the constraints by looking at the case were the MACHOs are made off a range of masses.

2 Dark Matter

2.1 History

One of the first scientist who mentioned the existence of dark matter was the Swiss astronomer Fritz Zwicky, he measured the velocity dispersion of galaxies in the Coma cloud cluster. He found that the velocity dispersion had a value around the 1000 km/s. If the total mass of this cluster only existed off stars than the total gravitational pull of this system wouldn’t be sufficient to keep the stars with such velocities bound. He concluded that a part a the galaxies mass may be in the form of ’dark bodies’ [7].

One of the major breakthroughs came in 1980 when Vera Rubin and Kent Ford published their paper : ”Rotational Properties of 21 Sc Galaxies with a Large Range of Luminosities and Radii from NGC 4605 (R=4kpc) to UGC 2885 (R=122kpc)” [8]. In this paper they measured the rotational curve of the galaxy and found that the rotational velocity remained relatively constant after a certain radius(figure 1). This doesn’t match with the expectations from Kepler’s law. This difference can be explained by assuming that there are big quantities of dark matter in the outer layers of a galaxy. since then a lot of studies found evidence for the existence of dark matter: measurements to gravitational lensing [9], X-ray measurements [10] and evidence coming from the Cosmic microwave background [11].

At this points most of the scientist agree that there is a lot of mass in the galaxies that we can’t see. The questions about how and when it was formed and what it is made of however still remain open.
2.2 Dark matter components

There are a lot of possible candidates for the fundamental components that form dark matter. These candidates even cover a mass range of 90 orders of magnitude [12]. All those candidates can be roughly divided into two groups: Baryonic and non-baryonic matter components. Baryonic matter is matter that is made from Baryons, a type of particle that is made of 3 quarks. Most of the matter we know, such as protons and neutrons, are baryonic matter. Under baryonic matter we also include astronomical bodies such as white and brown dwarfs, neutron stars and black holes. These objects are collectively known as Massive compact halo compact objects (MACHOs). Non Baryonic dark matter candidates are more theoretical particles like axions and WIMPS (Weakly interacting massive particles). Wimps have been subject to a lot of research and although the R-parity-conserving supersymmetry predicts a Wimp-like particle [13], they have yet to be found. If these Wimp particles indeed exist, than they have been formed in huge quantities in the early universe. These Wimp particles would than annihilate with each other to form two normal particles. As the universe expanded the chance that two Wimp particles would collide decreased and these annihilation’s would become more and more rare. Simulations to these events show that we would end up with around 5 times more mass in Wimps than in normal matter [14], this closely resembles the amount of dark matter mass predicted by measurements to the cosmic microwave background [15] [16]. This among other things makes wimps for a lot of scientist the most favorable explanation to the dark matter issue.

Another possible explanation for the missing matter is the axion. The axion was first mentioned as a solution to the strong CP problem in quantum chromodynamics (QCD) [17]. Violation of the CP symmetry would result in an electric dipole moment of the neutron of around $10^{-18} e\cdot m$, experiments however suggest a fraction of this value [18]. In contrary to Wimps, axions are relatively light particles, (Berenji 2016) even placed the upper limit of the axion mass at $7.9 \times 10^{-2}$ eV [19].

It is possible that the total dark matter in our universe consists of a combination of some of the suggested fundamental components.
2.3 MACHOs

During this project we look into the possibility that MACHOs are the fundamental components that form dark matter. The fact that they are both likely to exist and objects that emit little to no light makes them among the first mentioned explanations for the dark matter issue. However, since most of the ‘natural’ MACHOs (neutron stars and black holes) are the result of supernovae explosions, which are very rare, it is unlikely that there would be enough to form the total dark matter in the universe. One thing that could solve this problem is the existence of primordial black holes, black holes that formed at the earliest stages of the universe [20].

2.3.1 Primordial black holes

If we look at the Cosmic microwave background (CMB) we get an image which shows us the temperature fluctuations in the 379,000 years old universe [21]

![Figure 2: An image from scholarpedia [21] of the temperature fluctuations when the universe was 379,000 years old. The image has been obtained by measuring the Cosmic microwave background radiation.](image)

This image (figure 2) tells us that even at the earliest stages of the universe the matter was not equally distributed. These density fluctuations have started the formations of stars and galaxies. Some researchers suggest that if these density fluctuations were high enough, they could have resulted in the formation of primordial black holes (PBH) [22]. Since these black holes don’t need massive stars for their formation, they can span a broad range of masses. In 1971 Hawking even suggested that these PBH could have masses which range from $10^{-8}$ kg to thousands of solar masses [23]. The relatively light ones (with a mass below $10^{11}$ kg) however would have been evaporated by now due to hawking radiation.
3 Constraints on MACHO dark matter

3.1 Constraints by microlensing measurements

Since Einstein stated his theory of relativity, we know that mass curves the space around it and that even a massless light particle is effected by gravity [24]. Einstein also predicted a phenomenon called gravitational lensing; when matter passes in front of a light-source it can bend the light as it travels from the source to the observer. This can be very useful when we are looking for exoplanets. As an exoplanet moves in front of the star, it will temporarily bend the light towards us which we can measure as an increase in luminosity coming from the star (see figure 2).

![Figure 3: An image from Encyclopedia Britannica Online [25] showing that matter can act as a lens when it bends the light to an observer. The observer from earth will temporary measure an increase in luminosity coming from the star.](image)

Dark matter MACHOs can be found using the same technique. But since the duration of the luminosity increase depends on multiple variables; lens velocity, lens mass and lens distance, it isn’t possible to determine the lens mass for a unique event. However, K Griest made an estimate for the average duration of a microlensing event for a given halo model. He found that the mass of a lens could be given by [26]:

\[
\frac{m}{M_\odot} \simeq \left( \frac{t}{130d} \right)^2,
\]

with \( m/M_\odot \) being the mass of the lens in solar masses and \( t \) being the time it takes for this lens to cross the Einstein ring of the source star in days.
These microlensing effects can be measured from earth. The efficiency of detecting these effects have been calculated by Alcock 2001 [6], for their calculations they considered fluctuations caused by seeing, the weather, telescopic failures and effects caused by blending. They got the following efficiency for detecting long-duration microlensing effects:

![Graph showing efficiency of detecting long-duration microlensing effect.](image)

Figure 4: graph from Alcock 2001 [27] showing the efficiency of detecting long-duration microlensing effect.

If we follow Alcock 2001 [27] and we assume that the MACHO masses are distributed according to a $\delta$- function, we can get a figure which shows the expected amount of measurements set out to the mass of those MACHOs:

![Graph showing expected measurements of dark matter MACHOs.](image)

Figure 5: Number of expected dark matter MACHO measurements if dark matter is only made of MACHOs [6].

During their 5.7 year during survey to 11.9 million stars, Alcock et al found zero
long duration (> 150 days) microlensing effects caused by dark matter MACHOs [6]. To use this to put constraints on the mass of dark matter MACHOs, we calculate the probability of detecting 0 events. Since the expected measurements are distributed as a Poisson distribution, these probabilities can be given by:

$$P(\text{zero events}) = e^{-\alpha}.$$  \hspace{1cm} (2)

With $P(\text{zero events})$ being the chance that zero events are measured, $e$ being the natural logarithm and $\alpha$ being the amount of measurements expected. This means that all masses who have more than 3 expected measurements can be ruled out at a 95 percent certainty: $P(\text{zero events}) = e^{-3} = 0.05$. They however can still make up a fraction of the total dark matter. If we combine equation 2 with the data from figure 5, we get the following constraints on MACHO dark matter:

![Figure 6: Figure from Alcock (2001) showing the constraints on MACHO dark matter masses by microlensing measurements. The left y-axis represent the total fraction of the dark matter in the form of MACHOs and the x-axis gives the mass of those MACHOs. The right y-axis gives the total possible dark matter mass in the form of these MACHOs. The section above the line is excluded at an 95 percent certainty. MACHOs with masses between 0.3 and 20 solar masses are ruled out to be the only components of dark matter.](image-url)
3.2 Constraints from galactic dynamics

3.2.1 Dynamical heating

If the dark matter in galaxies is indeed made of MACHOs, than these MACHOs will have a dynamical impact on there surrounding. Gravitational interactions will lead to exchange of energy. These interactions will dynamically heat the system, causing it to expand. To calculate the growth of a star cluster we assume that the system can be treated as a diffusion problem, with weak scattering changing the velocities of the stars. If we assume that the velocities of the stars are distributed according to the Maxwell–Boltzmann distribution, we get that the diffusion coefficient can be given by [28]:

\[
D[(\Delta v)^2] = \frac{4\sqrt{2}\pi G^2 f_{dm} \rho m_a \ln(\Lambda)}{v_{disp} \left[ \frac{\text{erf}(X)}{X} \right]).
\] (3)

Where \(f_{dm}\) is the fraction of the dark matter that is in the form of MACHOs, \(\rho\) is the total dark matter density, \(v_{disp}\) is the velocity dispersion of the MACHOs and \(m_a\) is the mass of the MACHOs. \(X\) is the ratio between the velocities of the stars to the velocity of the MACHOs \((v_*/\sqrt{2}v_{disp})\), with \(\text{erf}(X)\) being the error function of \(X\). This error function can be given by:

\[
\text{erf}(X) = \frac{2}{\sqrt{\pi}} \int_0^X e^{-t^2} dt.
\] (4)

We will assume that the relative temperature of the MACHOs is much higher than that of the stars, so \(v_{disp} > v_*\) and \(X << 1\). If we take the Tayler expansion of the integral close to 0 we get:

\[
\text{erf}(X) \simeq x - \frac{x^3}{3} + \frac{x^5}{10} + O(x^6).
\] (5)

So when \(X\) is close to zero, \(\text{erf}(X)/X\) becomes \(\sim 1\). \(\ln(\Lambda)\) is known as the coulomb logarithm, a factor which describes the amount that small angle collisions are more effective than large angle collisions:

\[
\ln(\Lambda) = \ln \left( \frac{r_h v_{disp}}{G(m + m_a)} \right).
\] (6)

This diffusion coefficient describes the mean change of the velocities per unit time. The potential energy per unit mass of the star cluster can be given by [4]:

\[
\frac{U}{M} = C - \alpha \frac{GM_*}{r_h} + \beta G \rho r_h^2,
\] (7)

in which \(C\) is a constant, \(M_*\) is the total stellar mass of the cluster, \(\rho\) is the dark matter density, \(\alpha\) and \(\beta\) are constants that depend on the mass distribution of the system and \(r_h\) is the half light radius : the radius in which half of the clusters light is emitted.

If we now take the derivative of Equation 5 we get:
\[
\frac{1}{M} \frac{d}{dt} (U) = \left( \alpha \frac{GM_s}{r_h^2} + 2\beta G\rho r_h \right) \frac{dr_h}{dt}.
\]

We now have a formula which gives the change in the total kinetic energy over time (equation (3)) and a formula which gives the change in the total potential energy over time (equation (8)). We can combine these two with the use of the virial theorem, a theorem which gives a relation between the potential and the kinetic energy. In our case this virial theorem is:

\[ E_{tot} = -\frac{1}{2} U \]

If we use this, we get the following expression for the change in the half light radius:

\[
\frac{dr_h}{dt} = \frac{4\sqrt{2}\pi Gf_{dm} m_a}{v_{disp}} \ln(\Lambda) \left( \frac{M_s}{\rho r_h^2} + 2\beta r_h \right)^{-1}.
\]

To simulate this effect we used the known data from a star cluster near the centre of Eridanus II. Eridanus II is a Ultra-faint dwarf galaxy which are perfect for the study to dark matter because they are relatively high dark matter dominated \[29\]. We numerically solved this equation (using python), this gave us the following change in half-light radius due to dynamical heating:

![Graph showing change in half light radius due to dynamical heating](image)

Figure 7: Change in the half light radius of a 6000 \( M_\odot \) star cluster due to dynamical heating. We used that all of the dark matter mass was in the form off MACHOs with mass 30 \( M_\odot \) and that the velocity dispersion of the MACHOs was 5 \( kms^{-1} \). This figure gives the same results as the simulations done by (brandt 2016) \[4\]
If we now combine equation 7 with the observed data from Eridanus II and we require that the timescale for dynamical heating takes longer than the clusters current age, we can put constraints on the MACHO dark matter masses [4]. The system has an observed half-light radius of 13 parsec and we assume that at \( t = 0 \) its half-light radius was 2 pc. If we use this data, python gives us the following constraints for two different velocity dispersion’s and dark matter densities:

![Graph](image)

Figure 8: MACHO dark matter mass constraints for a 3 Gyr old star cluster with a total mass of 2000 \( M_\odot \). The units of the velocity dispersion and the density are given in m/s and in \( M_\odot/pc^3 \).
3.2.2 Dynamical heating for a range of MACHO masses

In the previous section we only looked at the case where all the MACHOs have the same mass. It is however more likely that those MACHOs will span a range of masses. In this section we look at the equations for dynamical heating when the dark matter is made of MACHOs with a range of masses and look at the effects that this will have on the mass constraints.

We already have the formula which describes the diffusion coefficient if all the MACHOs have the same mass $m_a$:

$$D[(\Delta v)^2] = \frac{4\sqrt{2\pi}G^2 f_m \rho m_a \ln(\Lambda)}{v_{disp}} \left[ \frac{\text{erf}(X)}{X} \right]. \quad (10)$$

The only parameters in this equation that depend on $m_a$ are $\rho$, $\Lambda$ and $m_a$. So to simplify equation (1) we write:

$$D[(\Delta v)^2] = A \rho m_a \ln(\Lambda), \quad (11)$$

with

$$A = \frac{4\sqrt{2\pi}G^2 f_m}{v_{disp}}, \quad (12)$$

now we split the density of the dark matter $\rho$ into:

$$\rho = n m_a, \quad (13)$$

with n the number density of the MACHOs and $m_a$ the mass of such an object. Equation (9) than becomes:

$$D[(\Delta v)^2] = An m_a^2 \ln(\Lambda). \quad (14)$$

If we assume that the MACHOs span a range of masses, we need to integrate over all the possible masses:

$$D[(\Delta v)^2] = An_{tot} \int_{m_{min}}^{m_{max}} P(m_a) m_a^2 \ln(\Lambda) dm_a. \quad (15)$$

Were $P(m_a)$ stands for the probability density function of the MACHO masses and $n_{tot}$ for the total number density of the MACHOs. For this project we assume that the MACHO masses are lognormaly distributed, in this case $P(m_a)$ can be given by:

$$P(m_a) = \frac{1}{\sigma \sqrt{2\pi m_a}} e^{-\frac{(\ln(m_a) - \mu)^2}{2\sigma^2}}. \quad (16)$$

With $\sigma$ being the standard deviation of the logarithmic mass and $\mu$ being the mean of the logarithmic mass. If we put this into equation (13) and integrate form 0 to infinity we get:
\[ D[(\Delta v)^2] = \frac{A_{n_{\text{tot}}}}{\sigma \sqrt{2\pi}} \int_0^\infty e^{-\frac{(\ln(m_a)-\mu)^2}{2\sigma^2}} m_a \ln(A) dm_a. \] (17)

This \(n_{\text{tot}}\) is not a known parameter so we substitute \(n_{\text{tot}}\) with:

\[ n_{\text{tot}} = \frac{\rho_{\text{tot}}}{<m_a>}. \] (18)

With \(\rho_{\text{tot}}\) being the total dark matter density and \(<m_a>\) being the average MACHO mass of the system. In appendix 1 I prove that we can write \(<m_a>\) as:

\[ <m_a> = e^{\mu+\frac{\sigma^2}{2}}. \] (19)

Equation (15) than becomes:

\[ D[(\Delta v)^2] = \frac{A\rho_{\text{tot}}}{\sigma \sqrt{2\pi}e^{\mu+\frac{\sigma^2}{2}}} \int_0^\infty e^{-\frac{(\ln(m_a)-\mu)^2}{2\sigma^2}} m_a \ln(A) dm_a. \] (20)

If we now write out \(A\) and \(\Lambda\) we get:

\[ D[(\Delta v)^2] = \frac{4\sqrt{\pi}G^2 f_{dm}\rho_{\text{tot}}}{\sigma e^{\mu+\frac{\sigma^2}{2}} v_{\text{disp}}} \int_0^\infty e^{-\frac{(\ln(m_a)-\mu)^2}{2\sigma^2}} \ln \left( \frac{r_h v_{\text{disp}}}{G(m+m_a)} \right) m_a dm_a, \] (21)

which describes the diffusion coefficient when a range of MACHO masses is present. The equation for the potential energy per unit mass is:

\[ \frac{U}{M} = C - \frac{\alpha GM_s}{r_h} + \beta G \rho r_h^2. \] (22)

So if we combine equation (19) with (20) and use the virial theorem \((E_{\text{tot}} = -\frac{1}{2}U)\) we get the following formula for the change in half-light radius over time:

\[ \frac{dr_h}{dt} = \frac{4\sqrt{\pi}G f_{dm}}{\sigma e^{\mu+\frac{\sigma^2}{2}} v_{\text{disp}}} I \left( \frac{M_s}{\alpha r_h^2 \rho} + 2\beta r_h \right)^{-1}, \] (23)

with \(I_1:\)

\[ I = \int_0^\infty e^{-\frac{(\ln(m_a)-\mu)^2}{2\sigma^2}} \ln \left( \frac{r_h v_{\text{disp}}}{G(m+m_a)} \right) m_a dm_a. \] (24)
If we simulate this equation for multiple values of sigma we get the following figures:

Figure 9: The change in the half light radius due to dynamical heating when a range of dark matter MACHO masses is present. The average MACHO mass is $30 \, M_\odot$ and the total Stellar mass is $6000 \, M_\odot$.

It becomes visible that the change in half-light radius increases as the standard deviation gets higher. If we use this in the same way as in the previous section to put constraints on the MACHO masses we get the following results:

Figure 10: mass constraints from galactical dynamics. I used that the average MACHO mass was $30 \, M_\odot$, the velocity dispersion was $10 \, \text{km/s}$, the dark matter density was $1 \, M_\odot/\text{pc}^3$ and the total system had a stellar mass of $2000 \, M_\odot$.

The figure shows that the mass constraints on the MACHOs becomes stronger if a range of masses is present.
3.2.3 Mass segregation

In the section about MACHOs, we assumed that most of the dark matter MACHO mass is in the form of primordial black holes, these PBHs have been around since the formation of the galaxies and star clusters. If this is the case than the exchange of kinetic energy will have led to mass segregation: A process where the kinetic energies of two objects tend to equalize during an encounter. This equalization will cause the relatively heavy object (MACHOs) to lose velocity and the relatively light ones (The stars) to gain in velocity. Kepler’s second law tells us that this will cause the MACHOs to move to the center of the galaxy while the stars will move away from the center.

In (Koushiappas 2017) it is calculated that this will lead to a mean change of kinetic energy of the stars: \[ \frac{dE_s}{dt} = \sqrt{\frac{96\pi G^2 m_s \rho_BH \ln \Lambda}{\left(\langle v_s^2 \rangle + \langle v_{BH}^2 \rangle \right)^3}} \left[ m_{BH}\langle v_{BH}^2 \rangle - m_s\langle v_s^2 \rangle \right]. \] (25)

With \( \langle v_s \rangle \) and \( \langle v_{BH} \rangle \) being the average velocities of the stars and Black holes.

If \( m_{BH}\langle v_{BH}^2 \rangle = m_s\langle v_s^2 \rangle \), the kinetic energies have equalized and the system has reached an equilibrium.

For the simulations to these mass segregation we use the known data from the Segue 1 dwarf galaxy. Segue 1 has a half-light radius of 29 pc. Within its half-light radius, a total mass of \( 2.6 \times 10^5 M_\odot \) is located. To calculate how the radial shells of Segue 1 would have evolved, we use equation (7):

\[ \frac{dr}{dt} = 4\sqrt{2\pi G \frac{f_{dm}}{m_a}} \ln(\Lambda) \left( \frac{M_s}{\rho r^2} + 2\beta r \right)^{-1}. \] (26)

If we simulate this (in python) we get that the radial shells have evolved as follows:

Figure 11: Change of the radius of radial shells due to mass segregation. Left shows the effects when 1 percent of the total dark matter is in the form of 30 \( M_\odot \) MACHOs and the right figure shows the effect when 10 percent of the dark matter is in the form of 30 \( M_\odot \) MACHOs.
The Figure shows that all the shells increase in radius with the change decreasing as the radius increases and that in the center of the galaxy a lack of stars becomes visible. To calculate the stellar number density as a function of radius, we assume that the stars were initially distributed according to Plummer's model:

\[
\rho(r) = \left(\frac{3M}{4\pi r_s^3}\right) \left(1 + \frac{r^2}{r_s^2}\right)^{-\frac{5}{2}}.
\]  

(27)

Were M stands for the total stellar mass in the star cluster and \( r_s \) for the scale radius, which depends on the size of the core. For Segue 1, this scale radius is 16 parsec [5]. If we use this distribution and use python to simulate how this distribution changes when mass segregation takes place, we get the following figure for the stellar distribution:

Figure 12: Number density of stars as a function of radius. The left figure shows the distribution when 10 percent of the dark matter is in 30 \( M_\odot \) MACHOs and the right figure shows the distribution when 1 percent of the dark matter is in 30 \( M_\odot \) MACHOs. The blue line shows the initial distribution.

From earth it isn’t possible to measure this distribution of stars, it is however possible the measure the projected stellar surface density from the measured light coming from Segue 1. So if we apply a line of sight integral over the data from figure 12 and do this for multiple fractions of MACHO dark matter we get the following figure:
Figure 13: figure from (Koushiappas 2017) showing the projected stellar surface densities. The black line represents the distribution where there is no dark matter in the form of MACHOs and the vertical black lines represent the data measured from observation [30].

If we make a 3d plot of these line of sight distribution we will get the following figure:

Figure 14: These figures from (Koushiappas 2017) show the simulated effects of MACHOs dark matter in a galaxy: The left figure shows the stellar distribution when there is no dark matter in the form of MACHOs, the one in the middle shows the effects when 1 percent of the dark matter is in $10 \, M_{\odot}$ MACHOs and the right figure shows the effects when 10 percent of the total dark matter is in the form of $30 \, M_{\odot}$ MACHOs. It becomes visible that, when a fraction of the dark matter was in the form of MACHOs, mass segregation resulted in a depletion of stars in the centre.

To put constraints on the MACHO masses and the fraction that it can form of the total dark matter ($f_{dm}$), we compare the simulated mass segregation effects with the observed data (shown in figure 13). We do this by applying the $\chi^2$ test:

$$\chi^2 = \sum \frac{(o - e)^2}{o},$$

(28)
where \( o \) stands for the observed values and \( e \) for the expected values. For each value of \( f_{dm} \) and \( m_{BH} \) we calculate how the line of sight integral looks and compare the 3 measured data points from figure 13 (\( o \) in equation 28) with the simulated points at the same radii (\( e \) in equation 28). To get the probability that this null hypothesis (a specific \( f_{dm} \) and \( m_{BH} \)) is true, we use the following graph:

![Graph showing the relation between \( \chi^2 \) value and the p-value.](image)

Figure 15: Line which shows the relation between \( \chi^2 \) value and the p-value: The probability of obtaining the data, or if the null hypothesis is true

If we use this graph, equation 28 and the observed and simulated data we get the following constraints on MACHO dark matter:

![Figure showing constraints on MACHO dark matter masses.](image)

Figure 16: figure from (Koushiappas 2017) showing the constraints on MACHO dark matter masses from measuring the star distribution. The black lines represent the p-values written above them. These measurements have excluded MACHO masses above 1 solar mass as the only component of dark matter.
4 Conclusion and outlook

During this project I tried to find mass constraints on dark matter MACHOs. By looking at the microlensing effects and at the dynamical impact on the evolution of a galaxy, we got the figures (6),(8),(10) and (16) which give constraints on the possible MACHO masses. These figures show that all MACHOs with masses from 0.15 \( M_\odot \) up to very high mass are excluded to form all of the dark matter in our universe and MACHOs with a few tens of solar masses are even excluded to form more than 4 percent of the total dark matter. Figure (10) shows that the constraints strengthen when we assume that the MACHOs have a whole range of masses instead of one mass.

There is certainly a lot of room for improvements on these constraints. For the simulations, a lot of assumptions had to be made, if more precise data were to be available, the constraints could strengthen. Another way to strengthen the constraints is by looking at other effects that the MACHOs would have on the universe. For example, I excluded the researches on the effects of MACHOs on the cosmic microwave background, which sets constraints on the low end of the mass spectrum [31]. So maybe in the future, when more data is available, MACHOs can be completely excluded as the main components of dark matter.

5 Acknowledgement

I would like to thank my supervisor Christoph Weniger for guiding me through this project. Despite his busy schedule, he made time to answer all of my questions. This project gave me an insight in the work he is doing and in the challenges he is facing.

6 References


7 Appendix

7.1 The mean value of The log-normal distribution.

We want to calculate the mean value of a random log-normally distributed function: \( Y = \exp(x) \) where \( x \) is a normally distributed variable.

For a normally distributed variable the probability density function is:

\[
PDF(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}. \tag{29}
\]

So to compute the mean value of \( Y \) we need to compute the following integral:

\[
< e^x > = \int_{-\infty}^{\infty} e^x \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx. \tag{30}
\]

In order to do this we need to carry out variable substitution, we take \( y = x - \mu \) with \( dy = dx \). The integral now still has the same limits so it becomes:

\[
< e^x > = \int_{-\infty}^{\infty} e^{y+\mu} \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{y^2}{2\sigma^2}} dy, \tag{31}
\]

\( \mu \) doesn’t depend on \( y \), so it can be taken outside of the integral:

\[
< e^x > = e^\mu \int_{-\infty}^{\infty} \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{y^2}{2\sigma^2}} dy. \tag{32}
\]

Now we can rewrite the exponent to get:

\[
< e^x > = e^\mu \int_{-\infty}^{\infty} \frac{1}{\sigma \sqrt{2\pi}} e^{\frac{y^2}{2\sigma^2}} e^{-\frac{(y-\sigma^2)^2}{2\sigma^2}} dy, \tag{33}
\]

\[
< e^x > = e^{\mu + \frac{\sigma^2}{2}} \int_{-\infty}^{\infty} \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(y-\sigma^2)^2}{2\sigma^2}} dy. \tag{34}
\]

The \( \sigma \) in this function doesn’t depend on \( y \) and it can have any value so if we replace \( \sigma^2 \) with \( \mu \) we end up with the probability density function inside the integral:

\[
< e^x > = e^{\mu + \frac{\sigma^2}{2}} \int_{-\infty}^{\infty} PDF(y) dy. \tag{35}
\]

And since this PDF is normalized, the integral from \(-\infty \) to \( \infty \) will be 1 and we end up with:

\[
< e^x > = e^{\mu + \frac{\sigma^2}{2}}. \tag{36}
\]
7.2 Populair dutch summary

Ongeveer 75 jaar geleden begon de Zwitserse astronoom Fritz Zwicky zijn onderzoek naar de spreiding van de snelheden van sterrenstelsels in een cluster van sterrenstelsels. Hij vond dat deze snelheden veel hoger waren als verwacht. Ze waren zelfs zo hoog dat als de massa van de cluster alleen uit sterren bestond dat de zwaartekracht nooit sterk genoeg zou kunnen zijn om de sterrenstelsels bij elkaar te houden. De verklaring hiervoor vond hij in donkere materie: materie die je niet ziet maar die wel zwaartekracht uitoefent op zijn omgeving. Sindsdien is hier heel veel onderzoek naar gedaan en zijn de meeste onderzoekers het er wel over eens dat er iets is met de eigenschappen van donkere materie. Een vraag die echter nog onbeantwoord is, is de vraag waar deze donkere materie uit bestaat. De verklaringen hiervoor lopen uiteen van superlichte neutrino deeltjes tot superzware zwarte gaten. Een mogelijke verklaring voor de ontbrekende donkere materie bestandsdelen zijn de MACHOs: Massive Compact Halo Objects. MACHOs zijn een verzameling van astronominische objecten die geen licht uitzenden, je kunt hierbij denken aan witte- en bruine dwergen, neutronensternen en zwarte gaten. Tijdens dit project nam ik aan dat donkere materie uit MACHOs bestaat en heb ik geprobeerd hier massa restricties voor te vinden. Een van de manieren waarop dat gedaan is, is door naar de microlensing effecten van MACHOs te kijken. Sinds Einstiens relativiteits theorie weten we namelijk dat ook massaloze lichtdeeltjes beïnvloed worden door zwaartekracht. Dit zorgt ervoor dat, wanneer een MACHO voor een ster langs beweegt, het licht van de ster naar de waarnemers op aarde wordt toegebogen. Dit kunnen we op aarde meten als een toenemen van de lichtkracht van de ster. Er zijn echter nul van deze microlensing effecten gemeten, met behulp van een statistische test kunnen hiermee MACHOs tussen 0.15 en 30 zonsmassa uitgesloten worden als het enige bestandsdeel van donkere materie. Een andere manier om massa restricties te vinden is door naar de impact van MACHOs op de evolutie van sterrenstelsels te kijken. Door voor meerdere MACHO massa's te simuleren wat de impact zou zijn en dit te vergelijken met de waargenomen data van sterrenstelsels, kunnen MACHOs met massa's tussen 1 en honderden zonmassa's uitgesloten worden als de enige bestandsdeel van donkere materie. MACHOs van enkele tientallen zonmassa's kunnen zelfs uitgesloten worden om meer dan 4 % van de totale donkere materie in ons heelal te maken. Met meer onderzoek zouden deze restricties in de toekomst nog versterkt kunnen worden waardoor MACHOs in de toekomst misschien helemaal uitgesloten kunnen worden als het enige bestandsdeel van donkere materie.